Middle School Mathematics
A Guide to the Connected Mathematics™ Series

Moving Straight Ahead

Prepared by members of
the Readington Middle
School math department
2004 Issue 1

Table of Contents

1 Introduction

This guide supports the Connected Mathematics™ student textbook *Moving Straight Ahead*. This book is in the Algebra strand. Its primary topic is linear relationships, including graphing and solving linear equations.

2 Goals/Objectives

This unit will help students:

- Further develop their understanding of variables and patterns.
- Recognize and represent relationships among variables in a variety of ways, including words, tables, graphs and symbols.
- Identify variables and determine an appropriate range of values of independent and dependent variables.
- Collect data and use patterns in table sand graphs to make predictions.
- Use graphing calculators to investigate linear relationships.
- Recognize that linear relationships have a constant rate of change (slope).
- Find the slope and y-intercept of a linear equation from its table, graph or equation and interpret its meaning.
- Recognizing a change in the slope or the y-intercepts and its effect on the various representations of data.
- Solve a linear function in the form $y = mx + b$ using tables, graphs and equations.
- Write a linear equation given the slope and y-intercept.
- Find a common solution between two linear equations by graphing or creating tables.

3 Vocabulary

The following words and concepts are used in this unit. All students are expected to understand these words and use them appropriately to communicate mathematical ideas. The Descriptive Glossary in the student text gives definitions for many of these words. There are other words that may arise during each student's course of study. These words will be identified by the teacher and added to the set of vocabulary for which the student is responsible.

™ Connected Mathematics was developed at Michigan State University with financial support from the Michigan State University Office of the Provost, Computing and Technology, and the College of Natural Science.

Terms

Coefficient
coordinate pair
linear
linear relationship
origin
point of intersection
rise
run
slope
y-intercept

4 Summary of Investigations

4.1 Investigation 1 – Predicting from Patterns

This investigation poses the unit's central question of how we can determine whether a relationship is linear. The class conducts an experiment and observes that the data collected seems to lie in a straight line. At this stage, a linear relationship is described as one whose graph is a straight line. Students use tables and graphs to make sense of and to make predictions from the data they collect.

In the ball bouncing experiment, students discover that there is a direct correlation between the "drop height" (the height from which a ball is bounced) and its "bounce height" (the height that the ball will rise in the air after it has hit the ground one time). The drop height can be selected by the students and is therefore identified as the independent variable and is graphed along the x-axis of a coordinate grid. The bounce height varies based on the independent variable (i.e., drop height) and is identified as the dependent variable. On a graph, it is located on the y-axis.

4.2 Investigation 2 – Walking Rates

Students look at rates in multiple representations. Using the rates at which people walk and receive pledges for such activities, students explore constant rate of change in tables, graphs, and equations. They investigate the change in the rate and its effect on the table, graph and equation. The y-intercept is introduced as students are asked to interpret the information represented by points on a line, entries in a table or solutions to an equation.

4.3 Investigation 3 – Exploring Lines with a Graphing Calculator

Students use graphing calculators to explore the effects of changing *m* and *b* in graphs of the form $y = mx + b$. They also use the calculator to find points on a graph and relate the information in the coordinates to the situation to which the graph relates. They also learn how the point on a graph can be shown on a table and in the equation for the line. The intersection of point two lines is identified. The students learn to explain what that point means in the context of the problem in which it was used.

4.4 Investigation 4 – Solving Equations

Students learn a symbolic method for solving equations in the form $y = mx + b$ using the idea of "undoing the operations." They are first asked to solve the problem in any way they choose, and then they apply mathematical reasoning to solve the equation. This method is developed more fully in the Say It With Symbols unit.

4.5 Investigation 5 – Exploring Slope

Students continue to develop their understanding of linear relationships, which by definition have a constant rate of change. For each unit change in *x*, there must be a constant rate of change in *y*. The steepness of a set of stairs is used to model the slope of a line. Students find constant rate or slope from a table, find slope of a line given two points and determine whether a line is increasing (has a positive slope), decreasing (has a negative slope), or neither (has a slope of 0).

4.6 Investigation 6 – Writing an Equation for a Line

Students develop methods for calculating y-intercept from a table or graph. Along with the slope, they are then able to write the equation of a line in the standard form $y = mx + b$. For example, the y-intercept can be found in the table as the point (0, b). The slope is found either by generating a table or using the ratio of the vertical change to the horizontal change.

5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Questions, page 9:

1a. The graph would consist of the following points: (20,10), (30,18), (25,40), (32,50), (38,60), (45,70), (50,80)

1b. Answers will vary. One possible answer: 28.5cm (I used the value ½ way between 25cm and 32cm in the table).

1c. Answers will vary. One possible answer: 92 cm.

1d. Answers will vary. One possible answer: 95 cm.

5.2 Investigation 2

ACE questions, page 24:

1a. Jose': 15/3 = 5 mph; Mario :21/3 = 7 mph; Melanie: 27/3 = 9 mph
1b. Jose': 7 x 5 = 35 mi; Mario :7 x 7 = 49 mi; Melanie: 7 x 9 = 63 mi

15.

13 h 35 min 18 sec = 13.59 hours
363.1/13.59 = 26.72 mph
(or convert time to minutes or seconds and calculate to produce: 0.45 mi/min or 0.0074 mi/sec)

5.3 Investigation 3

ACE Questions, page 44:

5a. 75 candy bars
5b. For 50 candy bars, the revenue would be $33. For 125 candy bars it would be $83.
5c. For a revenue of $200 the band would have to sell 300 candy bars. The cost would be $125, leaving a profit of $75.

18. values greater than –2.5
19. All values of x will make y negative.

29a. Point (3,2) is on the line. (3,3) is above the line. The other two points are below the line.

5.4 Investigation 4

ACE Questions, page 59

2. Find p if

$$\begin{aligned} 93 &= 16 - 5p \\ 93 - 16 &= 16 - 16 - 5p \\ 77 &= -5p \\ 77/(-5) &= -5p/(-5) \\ -15.4 &= p \end{aligned}$$

5. $y = 10 - 2.5x$. Find y if $x=3.2$

$$\begin{aligned} y &= 10 - 2.5(3.2) \\ y &= 10 - 8 \\ y &= 2 \end{aligned}$$

5.5 Investigation 5

ACE Questions, page 70.

1. slope = 3; y-intercept = 10
5. slope = 2; y-intercept = 0; $y=2x$

29a. $2.39P + P = 37{,}000$; $3.39P = 37{,}000$; $P = 10{,}914.45$. Therefore the approximate population in 1980 was 10,914.

5.6 Investigation 6

ACE Questions, page 84.

1. $y= -3x + 5$
5. $y = 6$

15 a. $d=1100t$, where d is the distance in feet to where the lightening struck and t is the time before the thunder is heard.
15b. The slope,100 is the speed of sound, and the y-intercept is 0.
15c. 1 mi = 5280 feet, so, $5280=1100t$, and $t = 4.8$s.
15d. $d = 1100(6.5) = 7150$ ft or 1.35 mi.