Middle School Mathematics
A Guide to the Connected Mathematics™ Series

Say It With Symbols

Prepared by members of
the Readington Middle
School math department
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Table of Contents

1 Introduction

This guide supports the Connected Mathematics™ student textbook *Say It With Symbols.* This book is in the Algebra strand. Its primary topic is using algebraic reasoning to solve problems, understanding order of operations, writing equivalent expressions through applying the distributive and commutative properties, and solving algebraic equations.

2 Goals/Objectives

This unit will help students:

- Strengthen their understanding of the order of operations rules and apply these rules to evaluate expressions.
- Write symbolic (algebraic) sentences to express ideas.
- Recognize and use applications of the distributive and commutative properties.
- Recognize and evaluate equivalent expressions.
- Apply the properties for manipulating expressions to solving linear equations.
- Solve simple quadratic equations requiring basic factoring.

3 Vocabulary

The following words and concepts are used in this unit. The concepts in the left column are those essential for student understanding in this and future units. The Descriptive Glossary in the student text gives definitions for many of these words.

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Essential Terms

algebraic expression
commutative property of addition and multiplication
distributive property
equivalent expressions
expanded form
factored form
function
linear relationship
order of operations
parabola
quadratic relationship
roots
surface area
term
x-intercept
y-intercept

4 Summary of Investigations

4.1 Investigation 1 – Order of Operations

Students continue to develop their intuition about the order in which operations in symbolic expressions should be performed. The problems highlight conventions of mathematical notation governing the order of operations and evaluating expressions that involve addition, subtraction, multiplication, division and exponents.

Students conclude that in a mathematical sentence, operations must be performed in the following order: (1) Parentheses; (2) Exponents; (3) Multiplication/Division; and finally (4) Addition/Subtraction

4.2 Investigation 2 – Equivalent Expressions

Students are presented an opportunity to justify, in their own ways, the equivalence of two or more symbolic expressions for the same situation. They look at the problems in a variety of ways including the comparison of graphs and tables. They will develop a strong sense of the distributive property.

The distributive property is the property by which the expression 4*(3 + 2) can be written as (4 * 3) + (4 * 2).

To solve 4*(3 + 2) a student would add 3 + 2 = 5 and then multiply 4 * 5 to produce a product of 20.

To solve (4 * 3) + (4 * 2), a student would multiply 4 * 3 and then 4 * 2 and then add the products together (12 + 8) to get a sum of 20. Because both of these problems have the same solution, we say that they are equivalent.

Students also use the distributive property with algebraic expressions. For example, they learn to apply the distributive property to write an expression equivalent to 4*(2x + 3). By performing the multiplication of the 4 before adding the terms inside parentheses, a student will produce an equivalent expression 8x + 12. Students know then that 4*(2x + 3) = 8x + 12.

4.3 Investigation 3 – Some Important Properties

Students acquire formal vocabulary for the work they've done previously with the distributive and commutative property. They understand that the commutative property for addition states that numbers can be added in any order, and that the commutative property for multiplication states that numbers can be multiplied in any order. For example, 3 + 5 = 5 + 3; 4 + x = x + 4; 6*8 = 8*6; 12*x= x*12.

Throughout the investigation, students use the properties of real numbers to write equivalent expressions as they continue to connect symbolic expressions with real-world contexts. Students work with both linear and quadratic equations.

A linear equation is one in the form $y = mx + b$, for example, $y = 3x+6$.
A quadratic equation is one in the form $y = ax^2 + bx$, for example, $y=3x^2 + 6x$.

4.4 Investigation 4 – Solving Equations

Students extend their earlier work to solve linear equations using a graph and a table, and then by the symbolic method. They learn more strategies for solving equations in the form $ax + b = cx + d$ and are introduced to strategies for equations in the form $y = ax^2 + bx$.

4.5 Investigation 5 – Writing Expressions for Surface Area

Students are asked to find an expression for the surface area of a staggered stack of logs. In this problem, they have the opportunity to approach it from several different angles and represent their reasoning in a variety of ways. Students will verify that all of the solutions are equivalent expressions using the strategies that have been learned to date. This activity provides a review of

order of operations, writing and comparing equivalent expressions and solving linear equations.

5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Questions, page 12:

1. $3(12) + 15 = 36 + 15 = 51$
6. $5(4^2) = 5(16) = 80$
10. $4(7^2) + 3(7) = 4(49) + 3(7) = 196 + 21 = 217$
18. $7(-3+8) = 7(5) = 35$
22. $(10-5)(10+2) = (5)(12) = 60$

5.2 Investigation 2

ACE questions, page 26:

6a. Rectangles i and iv have an area of $6(x+1)$
6b. $6x + 6$
19a. $2.50(76 + 49)$ and $2.50(76) + 2.50(49)$
19b. $2.50(43 + 57)$ and $2.50(43) + 2.50(57)$
19c. $2.50(x + y)$ and $2.50x + 2.50y$

5.3 Investigation 3

ACE Questions, page 44:

1a. $3x – 4 – 2x$ or $x – 4$
1b. each expression equals 1 when $x = 5$ and –5 when $x = -1$

4. $(3 + 5)x + 2$ and $8x + 2$; the last expression is easiest to use.
19. $2(4+5) = 18$
21a. Possible solutions include: $12 = 2(6) = 3(4) = 12(1) = 10(1.2)$ and $12=2(2)(3)=2(5)(1.2)$
21b. Possible solutions include: $12 = 5 + 7 = 6 + 6$ and $12 = 2 + 6 + 4 = 9 + 2 + 1$

5.4 Investigation 4

ACE Questions, page 59.

1. $6x + 10 = 4x + 18$
$-4x \quad = -4x$

$2x + 10 = 18$
$-10 = -10$
$2x = 8$
$2x/2 = 8/2$
$x = 4$

4. $-13x + 36 = 20x - 30$
$+ 13x \quad = +13x$

$36 = 33x - 30$
$+30 = +30$

$66 = 33x$

$66/33 = 33x/33$
$2 = x$

5.5 Investigation 5

ACE Questions, page 67.

1. The equation is correct
1b. $A = 18 + 12(N - 1) = 18 + 12(15 - 1) = 18 + 12(14) = 186$ sq units
1c.
$246 = 18 + 12(N - 1)$
$246 = 18 + 12N - 12$
$246 = 12N + 6$
$240 = 12N$
$20 = N$
The stack contains 20 rods.

4a. Answers will vary. One example is $A=2[12(N+1) + N] + 2(N-1)$. An equivalent expression is $A=2[(12N+12) + N] + 2(N-1) = 2[13N+12] + 2N - 2$

$A = 26N + 24 + 2N - 2 = 28N + 22$

4b.
$A = 28N + 22$
$162 = 28N + 22$
$140 = 28N$
$5 = N$

There are 5 rods in the stack.

4c. volume $= 12(N)(1) = 12N$