Middle School Mathematics
A Guide to the Connected Mathematics™ Series

Frogs, Fleas and Painted Cubes

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School math department
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1 Introduction

<u>Frogs, Fleas and Painted Cubes</u> is part of the **algebra** strand. This unit develops students' understanding of the most important nonlinear polynomial relationship: the second-degree polynomial, or the quadratic function.
By investigating a variety of problem situations that display quadratic patterns of change and making comparisons with linear and exponential patterns, students develop a knowledge of the characteristics of tables, graphs, and equations of quadratic relationships. Each of these three types of representations reveals important information.

Questions related to graphs of quadratic relationships emphasize their parabolic shape; the location and interpretation of intercepts and lines of symmetry; and the presence and location of max and min points.

Questions also focus on connecting the symbolic form to tabular and graphics forms. The problems in this unit lead naturally to both factored form and expanded form of quadratic equations. Students will discover the equivalency of the two forms and learn that they are convenient for gathering different types of information. While there is some simple algebraic manipulation of quadratic expressions, students are not expected to master symbolic-manipulation procedures in this unit.

2 Goals/Objectives

This unit will help students:

- Make connections among coordinates, slope, distance, and area
- Develop an awareness of quadratic relationships and how they can be recognized from patterns in tables, graphs, and equations
- Describe patterns in tables of quadratic functions and predict subsequent entries
- Recognize the characteristic shape of the graph of a quadratic function and identify its line of symmetry, vertex, and intercepts
- Detect quadratic relationships from the pattern of differences in tables
- Match quadratic equations to patterns in tables and graphs
- Find the maximum or minimum values of quadratic functions from tables and graphs
- Develop an understanding of equivalent expressions, that is, of two expressions that model the same relationship
- Recognize a quadratic function from an equation written as a product of two linear factors or in expanded form as $y = ax^2 + bx + c$
- Recognize that the same equation can model more than one situation
- Predict from tables, graphs, and equations, whether quadratic functions have maximum or minimum values

- Interpret maximum and minimum points and intercepts in projectile-motion problems
- Develop a deeper sense of the properties that characterize quadratic relationships by comparing quadratic relationships to linear and cubic relationships

3 Vocabulary

The following words and concepts are used in this unit. The concepts in the left column are those essential for student understanding in this and future units. The Descriptive Glossary (page 88) in the student text gives definitions for many of these words.

Essential Terms	Terms developed in previous units	Nonessential terms
constant term	Equation	Cubic relationships
expanded form	Exponential relationship	First differences
factored form	Linear relationship	Second differences
function		Third differences
like terms		
line of symmetry		
linear term		
maximum value		
minimum value		
parabola		
quadratic expression		
quadratic term		
term		
triangular number		

4 Summary of Investigations

4.1 Investigation 1 – Introduction to Quadratic Relationships (pp 5-18)

- Study important characteristics of quadratic relationships through an exploration of the area of rectangles with a fixed perimeter. They find that within such a family of rectangles, a square has the greatest, or maximum area.
- Focus on the parabolic shape of graphs of quadratic functions and how it is reflected in tables of data.
- Create and use tables and graphs, search for patterns, and write an equation expressing the relationship between area and side length of rectangles with a given perimeter.

4.2 Investigation 2 – Quadratic Expressions (pp 19-40)

- Transform a square into a rectangle by increasing or decreasing the length of the sides of the square leads to two equivalent expressions for the area of the rectangle. For example, if one dimension of a square with side length of x is increased by 2 and the other dimension is increased by 3, the area of the rectangle is $(x + 2)(x + 3)$, or $x^2 + 5x + 6$. This demonstrates an important characteristic of quadratic functions: they can be represented as the product of two linear expressions, called factored form, or as the sum or difference of one or more terms, called expanded form. The equivalence of these forms can be conceptualized by thinking of the area of a rectangle in two ways: as the product of the dimensions of the rectangle and as the sum of the areas of the subparts of the rectangle.

4.3 Investigation 3 – Quadratic Patterns of Change (pp 41-51)

- Explore the classic handshake problem and variations to further explore quadratic functions.
- Discover that a single model can describe more than one context.
- Examine rate of change between the two variables of a quadratic function.

4.4 Investigation 4 – What is a Quadratic Function? (pp 52-70)

- Extend understanding of quadratic equations and graphs with the classic projectile-motion problems.
- Explore the pattern of change over time in the height of a ball thrown into the air.
- Investigate quadratic equations describing the jumps of a frog, a flea, and a basketball player.
- Study the patterns of change in quadratic equations and discover that the differences between consecutive y values, called first differences, are constant for linear relationships and that second differences are constant for quadratic relationships.

4.5 Investigation 5 – Painted Cubes (pp 71-84)

- Compare quadratic relationships with cubic and linear relationships and discover that in cubic relationships, third differences are constant.

5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Question 5 page 14

ANSWER

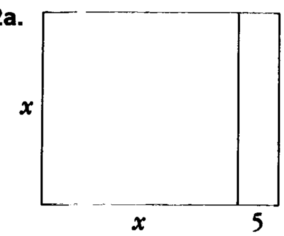
5a. The graph is a parabola; first it increases and then it decreases. It is symmetric about the vertical line through x = 7.5. It crosses the x-axis at the points 90,0) and (15,0).
5b. The max area is about 56 square cm.
5c. The area gets very close to zero.
5d. From the graph, the area is about 35 square cm.
5e. Use one set of dimensions to calculate the perimeter. For example, the square with a side length of 7.5 cm has a perimeter of 7.5 x 4 = 30 cm. Or, look at the point where the graph touches the x-axis on the right and double that number: 15 x 2 = 30 cm.

5.2 Investigation 2

ACE question 2 page 31.

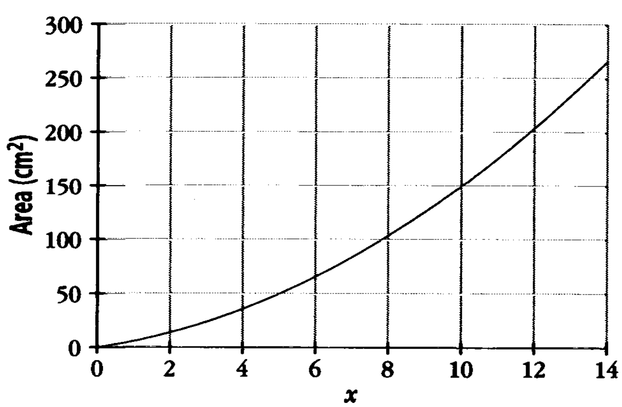
ANSWER

2a.



2b. factored form: x(x+5); expanded form: $x^2 + 5x$.

2c. $A = x(x + 5)$



5.3 Investigation 3

ACE Question 1 page 45

ANSWER

1a. The 8 people on each side shake hands with 8 others, so $8^2 = 64$ handshakes are exchanged.

1b. Each of the 8 people high-fives with other 7, but this counts as twice, 8(7)/2=28 high fives will be exchanged.

5.4 Investigation 4

ACE Question 1 page 60

ANSWER

1a. The ball is released at about 6.5 ft (the y-intercept).
1b. The ball reaches its max height, about 17.5 ft, at about 0.8 sec.
1c. The ball would reach the basket just after 1.5 sec.

5.5 Investigation 5

ACE Question 1 page 75.

ANSWER

1a. the 8 corners, or 8 cubes
1b. The cubes along the 12 edges that are not corner cubes, or 12 x 10 = 120 cubes.
1c. The large cube has 6 faces, and each face has 10 x 10 = 100 cubes with one face painted, a total of 6 x 100 = 600 cubes.
1d. Removing the external cubes leaves 10 x 10 x 10 = 1000 unpainted cubes.