



**FAIRFIELD  
PUBLIC SCHOOLS**

# Summer Packet for students entering Honors Calculus

Welcome to Honors Calculus. Honors Calculus is a demanding course that relies heavily upon a student's algebra, geometry and precalculus skills. You are expected to have a strong background in the skills reviewed in this packet. Resource links are listed below each section. This packet will be checked for completion and entered as a formative Infinite Campus grade.

**DUE:** 1<sup>st</sup> week of school.

Name \_\_\_\_\_

## COMPLEX FRACTIONS

[Simplifying Complex Rational Expressions \(youtube.com\)](#)

**Simplify each of the following complex fractions.**

$$1. \frac{2 + \frac{2}{x}}{3 - \frac{3}{x^2}}$$

$$2. \frac{x - \frac{1}{3}}{3 - \frac{1}{x}}$$

$$3. \frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}}$$

$$4. \frac{\frac{1}{3} + \frac{1}{3x}}{\frac{1}{x} + \frac{1}{3}}$$

$$5. \frac{\frac{2}{x} - 2}{\frac{1-x}{x}}$$

$$6. \frac{\frac{1}{4x^2} - 1}{2 - \frac{1}{x}}$$

$$7. \frac{2 + \frac{4}{x-2}}{\frac{2}{x-2}}$$

$$8. \frac{\frac{3}{x} - \frac{4}{x^2}}{\frac{2}{x^3} + \frac{5}{x^2}}$$

## FACTORING

[Example 6: Factoring quadratics with two variables | Algebra I | Khan Academy \(youtube.com\)](#)

[Factoring Trinomials The Easy Fast Way - YouTube](#)

[Factor By Grouping Polynomials - 4 Terms, Trinomials - 3 Terms, Algebra 2 \(youtube.com\)](#)

**Factor each of the following completely:**

1.  $3x^2 + 21xy - 54y^2$

7.  $8x^3 + 27$

13.  $1000 + 27a^3$

2.  $3x^2 + 24x + 48$

8.  $x^4 - 1$

14.  $s^3 - 64$

3.  $3m^2 - 18m + 27$

9.  $x^3 + 4x + x^2 + 4$

15.  $y^3 + 125$

4.  $8x^3 - 200x$

10.  $2x^3 + x^2 + 8x + 4$

16.  $3a^3 - 81x^3$

5.  $x^6 - 16x^4$

11.  $15x^3 + 5x^2 + 3x + 1$

17.  $(2x + 3)^3 - y^3$

6.  $4x^3 - 8x^2 - 25x + 50$

12.  $20n^3 + 12n^2 + 25n + 15$

18.  $r^3 + 8b^3$

## RATIONAL EXPRESSIONS

[Rational Expressions , Adding, Subtracting, Multiplying, Dividing, Simplifying Complex Fractions \(youtube.com\)](#)

**Simplify:**

$$1) \frac{x^3 - 9x}{x^2 - 7x + 12}$$

$$2) \frac{x^2 - 2x - 8}{x^3 + x^2 - 2x}$$

$$3) \frac{9 - x^{-2}}{3 + x^{-1}}$$

**Perform the indicated operation on the rational expression**

$$1. \frac{5}{8} - \frac{3}{8x}$$

$$2. \frac{2}{4x+12} + \frac{7}{x+3}$$

$$3. \frac{7}{x+2} - \frac{4}{x-5}$$

$$4. \frac{3}{y} + \frac{2}{y^4}$$

$$5. \frac{5}{4x^2} + \frac{3}{2x}$$

$$6. \frac{2}{x-3} - \frac{1}{x+7}$$

$$7. \frac{7}{3x} - \frac{2}{5}$$

$$8. \frac{3}{2x+6} + \frac{4}{6x+18}$$

$$9. \frac{3}{x+2} + \frac{4}{x-7}$$

$$10. \frac{1}{y+3} + \frac{4}{y^2+4y+3}$$

$$11. \frac{2}{5x} - \frac{3}{10x^3}$$

$$12. \frac{2x+3}{5x-30} - \frac{3x+4}{x-6}$$

## RADICALS AND RATIONAL EXPONENTS

[Fractional exponents with numerators other than 1 | Algebra I | Khan Academy \(youtube.com\)](#)

**REVIEW:**  $n^{\text{th}}$  Root: If  $b^n = a$ , then  $b$  is the  $n^{\text{th}}$  root of  $a$ . This is written  $\sqrt[n]{a} = b$ .  $n$  is called the **index** of the radical.  $a$  is called the **radicand**. Roots as Rational Exponents: The  $n^{\text{th}}$  root,  $\sqrt[n]{a}$ , can be written as an exponent  $a^{\frac{1}{n}}$ .  $a^{\frac{m}{n}} = \left(\sqrt[n]{a}\right)^m$ . Notice the placement of the  $m$  and  $n$ . The root index is the denominator and the exponent is the numerator.

**Evaluate.**

1.)  $16^{\frac{3}{4}}$

2.)  $9^{\frac{3}{2}}$

3.)  $27^{\frac{2}{3}}$

4.)  $125^{\frac{2}{3}}$

5.)  $1^{\frac{1}{3}}$

6.)  $3^{\frac{3}{2}}$

7.)  $4^{\frac{5}{2}}$

8.)  $81^{\frac{3}{2}}$

9.)  $(-64)^{\frac{2}{3}}$

## DOMAIN AND RANGE

[Horizontal and Vertical Asymptotes - Slant / Oblique - Holes - Rational Function - Domain & Range \(youtube.com\)](#)

**Find the domain, vertical and horizontal asymptotes of the functions:**

1)  $y = \frac{x^2+4x}{x^2-16}$

2)  $f(x) = \frac{5x-3}{2x+1}$

3)  $f(x) = \frac{x-3}{x^2-x-6}$

4)  $f(x) = \frac{2x^2-3x-5}{x+1}$

5)  $f(x) = \frac{x^2+9x+20}{x^2+15x+50}$

6)  $f(x) = \frac{3x^2-5x-2}{x^2-x-2}$

# Complex Fractions

$$1. \frac{2 + \frac{2}{x}}{3 - \frac{3}{x^2}} = \frac{\frac{2x}{x} + \frac{2}{x}}{\frac{3x^2}{x^2} - \frac{3}{x^2}} = \frac{\frac{2x+2}{x}}{\frac{3x^2-3}{x^2}} = \frac{2(x+1)}{x} \cdot \frac{x^2}{3(x+1)(x-1)} = \frac{2x}{3(x-1)}$$

$$2. \frac{x - \frac{1}{3}}{3 - \frac{1}{x}} = \frac{\frac{3x}{3} - \frac{1}{3}}{\frac{3x}{x} - \frac{1}{x}} = \frac{\frac{3x-1}{3}}{\frac{3x-1}{x}} = \frac{3x-1}{3} \cdot \frac{x}{3x-1} = \frac{x}{3}$$

$$3. \frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}} = \frac{\frac{5}{5x} - \frac{x}{5x}}{\frac{25}{25x^2} - \frac{x^2}{25x^2}} = \frac{\frac{5-x}{5x}}{\frac{25-x^2}{25x^2}} = \frac{5-x}{5x} \cdot \frac{25x^2}{(5-x)(5+x)} = \frac{5x}{5+x}$$

$$4. \frac{\frac{1}{3} + \frac{1}{3x}}{\frac{1}{x} + \frac{1}{3}} = \frac{\frac{x}{3x} + \frac{1}{3x}}{\frac{3}{3x} + \frac{x}{3x}} = \frac{\frac{x+1}{3x}}{\frac{3+x}{3x}} = \frac{x+1}{3x} \cdot \frac{3x}{3+x} = \frac{x+1}{3+x}$$

$$5. \frac{\frac{2}{x} - 2}{1-x} = \frac{\frac{2}{x} - \frac{2x}{x}}{1-x} = \frac{\frac{2-2x}{x}}{1-x} = \frac{2-2x}{x} \cdot \frac{x}{1-x} = \frac{2(1-x)}{1-x} = 2$$

$$6. \frac{\frac{1}{4x^2} - 1}{2 - \frac{1}{x}} = \frac{\frac{1}{4x^2} - \frac{4x^2}{4x^2}}{\frac{2x}{x} - \frac{1}{x}} = \frac{\frac{1-4x^2}{4x^2}}{\frac{2x-1}{x}} = \frac{(1-2x)(1+2x)}{4x^2} \cdot \frac{x}{2x-1} = \frac{-1(1+2x)}{4x}$$

$$7. \frac{2 + \frac{4}{x-2}}{\frac{2}{x-2}} = \frac{\frac{2(x-2)+4}{x-2}}{\frac{2}{x-2}} = \frac{2x-4+4}{x-2} \cdot \frac{x-2}{2} = \frac{2x}{2} = x$$

$$8. \frac{\frac{3}{x} - \frac{4}{x^2}}{\frac{2}{x^3} + \frac{5}{x^2}} = \frac{\frac{3x}{x^2} - \frac{4}{x^2}}{\frac{2}{x^3} + \frac{5x}{x^3}} = \frac{\frac{3x-4}{x^2}}{\frac{2+5x}{x^3}} = \frac{3x-4}{x^2} \cdot \frac{x^3}{2+5x} = \frac{x(3x-4)}{2+5x}$$

# Factoring

$$\begin{aligned} 1. \quad & 3x^2 + 21xy - 54y^2 \\ & 3(x^2 + 7xy - 18y^2) \\ & 3(x+9y)(x-2y) \end{aligned}$$

$$\begin{aligned} 2. \quad & 3x^2 + 24x + 48 \\ & 3(x^2 + 8x + 16) \\ & 3(x+4)(x+4) \end{aligned}$$

$$\begin{aligned} 3. \quad & 3m^2 - 18m + 27 \\ & 3(m^2 - 6m + 9) \\ & 3(m-3)(m-3) \end{aligned}$$

$$\begin{aligned} 4. \quad & 8x^3 - 200x \\ & 8x(x^2 - 25) \\ & 8x(x-5)(x+5) \end{aligned}$$

$$\begin{aligned} 5. \quad & x^6 - 16x^4 \\ & x^4(x^2 - 16) \\ & x^4(x-4)(x+4) \end{aligned}$$

$$\begin{aligned} 6. \quad & 4x^3 - 8x^2 - 25x + 50 \\ & 4x^2(x-2) - 25(x-2) \\ & (4x^2 - 25)(x-2) \\ & (2x-5)(2x+5)(x-2) \end{aligned}$$

$$\begin{aligned} 7. \quad & 8x^3 + 27 \\ & (2x+3)(4x^2 - 6x + 9) \end{aligned}$$

$$\begin{aligned} 8. \quad & x^4 - 1 \\ & (x^2 - 1)(x^2 + 1) \\ & (x+1)(x-1)(x^2 + 1) \end{aligned}$$

$$\begin{aligned} 9. \quad & x^3 + 4x + x^2 + 4 \\ & x(x^2 + 4) + 1(x^2 + 4) \\ & (x+1)(x^2 + 4) \end{aligned}$$

$$\begin{aligned} 10. \quad & 2x^3 + x^2 + 8x + 4 \\ & x^2(2x+1) + 4(2x+1) \\ & (x^2 + 4)(2x+1) \end{aligned}$$

$$\begin{aligned} 11. \quad & 15x^3 + 5x^2 + 3x + 1 \\ & 5x^2(3x+1) + 1(3x+1) \\ & (5x^2 + 1)(3x+1) \end{aligned}$$

$$\begin{aligned} 12. \quad & 20n^3 + 12n^2 + 25n + 15 \\ & 4n^2(5n+3) + 5(5n+3) \\ & (4n^2 + 5)(5n+3) \end{aligned}$$

# Factoring

$$13. 1000 + 27a^3$$

$$(10 + 3a)(100 - 30a + 9a^2)$$

$$14. s^3 - 64$$

$$(s - 4)(s^2 + 4s + 16)$$

$$15. y^3 + 125$$

$$(y + 5)(y^2 - 5y + 25)$$

$$16. 3a^3 - 81x^3$$

$$3(a^3 - 27x^3)$$

$$3(a - 3x)(a^2 + 3ax + 9x^2)$$

$$17. (2x + 3)^3 - y^3$$

$$(2x + 3 - y)((2x + 3)^2 + (2x + 3)(y) + y^2)$$

$$(2x + 3 - y)(4x^2 + 12x + 9 + 2xy + 3y + y^2)$$

$$18. r^3 + 8b^3$$

$$(r + 2b)(r^2 - 2br + 4b^2)$$

# Rational

$$1. \frac{x^3 - 9x}{x^2 - 7x + 12} = \frac{x(x^2 - 9)}{(x-3)(x-4)} = \frac{x(x+3)\cancel{(x-3)}}{\cancel{(x-3)}(x-4)} = \frac{x(x+3)}{(x-4)}$$

$$2. \frac{x^2 - 2x - 8}{x^3 + x^2 - 2x} = \frac{(x-4)(x+2)}{x(x^2 + x - 2)} = \frac{(x-4)\cancel{(x+2)}}{x\cancel{(x+2)}(x-1)} = \frac{x-4}{x(x-1)}$$

$$3. \frac{9 - x^{-2}}{3 + x^{-1}} = \frac{9 - \frac{1}{x^2}}{3 + \frac{1}{x}} = \frac{\frac{9x^2 - 1}{x^2}}{\frac{3x + 1}{x}} = \frac{(3x-1)\cancel{(3x+1)}}{x^{\cancel{2}}} \cdot \frac{\cancel{x}}{3x+1} = \frac{3x-1}{x}$$

$$1. \frac{5}{8} - \frac{3}{8x} = \frac{5x}{8x} - \frac{3}{8x} = \frac{5x-3}{8x}$$

$$2. \frac{2}{4x+12} + \frac{7}{x+3} = \frac{2}{4x+12} + \frac{7 \cdot 4}{4(x+3)} = \frac{2+28}{4(x+3)} = \frac{30}{4(x+3)} = \frac{15}{2(x+3)}$$

$$3. \frac{7}{x+2} - \frac{4}{x-5} = \frac{7(x-5) - 4(x+2)}{(x+2)(x-5)} = \frac{7x-35-4x-8}{(x+2)(x-5)} = \frac{3x-43}{(x+2)(x-5)}$$

$$4. \frac{3}{y} + \frac{2}{y^4} = \frac{3y^3}{y^4} + \frac{2}{y^4} = \frac{3y^3+2}{y^4}$$

$$5. \frac{5}{4x^2} + \frac{3}{2x} = \frac{5}{4x^2} + \frac{6x}{4x^2} = \frac{5+6x}{4x^2}$$

$$6. \frac{2}{x-3} - \frac{1}{x+7} = \frac{2(x+7) - 1(x-3)}{(x-3)(x+7)} = \frac{2x+14-x+3}{(x-3)(x+7)} = \frac{x+17}{(x-3)(x+7)}$$

$$7. \frac{7}{3x} - \frac{2}{5} = \frac{7(5) - 2(3x)}{5(3x)} = \frac{35 - 6x}{15x}$$

$$8. \frac{3}{2x+6} + \frac{4}{6x+18} = \frac{3(3)}{6(x+3)} + \frac{4}{6(x+3)} = \frac{13}{6(x+3)}$$

$$9. \frac{3}{x+2} + \frac{4}{x-7} = \frac{3(x-7) + 4(x+2)}{(x+2)(x-7)} = \frac{3x-21+4x+8}{(x+2)(x-7)} = \frac{7x-13}{(x+2)(x-7)}$$

$$10. \frac{1}{y+3} + \frac{4}{y^2+4y+3} = \frac{y+1+4}{(y+1)(y+3)} = \frac{y+5}{(y+1)(y+3)}$$

$$11. \frac{2}{5x} - \frac{3}{10x^3} = \frac{2(2x^2) - 3}{10x^3} = \frac{4x^2-3}{10x^3}$$

$$12. \frac{2x+3}{5(x-6)} - \frac{(3x+4)5}{5(x-6)} = \frac{2x+3-15x-20}{5(x-6)} = \frac{-13x-17}{5(x-6)}$$

# Radicals

$$1. 16^{3/4} = (\sqrt[4]{16})^3 = 2^3 = 8$$

$$2. 9^{3/2} = (\sqrt{9})^3 = 3^3 = 27$$

$$3. 27^{2/3} = (\sqrt[3]{27})^2 = 3^2 = 9$$

$$4. 125^{2/3} = (\sqrt[3]{125})^2 = 5^2 = 25$$

$$5. 1^{1/3} = 1$$

$$6. 3^{3/2} = \sqrt{3^3} = \sqrt{3 \cdot 3 \cdot 3} = 3\sqrt{3}$$

$$7. 4^{5/2} = (\sqrt{4})^5 = 2^5 = 32$$

$$8. 81^{3/2} = (\sqrt{81})^3 = 9^3 = 729$$

$$9. (-64)^{2/3} = (\sqrt[3]{-64})^2 = (-4)^2 = 16$$


