Middle School Mathematics
A Guide to the Connected Mathematics™ Series

Kaleidoscopes, Hubcaps, and Mirrors

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1 Introduction

Kaleidoscopes, Hubcaps, and Mirrors is part of the **geometry** strand. It helps students to recognize and refine their knowledge of symmetry. A design is symmetric if some part of it is repeated in a regular pattern. Symmetry transformations, or rigid motions, include reflections, rotations, and translations. Similarity transformations change the size of a figure while preserving its shape. In contrast, symmetry transformations preserve both angle measures and side lengths, resulting in an image that is congruent to the original figure.

The purpose of this unit, the last geometry and measurement unit in the CMP curriculum, is to stimulate and sharpen awareness of symmetry and to begin to develop understanding of the underlying mathematics.

2 Goals/Objectives

This unit will help students:

- Understand important properties of symmetry
- Recognize and describe symmetries of figures
- Use tools to examine symmetries and transformations
- Create figures with specified symmetries
- Identify basic design elements that can be used to replicate a given design
- Perform symmetry transformations of figures, including reflections, translations, and rotations
- Give precise mathematical directions for performing reflections, rotations, and translations
- Write coordinate rules for specifying the image of a general point *(x, y)* under particular transformations
- Combine transformations and find a single transformation that will produce the same result
- Find the symmetries of geometric figures and make tables showing the results of combining symmetry transformations
- Learn and appreciate the power of transformational geometry to describe motions, patterns, and designs in the real world

3 Vocabulary

The following words and concepts are used in this unit. The concepts in the left column are those essential for student understanding in this and future units. The Descriptive Glossary (page 72) in the student text gives definitions for many of these words.

Essential Terms	Terms developed in previous units	Nonessential terms
congruent figures		Angle of rotation
line reflection	Diagonal	Center of rotation
reflectional symmetry	Equilateral triangle	Commutative operation
rotation	Hexagon	Directions of translation
rotational symmetry	Parallel	Identity element
symmetry	Parallelogram	Image
transformation	Perpendicular	Inverse element
translation	Tessellation	Kaleidoscope
translational symmetry		Line of symmetry
		Reflection line
		Strip pattern

4 Summary of Investigations

4.1 Investigation 1 – Three Type of Symmetry (pp 5-23)

- Students are introduced to reflectional, rotational, and translational symmetry.
- Identify symmetries in several designs and create designs with given symmetries.
- Work with tools and procedures for testing symmetry and making symmetric figures.
- Heighten sensitivity to various forms of symmetry and develop geometric techniques for testing and drawing symmetric figures.

4.2 Investigation 2 – Symmetric Transformations (pp 24-41)

- Describe the motions involved in constructing symmetric designs.
- Explore the relationships between figures and their images under reflections, rotations, and translations.
- Use findings to write precise rules for finding images under each type of transformation.
- Look at combinations of reflections.
- Reflect a figure over intersecting and parallel lines.
- Determine whether a combination of reflections is equivalent to a single transformation.

4.3 Investigation 3 – Transforming Coordinates (pp 42-59)

- Work with figures on a coordinate grid.
- Develop rules for locating an image of a general point (x,y) under a particular reflection, rotation or translation.

- Work with the ideas of rigid motion to learn about congruent figures.

4.4 Investigation 4 – Symmetry and Algebra (pp 60-70)

- Explore combinations of symmetry transformations of an equilateral triangle and square.
- Create tables showing the results of every combination of two symmetry transformations for these figures.
- Use the tables to determine whether the combining operation satisfies important algebraic properties.

5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Question 1 page 15

ANSWER

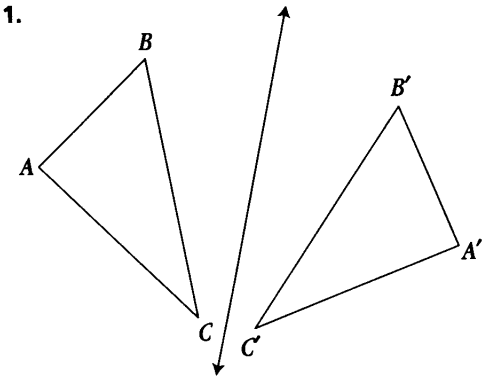
1. The design has rotational symmetry about the centerpoint with a 36-degree angle of rotation.

5.2 Investigation 2

ACE question 1 page 34.

ANSWER

1.figure 1.



Triangle A'B'C' is the image of triangle ABC. Each point on triangle ABC is matched to an image point on the other side of the line of reflection. The image point lies on a line passing through the original point and perpendicular to the line of reflection. The distance from the image point to the line of reflection is equal to the distance from the original point to the line of reflection.

5.3 Investigation 3

ACE Question 5 page 51

ANSWER

5a. P and R, and S, S and Q.
5b. A 180 degree rotation about the midpoint of segment QS.
5c. Sides: PQ and RS, PS and RQ, QS and SQ; angles: <P and <R, <PQS and <RSQ, < PSQ and <RQS.

5.4 Investigation 4

ACE Question 7 page 66

ANSWER

7a. The x represents the transformation that is performed before R_{120} to produce a result equivalent to L_1. From the table this must be L_2.
7b. The x represents the transformation that is performed after L_3 to produce a result that is equivalent to R_{240}. From the table this must be L_1.