Middle School Mathematics
A Guide to the Connected Mathematics™ Series

Shapes and Designs

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1 Introduction

This guide supports the Connected Mathematics™ student textbook ***Shapes and Designs***. This book is in the Geometry strand. Its primary topic is properties of polygons.

2 Goals/Objectives

This unit will help students:

- Acquire knowledge of important properties of polygons, recognize them in and out of the classroom, and describe decorative and structural applications in which polygons appear.
- Explain the property of a triangle that makes it useful as a stable structure.
- Explain the side and angle relationships that make parallelograms useful for a variety of designs and uses.
- Estimate the size of any angle using reference to a right angle and other benchmark angles.
- Use an angle rule for making accurate angle measurements.
- Develop a variety of strategies for solving problems involving polygons and their properties.

3 Vocabulary

The following words and concepts are used in this unit. The concepts in the left column are those essential for student understanding in this and future units. The Descriptive Glossary in the student text gives definitions for many of these words.

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Essential Terms	Non Essential Terms
angle	angle ruler
degree	benchmark
hexagon	central angle
octagon	decagon
parallelogram	diagonal
pentagon	equilateral triangle
polygon	heptagon
quadrilateral	interior angle
rectangle	irregular polygon
regular polygon	isosceles triangle
right angle	line symmetry
side	property
square	tiling
symmetry	trapezoid
triangle	turn symmetry
vertex	wedge

4 Summary of Investigations

4.1 Investigation 1 – Bees and Polygons

This investigation focuses on the key question: "Which polygons can be used to cover a plane (flat surface) completely?" Students begin with making a conjecture about why honeycombs are made with hexagons and use physical materials to explore various other possibilities.

Students discover that squares, triangles and hexagons can be used alone to cover a surface. Later, students learn that this is because the angle measures of these shapes are factors of 360. Examples of combinations that work to cover a surface without gaps are octagons and squares; hexagons and triangles; squares and triangles.

4.2 Investigation 2 – Building Polygons

This investigation centers around the question "Is the shape of a polygon determined exactly by the lengths of its sides and the order in which those sides are connected?" Three problems explore this question for triangles, quadrilaterals and parallelograms.

Students discover that to create a triangle, the sum of the lengths of any two sides must be greater than the length of its third side. Triangles are stable –

meaning that there is at most one triangle that can be created from any three side lengths.

Students discover that to create a quadrilateral, the sum of the lengths of any three sides must be greater than the length of its fourth side. Quadrilaterals are not stable, meaning that given any four side lengths, if a quadrilateral can be created, it is true that more than one unique quadrilateral can also be made from those same side lengths by stretching and shrinking the size of the angles, and by re-ordering the sides.

Students discover that to create a parallelogram, the sum of the lengths of any three sides must be greater than the length of its fourth side and that the angle measures must remain constant, or its sides will no longer be parallel. The sides have to be ordered such that the opposite sides are the same length.

4.3 Investigation 3 – Polygons and Angles

This investigation introduces students to three ways to conceptualize angles and the theory behind angle measurement. It give students practice estimating angles using benchmarks as well as accurately measuring with an angle ruler.

4.4 Investigation 4 – Polygons Properties and Tiling

The investigation focuses on basic properties of familiar quadrilaterals, using tiling as a context.

4.5 Investigation 5 – Side-Angle-Shape Connections

Students look at what changes and what remains constant as squares, rectangles and parallelograms are rotated and flipped. The symmetries of the figures is realized and comprehended.

5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Questions, page 11:

4. East Lansing: rectangle; stop: Octagon; School Zone: pentagon; Yield: triangle; Road Construction: square or rhombus.

5.2 Investigation 2

ACE questions, page 19:

1. The only possibility is a 5-5-3 isosceles triangle.
2. The only possibility is a 8-8-8 equilateral triangle.
4. These sides will make a quadrilateral with sides with order 5-5-6-14 or 5-6-5-14.

5.3 Investigation 3

ACE Questions, page 35:

13. 90 degrees
14. 180 degrees
15. 270 degrees
16. 45 degrees
25. f
26. e

5.4 Investigation 4

ACE Questions, page 47.

1a. 1440 degrees
1b. 144 degrees
2. 60 degrees
4. 65 degrees
5. 120 degrees

5.5 Investigation 5

ACE Questions, page 57.

2a. The shortest path from A to C along city streets is 325 meters.
2b. The shortest path from B to D along city streets is 325 meters.
2c. The flight from A to C is shorter than the flight from B to D.