Middle School Mathematics
A Guide to the Connected Mathematics™ Series

Looking For Pythagoras

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School math department
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1 Introduction

This guide supports the Connected Mathematics™ student textbook *Looking For Pythagoras*. This book is in the Geometry strand. Its primary topic is understanding and using the Pythagorean theorem and understanding irrational numbers.

2 Goals/Objectives

This unit will help students:

- Make connections between coordinates, distance between points, slope and area of polygons.
- Relate the length of a side of a triangle to the area of a square made on its side.
- Develop strategies for finding the distance between two points on a coordinate grid.
- Discover and apply the Pythagorean Theorem
- Understand irrational numbers and locate them on a number line
- Represent decimals as fractions and fractions as decimals
- Determine whether a decimal representation for a fraction repeats and/or terminates
- Use slopes to solve problems

3 Vocabulary

The following words and concepts are used in this unit. The concepts in the left column are those essential for student understanding in this and future units. The Descriptive Glossary in the student text gives definitions for many of these words.

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Essential Terms	Non-Essential Terms
equilateral triangle	leg (of a right triangle)
hypotenuse	midpoint
irrational number	theorem
isosceles triangle	
perpendicular	
Pythagorean Theorem	
rational number	
real numbers	
repeating decimal	
square root	
terminating decimal	

4 Summary of Investigations

4.1 Investigation 1 – Locating Points

Students review the concept of a coordinate grid and are introduced to finding the distance between pairs of points on a grid. They also investigate properties of geometric figures. Given two vertices they are to locate the other vertices that would define a square, a non-square rectangle, a right triangle and a non-rectangular parallelogram.

4.2 Investigation 2 – Finding Areas and Lengths

Students find areas of figures drawn on dot grids and then explore the relationship between the area of a square on the side of a triangle and the length of the side. They are introduced to the concept of square root. The square root is the number that is multiplied by itself to get another number. In this example, the square root is the side length of a square of a given area. For example, for a square to have an area of 4, its side length must be 2. We say that 2 is the square root of 4. If the square has an area of 16, its side length must be 4. We say that 4 is the square root of 16. Further, students are introduced to irrational numbers, such that, for a square with an area of 5, its side will have the length of the square root of 5 (written as $\sqrt{5}$).

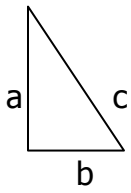
4.3 Investigation 3 – Pythagorean Theorem

Students discover the Pythagorean Theorem and explore its implications. They collect information about the areas of the squares on the sides of right triangles and conjecture that the sum of the areas of the two smaller triangles equals the area of the largest square. They apply the theorem to find the distance between

two dots on a dot grid. Then, they apply the converse of the theorem to determine whether a triangle is a right triangle.

The Pythagorean Theorem states that for any right triangle, $a^2 + b^2 = c^2$ where a and b are the lengths of the legs of the triangle and c is the length of its hypotenuse.

The hypotenuse of a right triangle is the side that is opposite the right angle. The legs of a right triangle are the sides that form the vertex at the right angle.



To investigate the converse of the Theorem, students take the three side lengths, and substitute them into the equation $a^2 + b^2 = c^2$ to determine if the statement is true. For example, if a triangle has side lengths 2, 3 and 4, is it a right triangle?

Let a = 2, b = 3 and c = 4. Let's determine if $2^2 + 3^2 = 4^2$.
$2^2 = 4$; $3^2 = 9$; and $4^2 = 16$. $4 + 9 = 13 \neq 16$. Therefore a triangle with side lengths 2, 3 and 4 is not a right triangle.

4.4 Investigation 4 – Using the Pythagorean Theorem

Students use the Pythagorean Theorem to explore a variety of applications including distances on a baseball diamond; properties of right triangles; and finding missing lengths and angles in a group of triangles.

4.5 Investigation 5 – Irrational Numbers

Students investigate irrational numbers (square roots) more closely. They express lengths (square roots) as decimals and study the relationship between fractions and decimals. They also study the classification of decimals as terminating or non-terminating; repeating or non-repeating. Irrational numbers are defined as those that can only be represented by non-terminating decimals. Rational numbers are defined as those that can be represented by terminating decimals. For example, ¼ is a rational number. Its decimal equivalent is 0.25 (terminating). $\sqrt{25}$ is a rational number. Its decimal equivalent is 5. As another example, $\sqrt{2}$ is an irrational number whose decimal equivalent is 1.414213562,

rounded to 9 decimal places. $\sqrt{2}$ is represented as a non-terminating decimal, and in middle school applications is most generally rounded to 1.414.

5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Questions, page 12:

2. (-3, -2)
3a. art museum and the cemetery
4. The hospital is 4 blocks from the greenhouse. There are 10 intersections on the map that are 4 blocks by car from the gas station: (1,5), (0,4), (1,3), (2,2), (3,1), (4,0), (5,1), (6,2), (7,3) and (7,5).

7a. 8 blocks x 150 meters/block = 1200 meters

5.2 Investigation 2

ACE questions, page 22:

6a. 2 square units
6b. about 1.414
7a. 5 square units
7b. about 2.236

5.3 Investigation 3

ACE Questions, page 34:

3. $h^2 = 4^2 + 3^2 = 25$
so $h = \sqrt{25} = 5$

5. $x^2 = 5^2 + 4^2 = 41$; so $x = \sqrt{41}$;
$y^2 = 20^2 + 4^2 = 416$; so $y = \sqrt{416}$

5.4 Investigation 4

ACE Questions, page 46.

1a. Since $500^2 + 600^2 = 610{,}000$, the distance is $\sqrt{610{,}000} \approx 781$m.
1b. 1100 – 781 = 319m

5.5 Investigation 5

ACE Questions, page 59.

9. = 12
10. = 0.6000
11. ≈ 3.8730
12. ≈ 31.6228
13. This is between 5 and 6 because 5 x 5 = 25 and 6 x 6 = 36; and 27 is between 25 and 36.