Middle School Mathematics
A Guide to the Connected Mathematics™ Series

How Likely Is It?

Prepared by members of the Readington Middle School math department
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1 Introduction

This guide supports the Connected Mathematics™ student textbook *How Likely Is It?* This book is in the Data Analysis and Probability strand. Its primary topic is determining the probability that an uncertain event may occur.

2 Goals/Objectives

This unit will help students:

- Understand that probabilities are useful for predicting what will happen over the long run
- Understand the two ways to obtain a probability: by gathering data from experiments (experimental probability) and by analyzing the possible equally likely outcomes mathematically (theoretical probability).
- Understand the relationship between experimental and theoretical probability: The experimental probabilities and the theoretical probability that a certain event will happen will become equal as the number of trials increases.
- Determine and critically interpret statements of probability
- Develop strategies for finding the experimental and the theoretical probability of an event.
- Organize data into lists or charts of equally likely outcomes as a strategy for finding theoretical probabilities.
- Use graphs and tallies to find experimental data.

3 Vocabulary

The following words and concepts are used in this unit. The concepts in the left column are those essential for student understanding in this and future units. The Descriptive Glossary in the student text gives definitions for many of these words.

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Essential Terms	Non Essential Terms
certain event	fair (fair coin; fair game)
chances	favorable outcome
equally likely event	possible
event	probably
experimental probability	random events
impossible event	trial
outcome	
probability	
theoretical probability	

4 Summary of Investigations

4.1 Investigation 1 – A First Look at Chance

Students flip a coin 30 times and then compute the experimental probability of a head occurring on a toss of a coin. They use their own data and class data and discover that as the number of trials increases, the probability of an outcome becomes more evident. For a coin, the chance of tossing a head is ½. In this experiment, students should most likely have approximately 15 heads in 30 tosses. A class of data may show that in the 500 tosses, the number of head tosses is 250.

4.2 Investigation 2 – More Experiments with Chance

Students experiment with marshmallows, finding the experimental probability that each of two sizes of marshmallow will land on an end or a side. Individual student data is combined with class data to improve the reliability of the results.

Problem 2.1 will have a variety of answers, depending on student experimental results. Students should be able to explain how they conducted the experiment and why they think they got the results they did.

In Problem 2.2 we see that it is possible for either player to win, however, Tat Ming is more likely to win because the chances of two coins matching are greater than the chances of three coins matching. The probability that two coins will match is ¾. We write, P(2 coins match) = ¾. The probability that three coins will match is ¼. We write, P(3 coins match) = ¼. This is not a fair game because one player has a better chance of winning than the other.

4.3 Investigation 3 – Using Spinners to Predict Chances

The pattern of collecting data, analyzing data and making predictions continues in this investigation with an emphasis on experimental data. Students also measure the area of each section of the spinner and use this information to calculate the theoretical probability of landing on a given section. Students combine their results to compare the class data for experimental and theoretical probability of a given event.

Students use an angle ruler to measure the angle of each section of the pie chart. They use this measurement as it compares to the whole circle (360 degrees) to find the probability of the spinner landing on a given section.

For example, if the angle of a section of the pie chart measures 180 degrees, we would know that there is a 180/360 chance of the spinner landing on that section. In lowest terms, we say that there is a ½ chance that the spinner will land on the section.

If the section is 120 degrees, we say there is a 120/360 or 1/3 probability that the spinner will land in that section.

This calculation provides the theoretical probability for the spinner. Students can perform and experiment (spinning the spinner and recording their results) to calculate the experimental probability. With a sufficient number of spins, the experimental probability will approximate or equal the theoretical probability.

4.4 Investigation 4 – Theoretical Probabilities

Probability is formally introduced in a game-show setting in which players need to determine the probability of drawing a certain color of block from a container in which the total number of each color and the total number of blocks is unknown. A working definition of *probability* is developed as the likelihood that an event will happen. Students come to understand probability as a number between 0 and 1 (inclusive); and that a probability of 0 or 1 has a particular significance. An event with a probability of 0 will never happen (i.e., an impossible event). An event with a probability of 1 will always happen (i.e., a certain event).

If the probability of selecting a green block from a bag is ½, this is written using the notation P(green) = ½.

4.5 Investigation 5 – Analyzing Games of Chance

Students play and analyze a game that involves possible outcomes of a roll of dice. They identify and analyze the possible outcomes of the roll and use this information to develop a winning strategy to the game.

4.6 Investigation 6 – More About Games of Chance

Students design a simulation to find an experimental probability of winning a prize in a promotional contest involving scratch-off game cards. They also find the theoretical probability of winning by making an organized list of the possible pairs of spots that could be scratched off. The ACE questions continue the theme of promotions and are designed to help students realize that there are many loopholes in advertising claims and contests.

4.7 Investigation 7 – Probability and Genetics

This investigation introduces biology as a source of application for probability. Students determine how many students in their class have particular traits and use this data to make predictions about the likelihood of a person having such a trait.

5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Questions, page 17:

1a. 28/50 = 14/25
1b. about 250
There are about 6 years x 365 days = 2190 days. Students may add a day or two for leap years, making the answer 2191 or 2192.
11. no
12. yes
13. no

5.2 Investigation 2

ACE Questions, page 17:

1. no. Answers will vary based on class data.

3. It is more likely to land on its side because it is better to base a conclusion on a larger number of trials. 5 trials is insufficient to predict performance.

5.3 Investigation 3

ACE Questions, page 24:

1a. about ½ for hearts; 1/3 for dots; 1/6 for stripes

1b. Possible answer: divide the large heart section in half, forming 6 equal sections. The hearts occupy 3/6, the dots 2/6, and the stripes 1/6.
1c. The answers will be close if we do many trials.
1d. Answers should be closer to the actual fraction because I'd be using more data to determine them.

5.4 Investigation 4

ACE Questions, page 35.

2a. 3/3=1
2b. 0/3 = 0
2c. 0/3 = 0

3a. P(green) = 12/25
3b. P(purple) = 6/25
3c. P(orange) = 2/25
3d. P(yellow) = 5/25

5.5 Investigation 5

ACE Questions, page 44.

1. Probability that Eleanor will win is 0; that Carlos will win is 1.
2. 6/36 = 1/6. Six of the possible outcomes are doubles.

10b. There are 9 ways to get an odd product; 27 ways to get an even product; so the probability Humberto will win is 27/36 = 3/4

5.6 Investigation 6

ACE Questions, page 51.

1. There are 15 possible outcomes. The probability of winning is 1/15.

2a. There are 9 possible outcomes.
2b. 1/9
2c. About 11 people
2d. About $45
5a. Brett: 30.9%; Puckett: 32.1%; Boggs: 33.8%

5.7 Investigation 7

ACE Questions, page 61.

1. 0; 2. 1; 3. no; 4. yes