Middle School Mathematics
A Guide to the Connected Mathematics™ Series

What Do You Expect?

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1 Introduction

This unit is part of the **Data Analysis and Probability** strand. Students will deepen their understanding of the basic concepts of probability, through work on this unit, ***What Do You Expect?*** They will develop several powerful strategies for finding and interpreting experimental and theoretical probabilities, such as unit simulations to gather experimental data; using counting trees and other listing techniques to determine all the possible outcomes in a situation; and creating area models in which the probability of each possible outcome is represented as portion of the whole.
In addition students will use area models to determine probability in two-stage situations and to informally develop an understanding of the concept of expected value, or long term average.
The work in this unit assumes that students are familiar with the basic ideas of probability that are presented in the unit, ***How Likely Is It?*** Investigations 1 and 2 in this unit provide a brief review of the basics.

2 Goals/Objectives

This unit will help students:

- Review and come to a deeper understanding of experimental and theoretical probabilities and the relationship between them
- Review and further develop an understanding of the possible outcomes in a situation
- Review and come to a deeper understanding of the distinction between equally likely and non-equally likely outcomes
- Understand the distinction between single, specific outcomes and sets of outcomes that comprise an event
- Analyze situations involving independent events
- Analyze situations involving dependent events
- Understand how to use probabilities and equivalent fractions to find expected value
- Determine whether games of chance are fair or unfair and find ways to make unfair games fair
- Develop strategies for analyzing probabilities such as using lists, counting trees, and area models
- Use counting trees for finding theoretical probabilities in binomial, or 50-50 probability situations
- Determine the expected value of a chance situation
- Use probability and expected value to make decisions
- Find probabilities in situations that involve drawing with and without replacement

3 Vocabulary

The following words and concepts are used in this unit. The concepts in the left column are those essential for student understanding in this and future units. The Descriptive Glossary (page 72) in the student text gives definitions for many of these words.

Essential Terms	Terms developed in previous units	Non Essential Terms
counting tree	Equally Likely	Payoff
expected value, long-term average	Event	Law of Large Numbers
	Experimental probability	
	Fair game	
	Outcome	
	Probability	
	Random	
	Theoretical Probability	

4 Summary of Investigations

4.1 Investigation 1 – Evaluating Games of Chance (pp 5-21)

- Students' natural interest in exploring the fairness of games is highlighted in this investigation.
- Students analyze several games of chance to review and reinforce the basic ideas of probability, which they are already familiar with.
- Counting trees and other types of lists are used to determine all possible outcomes.

4.2 Investigation 2 – Analyzing Number Cube Games (pp 22-31)

- Explore rolling two number cubes and compute sums and products.
- Review basic concepts of probability.
    - Find experimental probabilities,
    - List outcomes to find theoretical probabilities
    - Compare experimental and theoretical probabilities
    - Decide whether a game is fair.

4.3 Investigation 3 – Probability and Area (pp 32-40)

- Relate probabilities to the area of a grid.
- Think about the concepts of equally likely versus non-equally likely outcomes.

4.4 Investigation 4 – Analyzing Two-Stage Games (pp 41-49)

- Encounter probability situations in which one event depends on another.
- Use the area model to analyze two or more dependent events.

4.5 Investigation 5 – Expected Value (pp 50-58)

- Use spinners to simulate free-throw situations in basketball.
- Employ area models to find the probabilities involved with players who have different free-throw averages.
- Learn about the concept of expected value, or long-term average.

4.6 Investigation 6 – Carnival Games (pp 59-68)

- Analyze a variety of proposed carnival games.
- Apply and extend strategies and ideas.
- Develop techniques for conducting simulations.
- Analyze situations to find all possible outcomes.
- Find expected value to determine profitability of a game.
- Confront the issue of how the dependence of the outcome of one action on another action affects the probabilities in a situation.

4.7 Investigation 7 – Analyzing Sequences of Outcomes (pp 69-78)

- Investigate two different problems that are essentially the same mathematically-binomial event applications.
    - Predict the sex of puppies in a litter .
    - Guess on a true-false quiz.
- Use counting trees to analyze sequences of equally likely outcomes.

5 Sample Problems and Solutions

This section provides solutions for selected ACE questions for each investigation.

5.1 Investigation 1

ACE Question 18.

ANSWER
Red: 35% is 21 marbles; blue: 25% of 60 is 15 marbles; white: 60 – 36 is 24 marbles.

5.2 Investigation 2

ACE question 1.

ANSWER
3/36 or 1/12

5.3 Investigation 3

ACE Question 2

ANSWER
2a. P(A) = ¼; P(B)=1/4; P(C)=1/4;P(D)=1/4
2b. One possible answer - Assign each region 1 point.

5.4 Investigation 4

ACE Question 1

ANSWER
1a. A square can be divided to show the probability of Zark ending in each room. The theoretical probability of ending in room A is 1/6 + 1/6 +1/8 +1/8 = 7/12 and in room B is 1/6 + ¼ = 5/12
1b. If you played the game 100 times, you could expect Zark to end in room A 100 X 7/12 = about 58 times and in room B 100x 5/12 = about 42 times.

5.5 Investigation 5

ACE Question 7.

ANSWER
7a. Each section is equally likely, so there is a chance of 1 in 10, or 0%, chance of landing on bankrupt.
7b. Three of the 10 sections have a value of $500 or more, so the probability of getting at least $500 on one spin is 3/10 or 30%.
7c. It is still 10%. Each player has 10% chance of hitting $350 one each spin.

5.6 Investigation 6

ACE Question 1, page 64.

ANSWER
1. Of the 36 possible outcomes when a player rolls two number cubes, 6 are matches, so the probability of winning is 1/6. If a player plays 20 times, he/she will pay 20 tickets and could expect to win 20 x 1/ 6 = about 3.33 times, the player could expect to win about 16.67 tickets – an overall loss of about 3.33 tickets.

5.7 Investigation 7

ACE Question 3, page 74.
ANSWER
3. Since a male and a female are equally likely outcomes for each puppy in the litter, there is still an expected value of ½ x @250 + ½ x $200 = $225 for each puppy. He can expect to make 5 x $225 = $1125 on a litter of five puppies. (Another way to solve the problem is to make a list of the possible outcomes and look at each case.)