Middle School Mathematics
A Guide to the Connected Mathematics™ Series

Prime Time

Prepared by members of
the Readington Middle
School math department
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1 Introduction

This guide supports the Connected Mathematics™ student textbook *Prime Time*. This book is in the Number and Operations strand. Its primary topic is recognizing prime and composite numbers and understanding the concepts of factors, multiples, common factors and common multiples.

2 Goals/Objectives

This unit will help students:

- Understand the relationships among factors, multiples, divisors and products
- Recognize that factors come in pairs
- Link area and dimensions of rectangles with products and factors
- Recognize numbers as prime or composite and as odd or even based on their factors
- Use factors and multiples to explain some numerical facts of everyday life
- Develop strategies for finding factors and multiples of whole numbers
- Recognize that a number can be written in exactly one way as a product of primes (Fundamental Theorem of Arithmetic)
- Develop a variety of strategies to solve problems involving factors and multiples.

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3 Vocabulary

The following words and concepts are used in this unit. All students are expected to understand these words and use them appropriately to communicate mathematical ideas. The Descriptive Glossary in the student text gives definitions for many of these words. There are other words that may arise during each student's course of study. These words will be identified by the teacher and added to the set of vocabulary for which the student is responsible.

Terms

common factor
common multiple
composite number
divisor
even number
exponent
factor
multiple
odd number
prime factorization
prime number
square number

4 Summary of Investigations

4.1 Investigation 1 – The Factor Game

The Factor Game engages students in a contest in which winning strategies involve distinguishing between numbers with many factors and numbers with few factors. Students are then guided through an analysis of the game. The rules of the game are located on page 7 of the student textbook. Students are encouraged to play the game with family and friends. Factors are pairs of number which, when multiplied together, produce a whole number.

4.2 Investigation 2 – The Product Game

Students find products of factors. Students develop strategies for winning the game and focus on multiplication facts. Students create their own game by selecting factors, determining products and choosing appropriate dimensions for their game board. These products identified are considered multiples of the numbers which were multiplied together to form the product.

4.3 Investigation 3 – Factor Pairs

Students use square tiles to make all possible rectangles to represent the numbers 1 through 30. Finding rectangles with a given area helps students to visualize whole numbers and to list factor pairs. These rectangles also provide a foundation to discuss how many factors must be checked to find all the factor pairs of a number. Connecting factor pairs to area previews the study of measurement in the *Covering and Surrounding* unit. Square tiles are used to model even and odd numbers and then to prove conjectures about the sums and products of odd and even numbers. Some students find that the most effective way to find all the factors of a number is to begin with one and increment by 1 until all the factors have been identified. A number is a factor of another number if it can be multiplied by any whole number to form the number of which it is a factor.

4.4 Investigation 4 – Common Factors and Multiples

Real-life situations are used to motivate student interest in common factors and common multiples. The concepts of least common multiple and greatest common factor, though not formally introduced yet, are used naturally throughout the problems and in the ACE questions. The context of the problems and questions helps make clear whether a solution involves finding a common multiple, factor or least common multiple, greatest common factor.

Each number has an infinite number of multiples. As such, any pair of numbers may have an extensive list of common multiples. A common multiple is a number which is a multiple of both of the numbers being considered and can be found by listing all the multiples of each number and circling or marking those multiples that exist as multiples of both numbers. Because each number has an infinite number of multiples, finding the greatest common multiple is impossible. This investigation looks at reasonable problems in which the least common multiple is required. The least common multiple is the smallest multiple that the numbers share.

Common factors are all the factors that are the same between two numbers. The greatest factor is the largest of those that are in common.

4.5 Investigation 5 – Factorizations

Finding longer and longer factors strings of a number leads students to discover the Fundamental Theorem of Arithmetic which states that a whole number can be factored into a product of primes in exactly one way. Factor trees are used as a systematic way of finding the prime factorization of a number. The last problem in the investigation helps students use prime factorizations to find the

greatest common factor and least common multiple of two or more numbers. A discussion of why 1 is NOT a prime number occurs.

4.6 Investigation 6 – The Locker Problem

The Locker Problem provides a way to summarize the unit. Students organize data, look for patterns and solve problems using multiples and factors.

5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Questions, page 12:

1. Divide 24 by 6 to get 4; so, 6 x 4 = 24
2. 4
3. 9
4. 8

5.2 Investigation 2

ACE questions, page 22:

1. 2 and 9, 3 and 6
2. Possible answer: 16 and 36. 16 = 4x4 = 2x8; 36 = 12x3 = 4x9 = 6 x 6
3. 1, 3 and 9

5.3 Investigation 3

ACE Questions, page 30:

6. rectangles: 1 x 72; 2 x 36; 3 x 24; 4 x 18; 6 x 12; 8 x 9
   factor pairs: 1, 72; 2, 36; 3, 24; 4, 18; 6, 12; 8, 9

7. Every number has itself and 1 as factors. A number with exactly 2 factors has only itself and 1 as factors. Such numbers are called *prime numbers*. For example, 31 is a prime number. Its factors are 1 and 31.

5.4 Investigation 4

ACE Questions, page 40.

1. 24, 48, 72, 96. LCM = 24
8. 1, 2, 3, 6. GCF = 6
15. 24 days

21. 3 x 4 x 5 + 1 = 61
22. 14 and 35

5.5 Investigation 5

ACE Questions, page 52.

1. 2 x 2 x 3 x 3
2. 2 x 2 x 3 x 3 x 5
13. GCF = 9; LCM = 150
21. Number is a multiple of 2 and 7. Therefore, it is a multiple of 14.
    Multiples of 14 between 50 and 100 are:
    56 = 2 x 2 x 2 x 7
    70 = 2 x 5 x 7
    84 = 2 x 2 x 3 x 7 and
    98 = 2 x 7 x 7

    Of these, only 70 has three different primes in its prime factorization (Clue 3). Therefore the answer is 70.

5.6 Investigation 6

ACE Questions, page 61.

1. 53
2. yes; numbers that end in 0 are multiples of 10. All multiples of 10 have 2 and 5 as factors.

7a. 2, 4, 8, 16, 32, 64, 128, 256, 512
7b. 1024
20. 1, 2, 3, 5. If 10 and 6 are factors, the factors of 10 and 6 must also be factors.