Middle School Mathematics
A Guide to the Connected Mathematics™ Series

Covering and Surrounding

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School math department
2004 Issue 1

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1 Introduction

This guide supports the Connected Mathematics™ student textbook *Covering and Surrounding*. This book is in the Geometry strand. Its primary topic is calculating the area and perimeter of two-dimensional shapes: squares, rectangles, parallelograms, triangles, circles and irregular polygons.

2 Goals/Objectives

This unit will help students:

- Develop strategies and formulas for finding the area and perimeter of rectangular shapes and nonrectangular shapes.
- Discover relationships between perimeter and area
- Understand how the area of a parallelogram is related to the area of a triangle.
- Determine whether the area or perimeter is required to solve real world problems, and then solving the problem.
- Observe and reason from patterns in data by organizing tables to represent the data.
- Use multiple representations (e.g., physical, pictorial, tabular & symbolic models) and verbal descriptions of data.

3 Vocabulary

The following words and concepts are used in this unit. The concepts in the left column are those essential for student understanding in this and future units. The Descriptive Glossary in the student text gives definitions for many of these words.

Essential Terms	Non Essential Terms
area	base
center (of a circle)	height
circumference	length
diameter	width
perimeter	pentomino
radius (radii)	perpendicular
pi	trapezoid

™ Connected Mathematics was developed at Michigan State University with financial support from the Michigan State University office of the Provost, Computing and Technology, and the College of Natural Science.

4 Summary of Investigations

4.1 Investigation 1 – Measure Perimeter & Area

Students build a good understanding of the difference between perimeter and area. They use square tiles to create designs and to cover pictures of designs to find areas and perimeter. They transform designs to fit a prescribed perimeter or area. Perimeter is the distance around the outside of the shape. The area is the number of 'tiles' that are needed to cover the entire shape.

4.2 Investigation 2 – Measure Odd Shapes

Students learn to measure things with curved or irregular edges that can't be laid along a rule. They use string to help find an approximate. They trace their feet on grid paper and estimate the perimeter and the area of the tracings. The perimeter is measured using string; the area by counting the number of square centimeters enclosed by the outline of the foot. Students must come to understand that they must count the whole squares as well as the partial squares which fall around the edges. They will have to estimate to find the area of the partial squares.

4.3 Investigation 3 – Constant Area, Changing Perimeter

Students are to find the largest and smallest perimeter for a given area. Students construct tables to help them to see the patterns in the data. They will discover that there are many different shapes of a given area – each with a different perimeter. For each shape they create, they must calculate the area (the number of squares that cover the face of the surface) and the perimeter (the number of units or distance around the outside of the shape). Students learn that area is measured in square units (e.g., sq cm, sq in, sq ft, sq mi, sq unit) and perimeter is measured in straight line units (e.g., cm, in, ft, mi, unit)

4.4 Investigation 4 – Constant Perimeter, Changing Area

Students will find the largest and smallest area for a shape of a given perimeter. Students will construct tables to help them see the patterns in the data. They will discover that area and perimeter are not uniquely matched. Knowing the area of a shape does not imply that you will know its perimeter, and vice versa.

4.5 Investigation 5 – Measuring Parallelograms

Students cut and rearrange parallelograms to make rectangles, and use this information to develop strategies for finding the area of a parallelogram. They will discover that the area of every parallelogram can be found by multiplying its base by its height. This is true for all parallelograms, as each could be cut to form a rectangle. Students are familiar with finding the area of the rectangle by multiplying its base by its height. The perimeter of a parallelogram is found by

adding the lengths of its four sides. The height of a parallelogram is not used to find its perimeter, except in the case of a rectangle where its height is also its side length.

4.6 Investigation 6 – Measuring Triangles

Students find areas and perimeters of triangles by using grids, arranging triangles to form parallelograms and measuring with rules. Special triangles such as isosceles and 30-60-90 triangles are explored. They discover that every parallelogram can be cut into two identical triangles. Therefore the area of the triangle is one-half of the area of the parallelogram. To find the area of a triangle you would multiply its base by its height and then multiply by one-half. Students could also multiply its base by its height and then divide by 2. The perimeter of a triangle is calculated by finding the sum of the lengths of its sides. The height of a triangle is not used to find its perimeter, unless it is also the length of one of its sides (i.e., in a right triangle).

4.7 Investigation 7 – Measuring Circles

Students learn that the circumference of a circle is the length of its outline (as a perimeter is for a rectangle) – and is slightly more than three times its diameter; and that the circle's area is slightly more than three times the area of a square whose edges are equal to the circle's radius. These discoveries lead students to the idea of the value of pi (which we estimate in Middle School as 3.14). The circumference of a circle is found by multiplying the diameter of the circle by pi (or 3.14).

Area of a Circle = radius^2 x pi = radius x radius x pi
Circumference of a Circle = diameter x pi

5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Questions, page 13:

1. area = 12 square inches; perimeter = 20 inches
6. area = 6 square units; perimeter = 14 units
7. area = 20 square units; perimeter = 22 units

5.2 Investigation 2

ACE questions, page 21:

1a. The area is between 102 and 118 sq cm
1b. The perimeter is between 77 and 87 cm
1c. The area would be used to determine how much material would be needed to make a glove; the perimeter would be used to calculate how much thread would be needed.
2. Ghost Lake
3. Loon Lake
4. Ghost Lake
5. Ghost Lake
18. Caspian Sea (143,244 sq mi); area
19. Russia (6,592,800 sq mi); area

5.3 Investigation 3

ACE Questions, page 32:
6a. 1 x 60; 2 x 30; 3 x 20; 4 x 15; 5 x 12; 6 x 10
6b. 1 x 61
6c. 1 x 62; 2 x 31
6d. The factors are the same as the side lengths of the rectangles.

5.4 Investigation 4

ACE Questions, page 38.

2. a 5 x 5 rectangle.
3. a square would have the largest area, so you would need to cut the molding into 4 equal length pieces, each 18 cm long

5.5 Investigation 5

ACE Questions, page 52.

9. area = 6 sq cm; perimeter = 10.48 cm
10. area = 6 sq cm; perimeter = 16.48 cm
11. area = 16 sq cm; perimeter = 12.24 cm

5.6 Investigation 6

ACE Questions, page 60.

7a. Each triangle has an area of 10 sq units.
7b. All triangles have the same base, height and area
7c. They are called a family because they have the same base, height and area.

5.7 Investigation 7

ACE Questions, page 76.

1. circumference = about 28.3 in; perimeter = about 63.6 in
2. 3. circumference = about 7.1 in
10. The circumference of a bicycle wheel tells you how far the bike travels in one revolution.