Middle School Mathematics
A Guide to the Connected Mathematics Series

Introduction & General Guidelines

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Introduction

This guide is an overview written by the Readington Middle School mathematics staff to support parents/guardians in their child's use of the Connected Mathematics series. These handbooks provide additional background information and supporting material which may be of assistance to parents/ guardians as their student progresses through each unit.

Readington Middle School Handbook Series

In addition to this parent overview that provides an introduction and general guidelines for helping students through all areas of mathematics education, there is one handbook for each Connected Mathematics unit. Each guide provides the following information for the unit for which it is written:

- goals and objectives for each unit;
- key vocabulary; and
- relevant sample problems and solutions.

1 Connected Mathematics Unit Organization

1.1 Units by Grade Level

The Connected Mathematics program utilizes a series of workbooks – each could be considered equivalent to a chapter of a more traditional textbook. There are eight units designated per grade level. The sequence for presenting the topics is flexible; however, some units are based on prior learning and therefore need to be taught in order. This flexibility of the program allows educators to make implementation decisions to best influence their students' mathematical learning and academic achievement.

1.2 Units by Content Strands

The Connected Mathematics units are organized by strands. There are four strands which cross grade levels. These strands are as follows:

- Data Analysis and Probability
- Number and Operations
- Geometry and Measurement
- Algebra (7^{th} and 8^{th} grade)

1.3 Implementation Plan by Order of Units (2004-2005)

Grade 6

- Prime Time
- Bits and Pieces I
- Bits and Pieces II
- Shapes and Designs
- Covering and Surrounding
- Data About Us
- How Likely Is It?
- Ruins of Montarek

Grade 7

- Variables and Patterns
- Stretching and Shrinking
- Comparing and Scaling
- Accentuate the Negative
- Moving Straight Ahead
- Filling and Wrapping
- What Do You Expect?
- Data Around Us

Grade 8

- Accentuate the Negative
- Linear Relationships
- Thinking With Mathematical Models
- Clever Counting
- Looking for Pythagoras
- Growing, Growing, Growing
- Say It With Symbols
- Frogs, Fleas, & Painted Cubes

2 Major Ideas by Unit

Each Connected Mathematics textbook has been given a clever title. The table below provides a very brief description of the major topic covered in each textbook. This could be considered the skill from which the book got its title. It is, by no means, an exhaustive list of the skills taught in each book.

2.1 Sixth Grade

The following table identifies the major idea addressed in each sixth grade unit.

Strand	Connected Mathematics Title	Area of Focus
Data Analysis & Probability		
	Data About Us	Statistics; Measures of Center
	How Likely Is It?	Probability
Geometry		
	Shapes & Designs	Two Dimensional Geometry; Properties of Polygons
	Covering & Surrounding	Area & Perimeter of Polygons & Circles
	Ruins of Montarek	Spatial Visualization
Numbers and Operations		
	Prime Time	Prime Numbers, Multiples, Factors
	Bits & Pieces I	Fractions, Decimals, Percents
	Bits & Pieces II	Operations with Fractions & Decimals; Percents

2.2 Seventh Grade

The following table identifies the major idea addressed in each seventh grade unit.

Strand	Connected Mathematics Title	Focus Area
Algebra		
	Variables & Patterns	Introduction to Algebra
	Moving Straight Ahead	Linear Relationships
Data Analysis & Probability		
	What Do You Expect?	Probability & Expected Value
	Data Around Us	Number Sense with Large Numbers
Geometry		
	Stretching & Shrinking	Similarity and Scaling
	Filling & Wrapping	Three Dimensional Geometry; Volume & Surface Area
Numbers and Operations		
	Comparing & Scaling	Ratio, Proportion & Percent
	Accentuate the Negative	Negative Integers

2.3 Eighth Grade

The following table identifies the major idea addressed in each eighth grade unit.

Strand	Connected Mathematics Title	Focus Area
Algebra		
	Thinking with Mathematical Models	Representing Relationships with Equations
	Growing, Growing, Growing	Exponential Relationships
	Frogs, Fleas, & Painted Cubes	Quadratic Equations
	Say It With Symbols	Algebraic Expressions
Data Analysis & Probability		
	Sample & Population	Data & Statistics; Predictions from Samples
Geometry		
	Looking for Pythagoras	Pythagorean Theorem
	Kaleidoscopes, Hubcaps & Mirrors	Translations
Number & Operations		
	Clever Counting	Patterns in Counting

3 Educational Philosophy

3.1 What Does It Mean To Be "Standards Based"?

When a mathematics curriculum or program is described as "Standards Based," it means that it supports the National Council for Teachers of Mathematics ("NCTM") Standards. These are national standards which address the importance of arithmetic as well as skills in problem solving, communication, reasoning and making connections.

3.2 NCTM Standards, NJ Standards, RMS Standards

The NJ State Core Curriculum Standards in mathematics closely align with the NCTM standards. The RMS curriculum in mathematics supports these state standards. As such, Readington Middle School students are receiving a complete and full mathematics education. They are being instructed on the complete range of skills required by the national and state standards, including:

- *Arithmetic Calculations* where students must be able to perform arithmetic operations on real numbers, fractions & decimals;
- *Problem Solving Strategies* involving real world applications in which students must apply mathematics meaningfully;
- *Verbal and Written Communication* practice through the application of mathematical symbols, diagrams, graphs, pictures, materials and written paragraphs;
- *Reasoning* where students must apply logic to the problems they are assigned and justify or explain their answers and strategies. Students are encouraged only to use a strategy if it makes sense to them. They learn concrete strategies through direct teacher led instruction as well as from listening to and sharing with their peers; and
- *Connections*. Students are encouraged to make connections between strategies they applied in one situation to similar strategies they may be able to utilize in another similar or different scenario. They have opportunities to utilize math in situations that they do or will encounter in their daily lives.

Since all curriculum is written to align with state standards, all RMS programs, including Connected Mathematics, are selected based on their ability to meet or exceed national and state standards.

3.3 What Are the Benefits of the Connected Mathematics Program?

It is true that students today are doing more writing and problem solving in math class than students 10 or 15 or more years ago. What are they getting now that earlier students may not have experienced? They have the opportunity to apply the arithmetic rules to real life situations so that they may learn the real meaning on which the "rule" is based. Students gain the advantage of written expression

and the ability to teach someone else their strategy. It is true in all fields of study that once a person begins to teach a subject, their own understanding of it increases. Students who were in cooperative classrooms have the opportunity to explain themselves, justify their reasoning, challenge each other and teach each other new strategies. All of these tasks require a deeper comprehension of the skill being taught. Students have the opportunity to apply the skills they are learning to real life applications. In this way, mathematics becomes a meaningful part of their everyday life, and not an isolated event within the confines of a classroom.

3.4 Is This Middle School Math "New"?

Mathematics, by definition, is the study of relationships with numbers and the use of symbols in expressions with numbers. Historically, we considered mathematics to be comprised primarily of 'arithmetic' or basic operations with numbers. While it remains true that arithmetic is a critical portion of any education in mathematics, it is not the only skill that is important.

For the most part, there is nothing "new" about middle school mathematics. By the sixth grade, students should understand and have memorized their basic math facts. They should be able to perform the four basic operations (addition, subtraction, multiplication and division) on any set of whole numbers without a calculator. In addition, students will have had a significant opportunity to learn to apply these skills to solve problems.

In middle school, students are challenged to solve problems in meaningful ways. They can solve problems using the same strategies their parents or older siblings were taught, or by using another strategy that works and makes sense to them. In the end, any successful method will arrive at the same solution. You may have memorized 12 x 6 = 72. Your student may calculate 12 x 5 = 60 and then add 12 to obtain the answer 72. It doesn't matter, as long as both of you recognize that there are 72 eggs in 6 dozen!

John Dewey, a major American educational theorist, developed an educational philosophy that was practical and could be tested in schools and classrooms. At the time that he first tested his educational practices, the country's educators believed that students learned best if they sat quietly and learned by rote a classical curriculum, an approach that had been labeled the "formal discipline theory."[1] Instead of strengthening the mind, Dewey believed that this form of learning put the students in an exceptionally passive role. He instead developed learning environments which ensured that children would actively engage in learning[2]. Experience should precede, or at least be concurrent with, educational

[1] Dewey. page 16.
[2] Ibid. page 17.

concepts and ideas. His objective was to create an "experienced-based curriculum to promote both the more effective learning and greater competence in living."[3] Mathematical learning and reasoning that takes root through investigation and is applied to solving real world problems is more cogent to the middle level child. At RMS, these educational principles based on the idea that children must be active in their learning are those that guide our instructional practices in mathematics.

What has been newly introduced into the middle school curriculum in the past several years is the support and incorporation of technology. Software, graphing calculators, PCs, spreadsheets, and internet research are all tools and challenges by which previous generations were not affected. This support for technology is being incorporated across the curriculum for the benefit of our students.

3.5 Why Don't Students Just Memorize Facts and Do Worksheets?

Students in the past may have spent more than 75% of their time on paper-and-pencil drills, in which students practiced arithmetic skills in isolation from problem-solving situations.[4] This was often in the form of worksheets. However, in real life, mathematics does not arise in this way. Deciding what to do is the critical first step before doing any calculation.[5]

Research shows that creating understanding leads to long-term retention, more advanced application ability, and utilization and development of higher level thinking skills.

In the Connected Mathematics classroom, through guided investigations, students are encouraged to try a strategy and see how it works and then try another. The student gains experience and much deeper conceptual understanding as the appropriate strategy is identified. There are multiple ways to arrive at a correct answer but . . . Which works better? Which is quicker? Easier? Longer? More accurate? They then pick the strategy that works for the situation. What if it doesn't work at all? Great! Now we know what not to try. Or maybe we know a part of what works and can combine it with a part of a neighbor's strategy for the perfect solution that works every time. Trial and error. Team work. Discussion. Supporting decisions with explanations and sound reasoning. These are skills that carry throughout one's life.

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[3] Ibid. pg 17
[4] Burns, Marilyn. About Teaching Mathematics. pg 5.
[5] Ibid. pg 6.

"The real power is in being able to talk about your ideas and strategies."[6] This is true in mathematics and most other fields. The ability to express clearly the reasoning behind solving a mathematical problem is an identified element of both state and achievement testing. The ability to read, write, and communicate are important aspects of any mathematics program and a precursor to the working world.

3.6 Educational Philosophy of the Connected Mathematics Program

The Connected Mathematics program presents material for students to learn according to the constructivist theories described above. Students (and those helping at home) are encouraged to experiment with new ideas and share strategies in order to try to figure out a solution or algorithm, before it is told to them. Encouragement in the sharing of ideas is welcome, and it is often interesting to explore the creative and innovative thinking of the student.

The text provides real life examples. Students no longer study mathematics in isolation. They learn to calculate percentages in shopping and tipping situations; they learn about linear equations in the context of pledges and fund raisers. As such, students gain knowledge of mathematics and also about society customs, traditions and applications.

The Connected Mathematics program meets all the NCTM standards – the content standards (involving the mathematical computations) as well as the process standards (involving communicating, problem solving and the like.)

Additionally, by its design, it incorporates other life skills such as cooperative group work, written expression, and the development of problem solving skills. Each of these skills, and the others that are naturally developed, are core strengths which students will take with them and apply long after their middle school days are completed.

3.7 Vertical Articulation

The K-5 Everyday Mathematics Programs builds a foundation of skills that is applied and expanded in the middle level program. Through in-district articulation, the teachers are coordinating the programs and assuring that the differentiated needs of the students are addressed. The Connected Math Program covers all the skills that are tested on the 8th grade GEPA and the skills that are required for success at the High School level. On-going articulation with the high school is helping to assure that students are prepared for the demands of the high school curriculum when they leave RMS.

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[6] Lappan. Bits & Pieces I. pg 3.

4 Student Textbook Format

4.1 Title Page

The title page is a two-page spread which describes the content of the book and gives some sample problems or questions that the material in the book is designed to prepare students to answer.

4.2 Mathematical Highlights

This page of each student workbook lists the goals and objectives of each unit.

4.3 Investigations

Each unit contains between four and seven investigations. Each investigation is broken down into problems, each with its own "follow-up" questions.

For example, Bits & Pieces II has seven investigations, numbered sequentially. Investigation 1 is comprised of Problems 1.1, 1.2, 1.3 and 1.4. Each problem is printed in an orange box and has one or more questions. The follow-up question(s) follow.

Each investigation in each book provides background information, sets the stage for a problem that will be presented and then presents an open ended problem for students to solve. These problems are solved in class – generally in pairs or small groups.

The follow-up question(s) generally extend or expand on the strategy developed in the problem. Generally, the problem is reviewed in class and strategies and solutions are discussed and taught. Then, students can apply this to the follow-up. At times, it makes more sense for the follow-up to be completed by the groups prior to full class review.

Prior to moving to the next problem, all strategies for that problem are reviewed through whole-class discussion. Students are encouraged to take notes during their working together as well as during this period of review.

4.4 Application, Connection, Extension (ACE)

After the problems in each investigation, there is a section of practice problems referred to as ACE questions – Application, Connection, Extension.

The questions in the application section require a direct application of the skill learned in class. All students should be able to answer these questions.

The connections questions are more challenging and involve at least one additional step or problem solving technique to solve. These questions generally involve real life applications in which students sometimes need background information. Most students should be able to answer these questions, but may need guidance or background information prior to completing a problem.

The extension questions provide challenges and enrichment opportunities for all students. These challenge problems involve more complex problem solving ability. Since it is not likely that all students will be able to answer these questions, they offer an opportunity for differentiation.

Unlike traditional textbooks, however, the questions in each of the three sections (Application, Connections & Extensions) are not written in order of increasing difficulty. Traditional textbooks often put the easiest practice questions first, and the most difficult at the end. That is not the case in this text book. Questions in each section appear to be arranged almost randomly. Each question relates to skills specifically learned in each problem in the investigation, but they are not put in the ACE section sequentially. It is possible that students complete Problem 1.1 and 1.2 in class on Monday and are assigned ACE questions # 3, 5, 10 and 23. These questions were assigned because the teacher knows that they directly relate to the problem completed in class. Perhaps questions 3 and 5 are in the Application section, 10 is in the Connections section, and 23 is an Extension question. It is highly probable that the students have not yet learned the skill required for some of the other problems. These questions will be assigned after the pre-requisite problems in the investigation have been learned. By the end of each investigation, the students should be able to complete all of the application and connections problems, and some may also be able to complete all of the extension problems as well.

4.5 Mathematical Reflections

After the ACE questions is a page entitled “Mathematical Reflections”. This section of questions requires contemplation and assimilation of the lessons learned throughout all the Problems in the Investigation. The Mathematical Reflections require both verbal and written expression. These questions provide them opportunities to answer open ended questions. Please talk to your students about these questions. Is this information she can explain to you? Are you clear on what she means? If not – she has questions to bring to class the next day! Please encourage her to do so!

4.6 Looking Back/Looking Ahead

This section is at the end of every unit. It contains practice problems that require a variety of skills – from application of lessons learned to problem solving and

written expression. It includes problems from the past as well as a tie in to problems that will be answered in the future.

4.7 Glossary & Index

Each student textbook ends with a descriptive Glossary & Index. This Glossary contains all key words used in the unit. It contains a description or definition and a sample, where applicable.

The Index may provide helpful pointers to information within the textbook.

5 How Can I Help My Student At Home?

The following guidelines are offered to support you in your desire to assist your student. It is comforting to know that you do not have to know everything about the math topic or the assigned problems to help your child with their math homework. Most students can benefit from assistance that you are well prepared to provide. Areas of assistance include, but are not limited to, organization, question comprehension, and positive attitude.

In general, students benefit from structure and guided practice. They are not expected to learn new skills through their homework assignments. Rather, they are to apply skills that they learned in class to a variety of situations which may be similar to or somewhat different than the scenario they learned in class. If they are struggling, gentle guidance in organization and attitude can be of great benefit.

5.1 Organization

Helping students maintain organization skills may result in benefits across all areas of the curriculum.

- **Provide a quiet study place** with pencils, paper and a calculator. Be sure they have their math book and notebook prior to beginning.

- **Help your student organize their notebook**. All students are to take notes in math class as well as record the work they do in groups or individually as they solve the problems in each investigation.

- **Help your student accurately record their homework assignments** and their due dates. Check their agendas to be sure they're using it clearly to record all assignments. Students can also phone their teachers' voicemail to hear a recorded message of the night's homework.

- **Review your student's notebook** to ensure that their notes make sense. Students should be able to look at their notes and work from class and explain to you which problem they completed in class and give you the correct answer to the problem and follow-up questions. **Tell your students which area(s) of their notes are clear.** Also help them to see where clearer notes would increase their ability to recall and explain information. This is a valuable skill that will be reinforced throughout their academic year. If you or your student are unclear about their notes, it may be evidence that they need to improve their note taking habits.

- **Ask your student to explain the strategies they utilized in class as well as those of their classmates**. Through teaching you and explaining what they are learning, they are better able to reinforce the new ideas and concepts, as well as increase their ability to communicate mathematically. Such communication will also help them to identify areas in which they need clarification. Encourage them to ask about these areas in class on the following day.

5.2 Question Comprehension and Problem Solving Strategies

- **Check comprehension of basic math facts.** In general, middle school students should know the basic addition, subtraction, multiplication and division facts. If your student is struggling in this area, you can encourage them to become proficient in these important areas through repeated practice, flashcards and speed drills. Knowing basic facts continues to be extremely important. There are computer games and activities that will also help in reinforcing the knowledge of basic facts.

- **Check your student's understanding of the question.** Read the question with your student – do they understand all the vocabulary and what each word means in the context of the question being asked? For example, students may be asked to make a "scaled model" of their home. While they may understand the mathematics behind the problem, they may not necessarily understand what is meant by "scaled model".

- **Ask students to explain what is being asked of them**. Perhaps they know that they are being asked to find the number of hours that it will take someone to travel a certain distance, but are only given distances in feet and their average rate per mile. Ask them what they need to know before they can calculate the answer. Do they understand the intermediate steps required to solve the problem? Do they understand that they may need to convert feet to miles? Do they know how many feet are in a mile? Ask them to tell you the steps they would need to take. Perhaps have them write key words for the steps in their notebook. This organization may be the piece they need to successfully complete multiple step problems.

- When working on a problem with your student, **share your problem-solving strategies, mental computations and any estimation strategies** that you use. Ask them to share some with you. Together you can find the solution to the problem. The point in doing this is not to do their homework for them, but to show that there are multiple strategies that people use in solving mathematical problems. The

students are on their way to developing what strategies work best for them.

- **Encourage students to get help the following day**. Students who cannot complete their homework assignments are encouraged to ask about the incomplete work in class the following day. Students are always encouraged to speak to the math teacher that is available to them during their tutorial period. They may have a quick question or may rather spend more time getting further assistance. There is always a math teacher available to help students during each tutorial period – even if it is not the student's own math teacher.

5.3 Positive Attitude

Most parents/guardians enjoy reading to young children, and as such children learn to value and enjoy reading from a young age. Similarly, students can learn to value and enjoy mathematics.

- **Maintain a peaceful and positive attitude** with your student as they work through their math homework. If they have questions they cannot solve, they are likely not to be alone. No student is expected to be able to answer all homework questions completely and correctly every time. Homework is practice and, as such, will sometimes present unique challenges that need to be raised in class. All students have the opportunity and encouragement to ask these questions in each class.
- **Portray positive images and values about mathematics.** It's important for children to have positive role models. You have the opportunity to portray math as relevant and useful.
- **Show students where you use mathematics in your every day life or profession.** Involve your children in decisions that require measurement, time or distance calculations, problem solving or reasoning.
- **Encourage your student to play math games** which involve computation, puzzles, logic or problem solving. Some websites can be found in the next section of this document.

6 Useful Websites

The following websites may be helpful in presenting games and exercises for students to practice and extend math skills that are learned in the classroom. The Readington School District is not affiliated with any of the website authors and is not responsible for its content.

Website	***Possible Use***
Shodor.org/interactivate/piindex.html	***Connected Math Skills by Unit:*** *Games; Practicing new skills*
Ettech.org/resources.asp?cat=math	*Reviewing Concepts; studying*
harcourtschool.com	*Advanced learning, practice; games*
edhelper.com	*learning new concepts*
aplusmath.com	*Independent practice, games, worksheets, activities*
funbrain.com/math/index.html	*learning new skills; practice; games*
mathforum.org	*advanced math; learning new concepts*
Quia.com/dir/math	*Independent practice; games; tests*
Puzzlemaker.com	*games; activities; projects*
math.com	*practice; games; tests; activities*
aaamath.com	*practice, games, activities; tests; learning new concepts*
edhelper.com	*independent practice; learning new concepts; tests*
archives.math.utk.edu	*independent practice; learning new concepts; tests*
Mcsdtechcenter.org/links/math.html	*learning new concepts*

Parent Feedback – We value your input

If you have feedback or you have some suggestions that you would like to have incorporated in this overview or in the unit handbooks, please contact Ms. Colleen Cirelli or Mr. John Casey of the RMS Mathematics Department. All such feedback is greatly appreciated.

7 References

The following resources were utilized in writing this manual.

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