Middle School Mathematics
A Guide to the Connected Mathematics™ Series

Filling and Wrapping

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2004 Issue 1

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1 Introduction

Students developed strategies for measuring the perimeter and area of polygons and irregular figures in a previous unit (*Covering and Surrounding)*. In this unit, ***Filling and Wrapping***, part of the **geometry** strand, students use their understanding of perimeter and area in two dimensions to help them explore and develop the concepts of surface area and volume in three dimensions. Students are introduced to volume as a measure of filling and to surface area as a measure of wrapping.

Students explore the surface areas and volumes of rectangular prisms and cylinders in depth. They look informally at how changing the scale of a box affects its surface area and volume. They also informally investigate other solids – including cones, spheres, and irregular shapes - to develop volume relationships.

2 Goals/Objectives

This unit will help students:

- Conceptualize volume as a measure of filling an object
- Develop the concept of volumes of prisms and cylinders as stacking layers of unit cubes to fill the object
- Conceptualize surface area as a measure of wrapping an object
- Determine that the total number of blocks in a prism is equal to the area of the base multiplied by the height (the volume)
- Discover that strategies for finding the volume of a rectangular prism will work for any prism
- Explore the relationship of the surface areas of rectangular prisms and cylinders to the total area of a flat pattern needed to wrap the solid
- Discover the relationship among the volumes of cylinders, cones, and spheres
- Apply the strategies for finding the volumes of rectangular prisms and cylinders to designing boxes with given specifications
- Reason about problems involving the surface areas and volumes of rectangular prisms, cylinders, cones, and spheres
- Determine which rectangular prism has the least (greatest) surface area for a fixed volume
- Investigate the effects of varying dimensions of rectangular prisms and cylinders on volume and surface area and vice versa
- Estimate the volume of an irregular shape by measuring the amount of water displaced by the solid
- Understand the relationship between a cubic centimeter and a milliliter

3 Vocabulary

The following words and concepts are used in this unit. The concepts in the left column are those essential for student understanding in this and future units. The Descriptive Glossary (page 77) in the student text gives definitions for many of these words.

Essential Terms	Terms developed in previous units	Non Essential Terms
Base	Area	Right prism
Cone	Circumference	Oblique prism
Cube	Congruent	
Cylinder	Dimensions	
Edge	Height	
Face	Length	
Flat pattern	Perimeter	
Prism	Radius	
Rectangular prism	Width	
Sphere		
Surface area		
Unit cube		
Volume		

4 Summary of Investigations

4.1 Investigation 1 – Building Boxes (pp 5-14)

Students are introduced to the concepts of volume and surface area through the ideas of wrapping and filling, building on their knowledge of area and perimeter of two-dimensional figures. Rectangular prisms are described by their dimensions: length, width, and height. Students design flat patterns for cubic and rectangular boxes, cut them out and fold them into boxes. They find the area of flat patterns and discover the association between this area and the surface area of the related box. They are introduced to the concept of volume by determining how many unit cubes it would take to fill particular boxes.

4.2 Investigation 2 – Designing Packages (pp 15-23)

Volume is defined as the number of unit cubes it takes to fill a rectangular box; surface area is defined as the amount of wrapping it takes to enclose a box. The focus in this investigation is on the surface areas of rectangular prisms. Students examine the amount of packaging material needed to enclose various arrangements of 24 cubic blocks; they find that a 2 by 3 by 4 arrangement has the least surface area, then generalize their findings to any number of blocks. In the process of solving these problems, they develop strategies for finding the surface area of a rectangular box.

4.3 Investigation 3 – Finding Volumes of Boxes (pp 24-36)

Students seek more efficient ways to determine the number of cubes a right prism would hold. They discover that the volume of the box is the number of blocks in the bottom layer multiplied by the number of layers - the area of the base times the height of the prism. (This holds for all prisms.) Students apply their strategy to find the volume of a waste site and how long it will take the residents of a certain city to fill the site. Then they develop a general strategy for finding the volume of any rectangular prism.

4.4 Investigation 4 – Cylinders (pp 37-45)

Students find the volume and surface area of a cylinder by following the same procedure they used for prisms. First, students estimate the volume of a cylinder by determining how many unit cubes would fill the cylinder. The volume is the area of the base multiplied by its height. The concept of the surface area of a cylinder is developed by having students cut out a flat pattern, think about what the dimensions and surface area of the cylinder made from the pattern will be, and then form the cylinder and determine its volume by finding how many unit cubes would fill it. Then students design a rectangular box with the same volume as the given cylinder, and they discover that the surface area of he box is greater than the surface area of the cylinder.

4.5 Investigation 5 – Cones and Spheres (pp 46-56)

Students compare the volumes of a cone, a sphere, and a cylinder of equal radius and height. They construct a transparent plastic cylinder and a clay sphere with the same radius and height. They compress the sphere until it fills the bottom of the cylinder, then compute and compare the volumes of the two shapes. Next, they construct a cone with the same height and radius as the cylinder. They compare the volumes of the two shapes by finding how many cones full of rice or sand it takes to fill the cylinder. In these hands-on activities, students determine how many times the volume of the cone or sphere will fill the cylinder and then look for the relationships among the three volumes. The investigation ends in a neat problem in which the students compare the volumes of cones, cylinders, and sphere.

4.6 Investigation 6 – Scaling Boxes (pp 57-67)

Students study the effects of changing the dimensions of the volume of a rectangular prism in the context of designing compost containers. They explore two central ideas: How do you build a rectangular container with twice the volume of a given box? What effect does doubling each dimension of a rectangular container have on its volume and surface area? After investigating these questions, students apply their knowledge of similarity and scale factors to rectangular boxes.

4.7 Investigation 7 – Finding Volume of Irregular Objects (pp 68-72)

Students explore how to find the volume of an irregular shape by measuring the amount of liquid it displaces when placed in a container of water. In the process they look at the relationship between milliliters and cubic centimeters.

5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Question 11, page 12.

ANSWER
11i. 14 units
ii. 14 units
iii. 14 units
iv. 12 units
v. 14 units

5.2 Investigation 2

ACE question 1, page 19.

ANSWER
1a. l=5 in, w = 3 in, h= 1 in
1b. 46 in^2
1c. 15 blocks

5.3 Investigation 3

ACE Question 5, pages 30

ANSWER
5. Volume = 32 cubic inches (in^3)
Surface area = 64 square inches (in^3)

5.4 Investigation 4

ACE Question 10 page 43

ANSWER
10. The 12-oz drink is about $0.1042 per ounce, the 18-oz drink is about $0.0972 per ounce, and the 32-oz drink is about $0.0938 per ounce. The 32 oz-drink is the best buy because it costs the least per ounce.

5.5 Investigation 5

ACE Question 1, page 51.

ANSWER
1b. About 50,265 ft^3
1c. 1/3 x 50265 = about 16,755 ft^3
1d. 2/3 x 50265 = about 33,510 ft^3

5.6 Investigation 6

ACE Question 3, page 61.

ANSWER
3a. 8 ft^3
3b. 28 ft^3

5.7 Investigation 7

ACE Question 3, page 70.

ANSWER
3. 100 ft^2