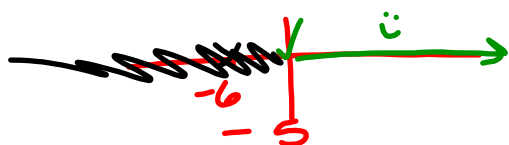


Find the domain of the function: $f(x) = \sqrt{2x+10} - 4$

- a. $(-\infty, -5] \cup [-4, \infty)$ b. $(-5, -\infty)$ c. $[-5, \infty)$ d. $[-4, \infty)$



Condense the logarithmic expression to a single quantity:

$$2 \log_3 x + \log_3 y - \frac{1}{3} \log_3 z$$

- a. $\log_3 \left(\frac{2xy}{3z} \right)$ b. $\log_3 \left(\frac{x^2 y}{\sqrt[3]{z}} \right)$ c. $\log \left(\frac{x^2 y}{\sqrt[3]{z}} \right)$ d. $\log_3 \sqrt[3]{\frac{x^2 y}{z}}$

$$\log_3 x^2 + \log_3 y - \log_3 z^{\frac{1}{3}}$$

$$\log_3 (x^2 \cdot y) - \log_3 \sqrt[3]{z}$$

$$\frac{x^2 \cdot y}{\sqrt[3]{z}}$$

Use these tables. What is the value of $f(g(0))$?

x	$f(x)$
-3	16
-2	9
-1	4
0	1
1	0
2	1
3	4

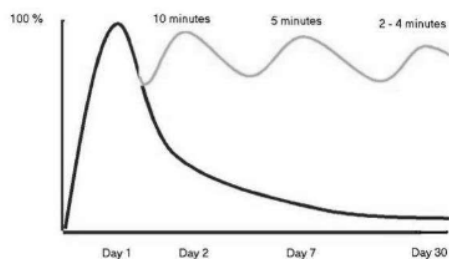
x	$g(x)$
-3	2
-2	1
-1	0
0	-1
1	-2
2	-3
3	-4

$$g(0) = -1$$

$$f(-1)$$

- A. 1
B. -1

- C. -2
D. 4



This is the Curve of Forgetting (AVID). This trend says if we do NOT revisit what we have learned within 24 hours of learning it, we will forget 50-80% of the new information.

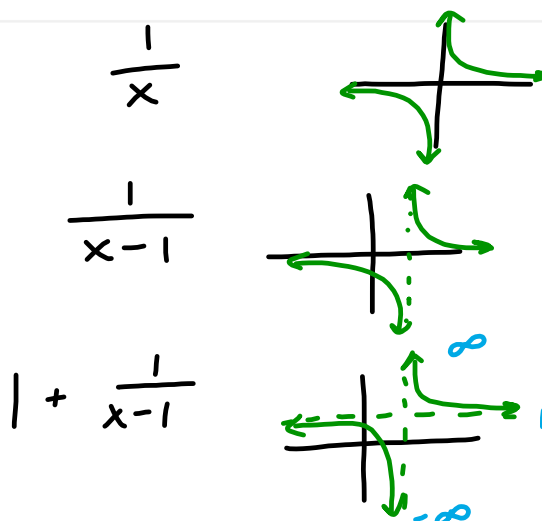
Our brain is always taking in new information. If we do not revisit the new information in 24 hours, our brain will dump this information, because it is scrap, and replace it with new. Just reviewing my schoology notes for 20 minutes a day is a great way to retain what you learn daily.

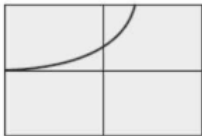
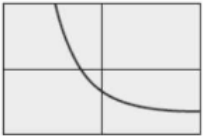
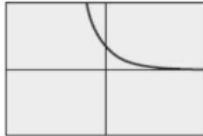
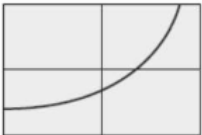
The following pages contain the functions you need to know, some general terms, formulas, or concepts, as well as practice problems. Please work on this packet throughout the summer and bring it COMPLETED on the first day of school in Fall 2019.

Calculus Prerequisite Assignment

Directions: It is expected that you answer each of these questions without a calculator! Simplify each expression:

1) True or False: The function $f(x) = x^4 + x^2 + x$ is an even function. PROVE! NOT EVEN! $f(a) = a^4 + a^2 + a$ $f(-a) = (-a)^4 + (-a)^2 + (-a)$ $a^4 + a^2 - a$	2) True or False: The function $f(x) = x^{-3}$ is an odd function. PROVE! $f(x) = \frac{1}{x^3}$ $f(-a) = \frac{1}{(-a)^3}$ $f(a) = \frac{1}{a^3}$ YES $-\frac{1}{a^3} = -\frac{1}{a^3}$
3) $\frac{(9x^2 - 3x - 2)(3x^2 - 10x - 8)}{(9x^2 - 4)(27x^3 + 1)} =$ ~~$(3x-2)(3x+1)$~~ ~~$(3x+2)(x-4)$~~ ~~$(3x-2)(3x+2)$~~ ~~$(3x+1)(9x^2-3x+1)$~~ $\frac{x-4}{9x^2-3x+1}$	4) Which of the following gives the domain of $f(x) = \frac{x}{\sqrt{9-x^2}}$? (A) $x \neq \pm 3$ (B) $(-3, 3)$ (C) $[-3, 3]$ (D) $(-\infty, -3) \cup (3, \infty)$ (E) $(3, \infty)$
5) $1 - \frac{4}{x} = \frac{5}{6}$ $-\frac{4}{x} = -\frac{1}{6}$ $\frac{x}{4} = \frac{6}{1}$ $x = 24$	6) Which of the following gives the range of $f(x) = 1 + \frac{1}{x-1}$? (A) $(-\infty, 1) \cup (1, \infty)$ (B) $x \neq 1$ (C) all real numbers (D) $(-\infty, 0) \cup (0, \infty)$ (E) $x \neq 0$



7) Solve: $x^4 - 10x^2 + 29 = 8$ $x^4 - 10x^2 + 21 = 0$ $(x^2 - 7)(x^2 - 3) = 0$ $x^2 - 7 = 0 \Rightarrow x = \pm\sqrt{7}$ $x^2 - 3 = 0 \Rightarrow x = \pm\sqrt{3}$	8) Solve: $2x^3 - 10x^2 - 48x = 0$ $2x(x^2 - 5x - 24) = 0$ $2x(x - 8)(x + 3) = 0$ $x = 0, x = 8, x = -3$		
9) If $f(x) = 2x - 1$ and $g(x) = x + 3$, which of the following gives $f(g(2))$? a) 2 b) 6 c) 7 d) 9 e) 10 $g(2) = 2 + 3 = 5$ $f(5) = 2(5) - 1 = 9$	10) The length L of a rectangle is twice as long as its width W. Which of the following gives the area A of the rectangle as a function of its width? $2W = L$ $A = L \cdot W = 2W \cdot W$ a) $A(W) = 3W$ b) $A(W) = \frac{1}{2}W^2$ c) $A(W) = 2W^2$ d) $A(W) = W^2 + 2W$ e) $A(W) = W^2 - 2W$		
Match the function with its graph. Do not use a graphing calculator.			
11) $y = 2^x$ <u>A</u> 14) $y = -0.5^{-x}$ <u>C</u>	12) $y = 3^{-x}$ <u>D</u> 15) $y = 2^{-x} - 2$ <u>B</u> 13) $y = -3^{-x}$ <u>E</u> 16) $y = 1.5^x - 2$ <u>F</u>		
 (a)	 (b)	 (c)	 (d)
 (e)	 (f)		
Exponential and Natural Log Functions: The natural log and exponential function ($\ln$ and e^x) are commonly used functions in calculus. Some of these items you need to know by heart:			
17) $\ln e = 1$ 19) $\ln 1 = 0$	18) $e^{\ln x} = x$ 20) $\ln e^x = x$		
<u>Write out the rules for logarithms</u> $\ln(a) + \ln(b) = \ln(a \cdot b)$ $\ln(a) - \ln(b) = \ln\left(\frac{a}{b}\right)$ $a \cdot \ln(b) = \ln(b^a)$			

You also need to be able to manipulate natural log and exponential equations. This means you must be able to solve them. Solve for x:

21) $32 = 8e^x$

$$4 = e^x$$

$$\ln(4) = \ln(e^x)$$

$$x = \ln(4)$$

22) $\ln 35^x = 9$

$$x \cdot \ln(35) = 9$$

$$x = \frac{9}{\ln 35}$$

23) $8 \ln 2x = 48$

$$\ln(2x) = 6$$

$$2x = e^6$$

$$x = \frac{e^6}{2}$$

24) $16e^{4x} = 80$

$$\ln(e^{4x}) = \ln(5)$$

$$4x = \ln(5)$$

$$x = \frac{\ln(5)}{4}$$

25) $\ln(1) - 4x^2 = \ln\left(\frac{1}{e}\right)$

$$-4x^2 = -1$$

$$x^2 = \frac{1}{4}$$

$$\ln(e^{-1})$$

$$-1 \cdot \ln(e) = -1$$

$$x = \frac{1}{2}, -\frac{1}{2}$$

26) $2 - \ln(7-x) = 0$

$$-\ln(7-x) = -2$$

$$\ln(7-x) = 2$$

$$7-x = e^2$$

$$x = 7 - e^2$$

27) $\frac{27}{1+e^{-x}} = 2$

$$27 = 2 + 2e^{-x}$$

$$25 = 2e^{-x}$$

$$\ln\left(\frac{25}{2}\right) = \ln(e^{-x})$$

$$\ln\left(\frac{25}{2}\right) = -x$$

$$x = -\ln\left(\frac{25}{2}\right)$$

$$\text{or } \ln\left(\frac{2}{25}\right)$$

28) $\frac{3}{4} = \frac{2e^x}{8}$

$$\frac{24}{4} = 2e^x$$

$$3 = e^x$$

$$x = \ln(3)$$

29) **Calculator permitted:** The number of bacteria in a petri dish culture after t hours is

$$B = 100e^{0.693t}$$

a) What is the initial number of bacteria present? **100**

b) How many bacteria are present after 6 hours? **6394**

c) Approximately when will the number of bacteria be 200? Estimate the doubling time of the bacteria.

$$200 = 100e^{0.693t}$$

Doubles in ≈ 1 Hr

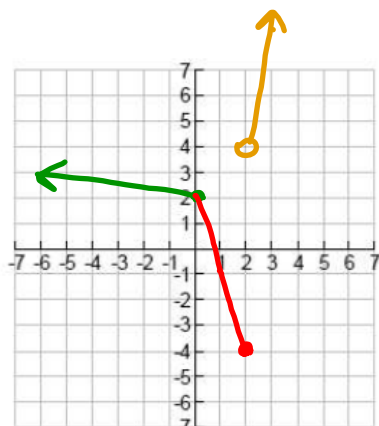
4 | Page

$$\ln 2 = \ln e^{0.693t}$$

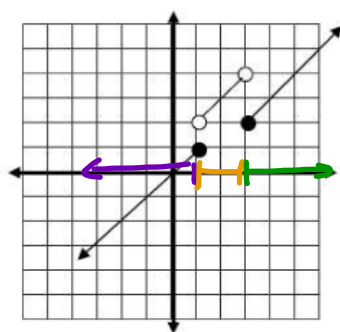
$$\ln(2) = 0.693t$$

$$t = 1.0002$$

30) Sketch the graph of $f(x) = \begin{cases} \sqrt{4-x} & x \leq 0 \\ 2-3x & 0 < x \leq 2 \\ x^2 & x > 2 \end{cases}$



31) For the graphed piecewise function below, state the equation and its domain.



$$f(x) = \begin{cases} x & x \leq 1 \\ x+1 & 1 < x < 3 \\ x-1 & x \geq 3 \end{cases}$$

Trigonometry Practice

You should know your unit circle coordinates like the back of your hand. It should not be a guess – you need to KNOW it!

Answer the following and commit to memory! Find domains and ranges. No calculator!

32) $y = \sin x$ D: $\mathbb{R}$ R: $[-1, 1]$

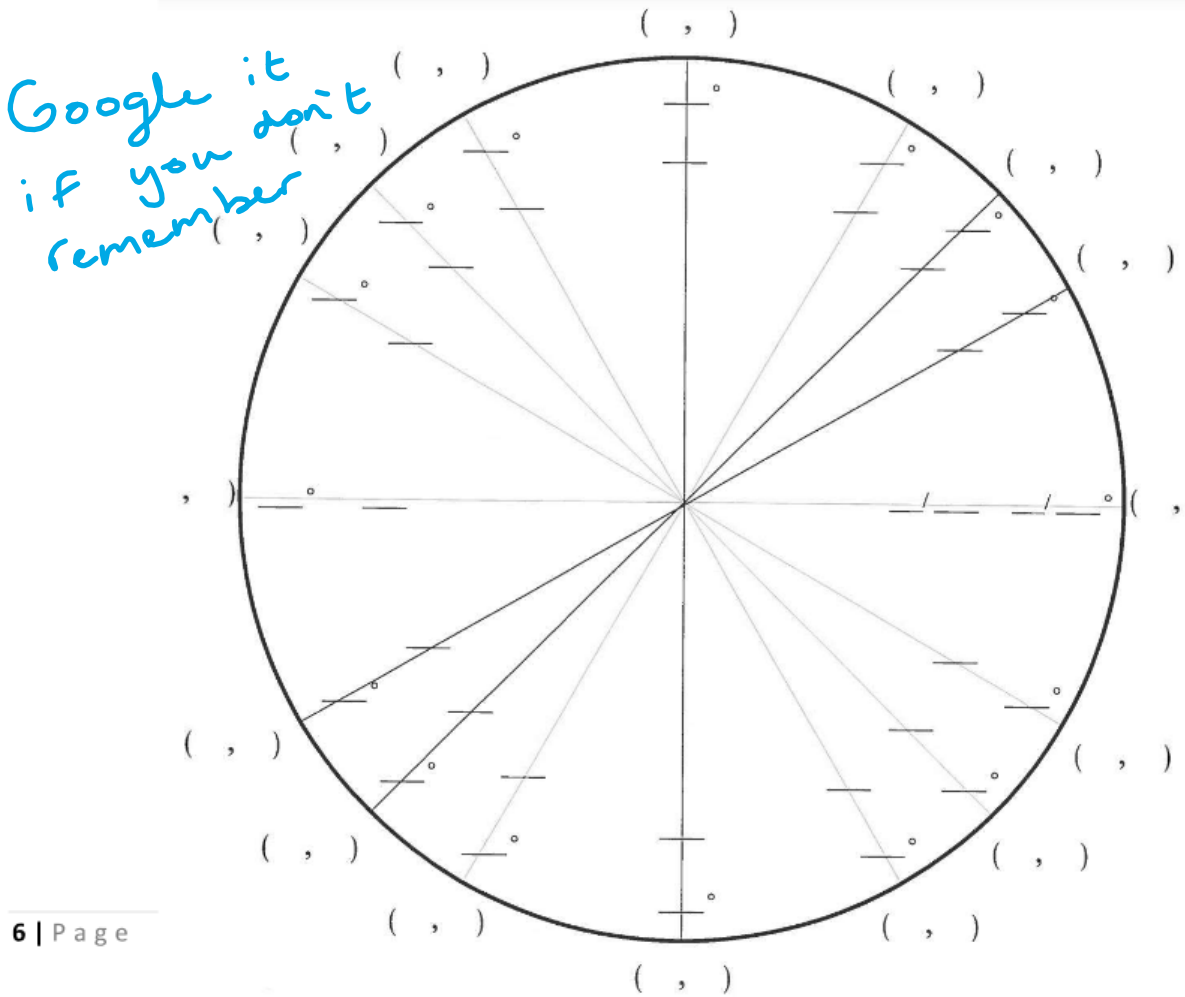
33) $y = \cos x$ D: $\mathbb{R}$ R: $[-1, 1]$

34) $y = \tan x$ Range: $\mathbb{R}$ Undefined at: $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{5\pi}{2}, \dots$

Give the values – no calculator.

35) $\sin\left(\frac{\pi}{2}\right) = 1$	36) $\cos\left(\frac{\pi}{3}\right) = \frac{1}{2}$	37) $\tan\left(\frac{\pi}{4}\right) = 1$	38) $\sin(\pi) = 0$
39) $\sin\left(\frac{\pi}{6}\right) = \frac{1}{2}$	40) $\cos(2\pi) = 1$	41) $\sin\left(\frac{\pi}{3}\right) = \frac{\sqrt{3}}{2}$	42) $\cos\left(\frac{2\pi}{3}\right) = -\frac{1}{2}$
43) $\tan\left(\frac{\pi}{2}\right) = \text{UND}$	44) $\sin\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$	45) $\tan(\pi) = 0$	46) $\cos\left(\frac{3\pi}{4}\right) = -\frac{\sqrt{2}}{2}$
47) $\tan(2\pi) = 0$	48) $\tan\left(\frac{3\pi}{4}\right) = -1$	49) $\cos\left(\frac{3\pi}{2}\right) = 0$	50) $\cos\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2}$
51) $\cos\left(\frac{\pi}{4}\right) = \frac{\sqrt{2}}{2}$	52) $\cos\left(\frac{\pi}{2}\right) = 0$	53) $\cos(\pi) = -1$	54) $\sin(2\pi) = 0$
55) $\sin\left(\frac{2\pi}{3}\right) = \frac{\sqrt{3}}{2}$	56) $\sin\left(\frac{3\pi}{4}\right) = \frac{\sqrt{2}}{2}$	57) $\sin\left(\frac{3\pi}{2}\right) = -1$	58) $\tan\left(\frac{3\pi}{2}\right) = \text{UND}$

Fill out the unit circle. Avoid using a calculator



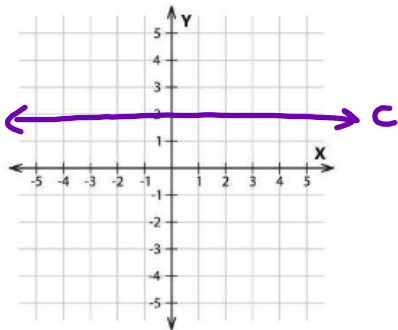
The Radian Measures and Coordinates **MUST** be memorized

Remember: $\sin \theta = \frac{y}{r} = y - \text{coordinate}$, $\cos \theta = \frac{x}{r} = x - \text{coordinate}$, and $\tan \theta = \frac{y}{x} = \frac{y - \text{coordinate}}{x - \text{coordinate}}$

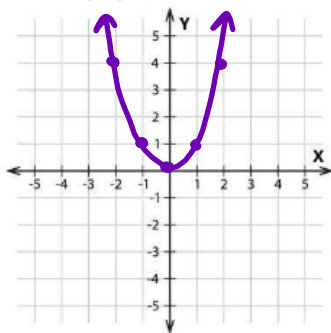
Reciprocal Identities	Quotient Identities	Pythagorean Identities
$\sin x = \frac{1}{\csc x}$ $\csc x = \frac{1}{\sin x}$	$\tan x = \frac{\sin x}{\cos x}$	$\sin^2 x + \cos^2 x = 1$
$\cos x = \frac{1}{\sec x}$ $\sec x = \frac{1}{\cos x}$	$\cot x = \frac{\cos x}{\sin x}$	$\tan^2 x + 1 = \sec^2 x$
$\tan x = \frac{1}{\cot x}$ $\cot x = \frac{1}{\tan x}$		$1 + \cot^2 x = \csc^2 x$

These are the 14 parent functions. Be sure you know their graphs, x and y intercepts, and how their transformations (ex: $f(x) = -4(x-3)^2 - 8$ ← this has stretch of 4, reflected over the x-axis, shift right 3, and shift down 8) changes their parent graph. Graph a sketch of each of the graphs.

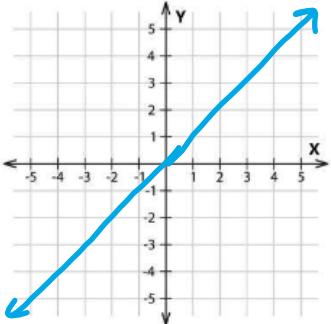
1) $f(x) = c$ (c is a constant, like 4 or -2)



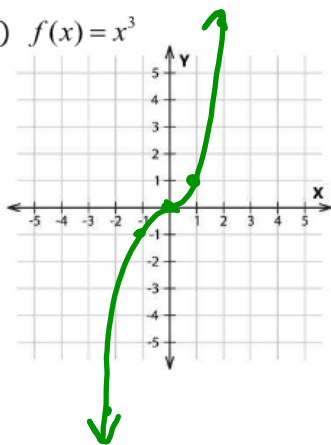
3) $f(x) = x^2$



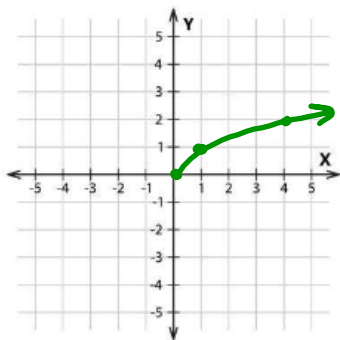
2) $f(x) = x$



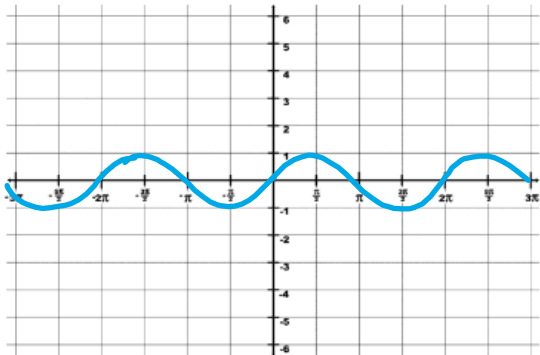
4) $f(x) = x^3$



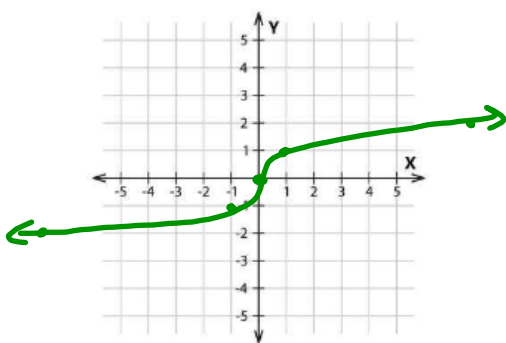
5) $f(x) = \sqrt{x}$



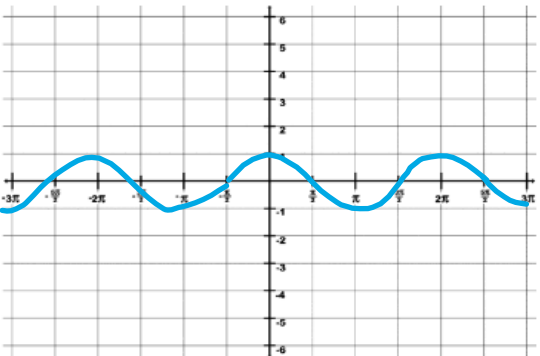
9) $f(x) = \sin x$



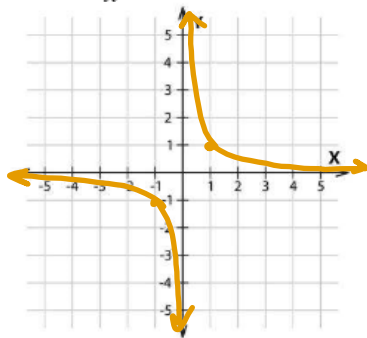
6) $f(x) = \sqrt[3]{x}$



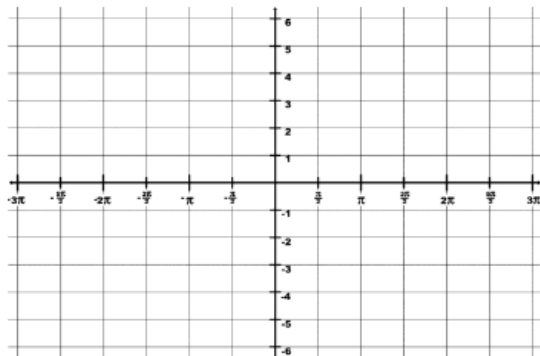
10) $f(x) = \cos x$



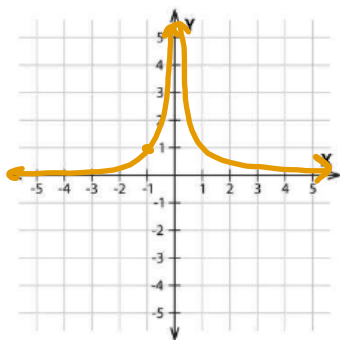
7) $f(x) = \frac{1}{x}$



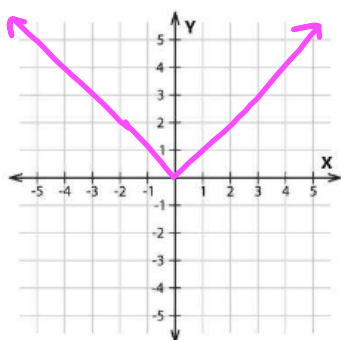
11) $f(x) = \tan x$



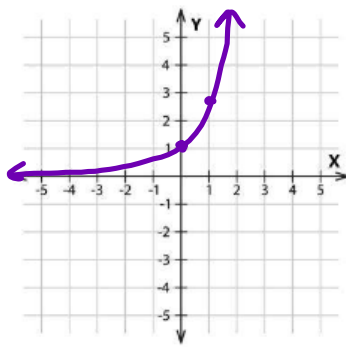
8) $f(x) = \frac{1}{x^2}$



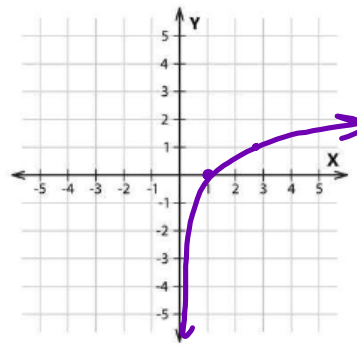
12) $f(x) = |x|$



13) $f(x) = e^x$



14) $f(x) = \ln x$



Please answer the following miscellaneous questions

1. Fifteen students take a test. How would you calculate the average score?

Add all scores then $\div$ by 15

2. A car travels at 55 mph for 4 hours. How far will it travel?

$$55 \times 4 = 220 \text{ mi}$$

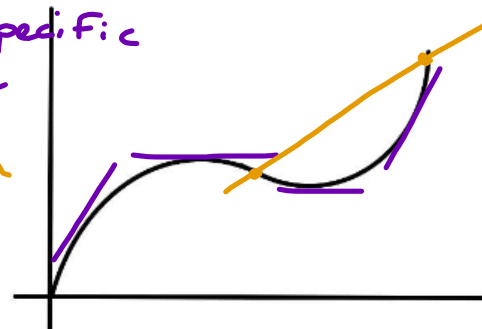
3. Is there a difference between $\frac{4x^3}{7}$ and $\frac{4}{7}x^3$? Explain yes or no

No, multiplying by 4 and $\div 7$ is the same as mult $\frac{4}{7}$

4. What is a tangent line? What is a secant line? (You might have to google this) Draw an example of each one using the curve below.

Touch a graph at 1 specific spot with the same slope

Intersects a graph at 2 points



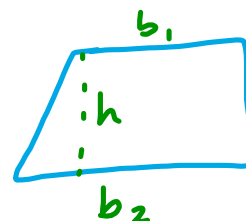
5. What is the formula for the area of the following shapes?

a. Circle πr^2

b. Rectangle $L \cdot W$

c. Triangle $\frac{1}{2}bh$

d. Trapezoid $\frac{1}{2}h(b_1 + b_2)$



6. Given $f(x) = x^2 + 4x - \sqrt{x+3}$ find the following

a. $f(1) = 3$

b. $f(6) = 57$

c. $f(k) = k^2 + 4k - \sqrt{k+3}$

d. $f(3h) = (3h)^2 + 4(3h) - \sqrt{3h+3}$
 $9h^2 + 12h - \sqrt{3h+3}$

e. $f(m-4) = (m-4)^2 + 4(m-4) - \sqrt{m-4+3}$
 $m^2 - 8m + 16 + 4m - 16 - \sqrt{m-1}$
 $m^2 - 4m - \sqrt{m-1}$

f. $f(x+h) = (x+h)^2 + 4(x+h) - \sqrt{x+h+3}$
 $x^2 + 2xh + h^2 + 4x + 4h - \sqrt{x+h+3}$

7. How do you find a horizontal asymptote of a rational function? List the 3 situations that occur.

① Top Deg = Bottom Deg

H.A. $y = \frac{a}{b}$ a & b are coeff.

② Top Deg < Bot Deg
H.A. of $y = 0$

③ Top Deg > Bottom Deg
No H.A.

8. What are the definitions of the following things

a. Velocity: Speed in a direction

b. Acceleration: Change in velocity

c. Speed: How fast something moves $\left(\frac{\text{Distance}}{\text{Time}} \right)$

9. Do the following without a calculator

$$\frac{2}{3} + 1 = \frac{5}{3}$$

$$\frac{2}{3} - 1 = -\frac{1}{3}$$

$$\frac{8}{5} + 1 = \frac{13}{5}$$

$$\frac{3}{5} - 1 = -\frac{2}{5}$$

$$\frac{9}{4} - 1 = \frac{5}{4}$$

10. Evaluate the following exponents

$$3^{-2} = \frac{1}{9}$$

$$8^{-1} = \frac{1}{8}$$

$$9^{\frac{1}{2}} = 3$$

$$4^{\frac{3}{2}} = 8$$

$$\left(\frac{2}{5}\right)^{-2} = \frac{25}{4}$$

$$3^{-2} = \frac{1}{9}$$

$$27^{\frac{2}{3}} = 9$$

$$125^{-\frac{2}{3}} = \frac{1}{25}$$

$$\left(\frac{4}{3}\right)^3 = \frac{64}{27}$$

$$100^{\frac{3}{2}} = 1000$$

$$\left(\frac{25}{49}\right)^{-\frac{1}{2}}$$

$$\downarrow$$

$$\frac{7}{5}$$

$$\left(\frac{1}{144}\right)^{-\frac{3}{2}}$$

$$\downarrow$$

$$1296$$