Middle School Mathematics
A Guide to the Connected Mathematics™ Series

Variables and Patterns

Prepared by members of
the Readington Middle
School math department
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1 Introduction

Variables and Patterns, the first unit of the Connected Mathematics **algebra** strand, develops students' ability to explore a variety of situations in which changes occur. They learn to observe, describe, and record changes, which is the first step in analyzing and searching for **patterns** in real-world situations. The setting for many of the investigations in this unit is the formation of a company that arranges bike tours.

The relationship between two variables – in particular, the way one variable changes in relation to another – is an important idea which is central to this unit. Methods for representing these relationships and patterns of change using verbal descriptions, tables, and graphs are developed. Towards the end of the unit, written and symbolic rules are introduced.

Each representation has its advantages and disadvantages in promoting understanding of relationships and patterns of change. It is important for students to move freely among the various representations.

By the end of the unit, students should feel comfortable with tables and graphs and with some simple symbolic rules.

2 Goals/Objectives

This unit will help students:

- Understand that variables in a situation are those quantities that change, such as time, temperature, feelings, a TV show's popularity, distance traveled, and speed.
- Understand that patterns describe a regular or predictable change in data.
- Search for patterns of change that show relationships among the variables.
- Select an appropriate range of values for the variables.
- Create tables, graphs, and simple symbolic rules that describe the patterns of change.
- Understand the relationships among forms of representation — words, tables, graphs, and symbolic rules.
- Make decisions using tables, graphs, and rules.
- Use a graphing calculator for making tables and graphs to find information about a situation.

3 Vocabulary

The following words and concepts are used in this unit. All students are expected to understand these words and use them appropriately to communicate mathematical ideas. The Descriptive Glossary in the student text gives definitions for many of these words. There are other words that may arise during each

student's course of study. These words will be identified by the teacher and added to the set of vocabulary for which the student is responsible.

Change	Area
Coordinate graph	Circumference
Coordinate pair	Diameter
Distance/time/rate of speed	Line plot
Income/cost/profit	Mean
Pattern	Median
Relationship	Mode
Rule	Perimeter
Scale	Polygon
Table	Radius
Variable	Symbolic form
X -axis	
X -coordinate	
Y -axis	
Y -coordinate	

4 Summary of Investigations

4.1 Investigation 1 – Variables and Coordinate Graphs

Students conduct a jumping jack experiment to explore what happens to a person's ability to perform (endurance) after exerting energy over a period of time. As they graph their jumping jack data, students learn about variables, coordinate axes, choosing appropriate scales for graphs and plotting data points. Then they begin to make interpretations from their graphs.

4.2 Investigation 2 – Graphing Change

Students look at data collected over a five-day trial run of a bike tour. They learn to examine data and create summary reports, tables, and graphs that show the relationship between distance and time (or speed and time for the fifth day). Students learn more about interpreting tables and graphs, including identifying and explaining changes in intervals and any interesting points, such as maximum and minimum points. They also examine what it means to connect a graph's data points in various ways, such as with straight-line segments and curved-line segments. They also learn appropriate situations in which they would connect the points. The advantages and disadvantages of using verbal descriptions, tables, or graphs to represent a situation are explored. Students understand the relationships among these representations and move freely among them.

4.3 Investigation 3 – Analyzing Graphs and Tables

Students consider business questions involved with running bicycle tours. They learn to make and interpret graphs and tables for a variety of situations such as:

- Number of customers
- Cost
- Income
- Profit

Students analyze and compare information presented in tables and graphs to make good business decisions. The patterns of change are more regular, and students are encouraged to describe these patterns in words. This is a precursor to the next investigation, in which students use symbolic rules to describe the patterns.

4.4 Investigation 4 – Patterns and Rules

Students create and analyze tables and graphs of situations involving distance, rate and time to find a pattern that relates the distance and time for a given rate. They are then asked to express this pattern as a rule, first in words and then in symbols. Students also compare the graphs, tables and symbolic rules for situations of the form $d = rt$ *(distance = rate x time)* for various rates. The objective is for students to develop a deeper understanding of the relationships among the various representations, including symbols, and use them all with ease.

4.5 Investigation 5 – Using a Graphing Calculator

Students use graphing calculators to make tables and graphs. They begin by exploring the shapes of graphs, then describing the similarities and differences of rules.

The graphing calculator is a tool which allows students to look at many examples quickly and helps them observe patterns and make conjectures about functions.

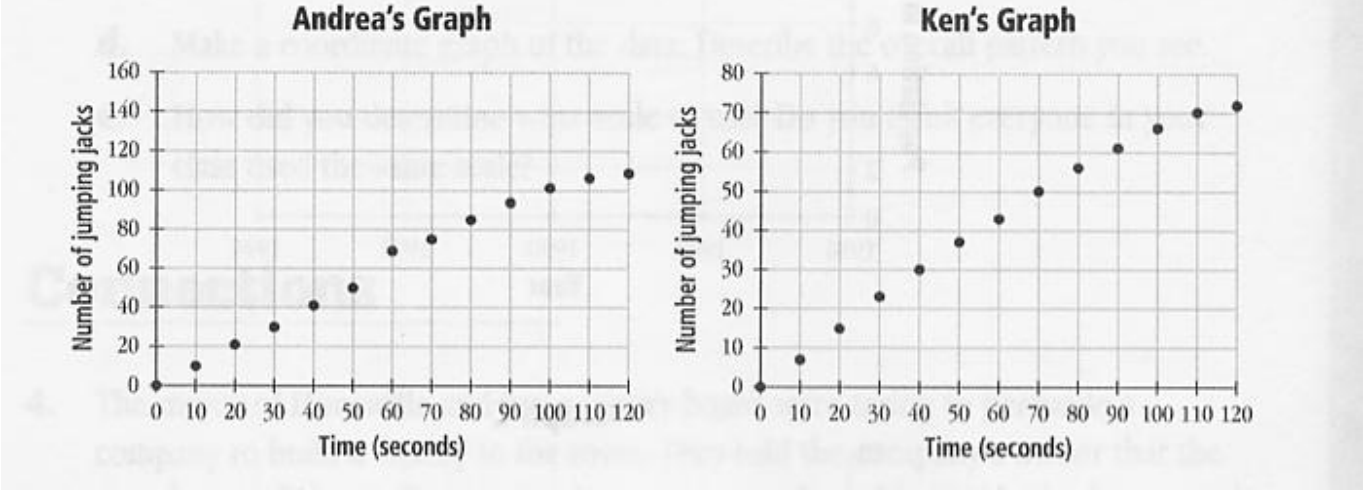
5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Question 5, page 14.

5. After doing the jumping jack experiment, Andrea and Ken compared their graphs. Because his points were higher, Ken said he did more jumping jacks than Andrea in the 120 seconds. Do you agree? Why or why not?



ANSWER

Although Ken's points look higher, on the two graphs. He did NOT do more jumping jacks in 120 seconds. The scales are different. Andrea did about 110 jumping jacks in 120 seconds. Ken only did about 72.

5.2 Investigation 2

ACE question 5, page 29.
ANSWER
5a. The speed never changes.

5b. Answers will vary. The graph is not reasonable for the cyclist or the wind. A rider's speed can be affected by fatigue or environmental factors such as temperature, wind speed and direction, and terrain. A van could travel close to a constant speed on level surface with cruise control set. The wind usually comes in gusts. It is not likely that it would remain constant over a long period of time.

5.3 Investigation 3

ACE Question 3, pages 42-43

ANSWER

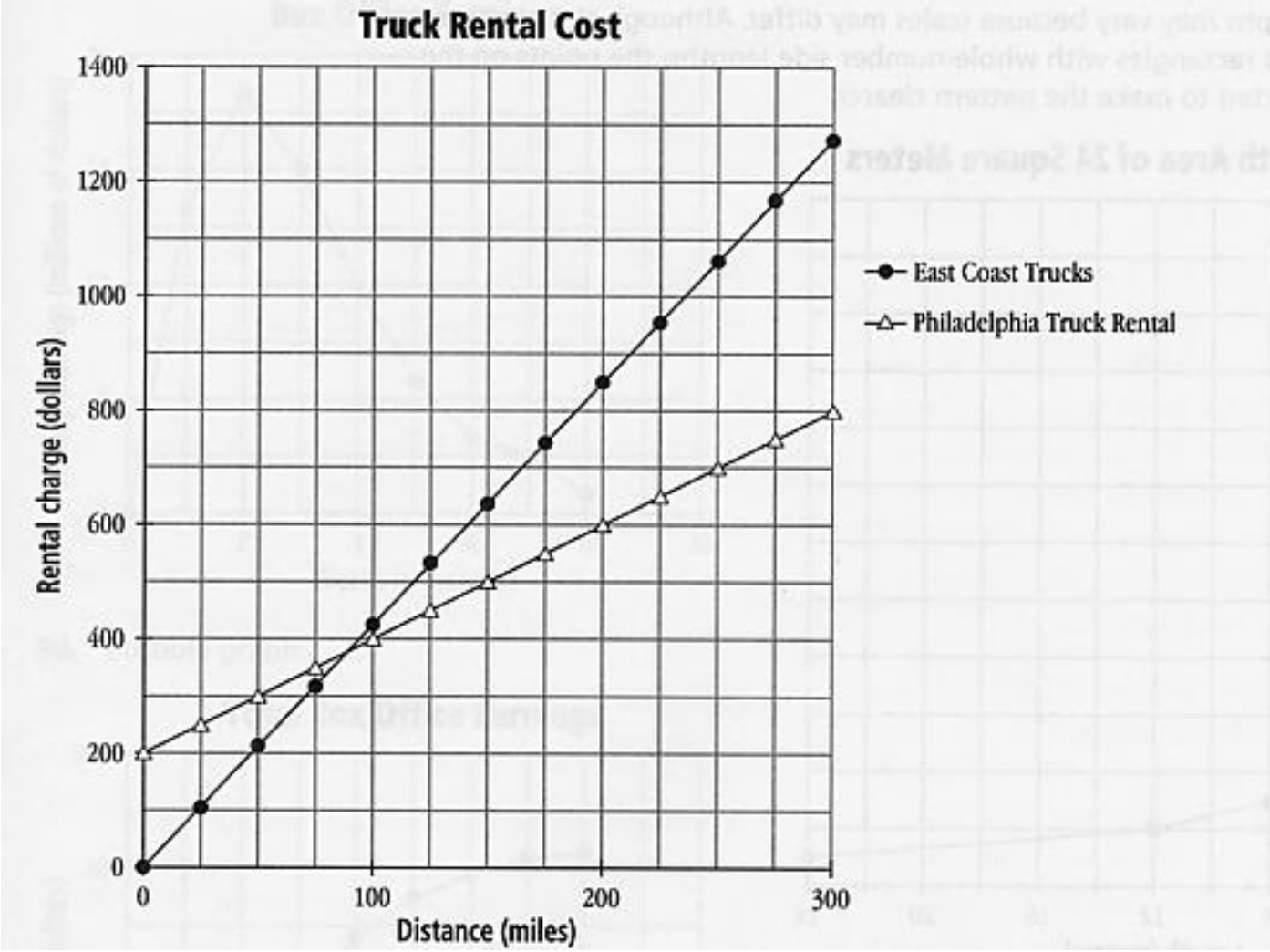
3a. The fees in the table are rounded to the nearest dollar.

Distance (miles)	Fee ($)
0	0
25	106
50	213
75	319
100	425
125	531
150	638
175	744
200	850
225	956
250	1063
275	1169
300	1275

3b. The fees in the table are rounded to the nearest dollar.

Distance (miles)	Fee ($)
0	200
25	250
50	300
75	350
100	400
125	450
150	500
175	550
200	600
225	650
250	700
275	750
300	800

3c. Appearance of graphs may vary because scales may differ.



3d. If the truck is driven for less than about 89 miles, East Coast Trucks is the firm to use. If it is driven more than 89 miles, Philadelphia Truck Rental is the better choice.

5.4 Investigation 4

ACE Question 9 page 57

9a. 2.5 feet

9b. The bike will travel 3.14 x 5 = 15.7 feet in one turn, so in 100 turns it will travel 1570 feet.

9c. Since 3 miles =15,840 feet, it will take 15,840 / 15.7 = about 1008.9 turns.

9d. The big wheel can go three time as far for the same number of turns because its diameter is three times the diameter of Masako's wheel (20 in. = 1 2/3 feet compared with 5 ft).

5.5 Investigation 5

ACE Question 3, page 65.

ANSWER

3a. $B = 7A$

A	0	1	2	3	4	8	20	100
B	0	7	14	21	28	**56**	**140**	**700**

3b. $Y = X + 6$

X	0	1	2	3	4	8	20	100
Y	6	7	8	9	**10**	**14**	**26**	**106**

3c. $Y = 2X + 1$

X	0	1	2	3	4	8	20	100
Y	1	3	5	7	**9**	**17**	**41**	**201**

3d. $S = R^2$

R	0	1	2	3	4	6	10	20
S	0	1	4	9	16	**36**	**100**	**400**