Summer Packet for students entering Honors Pre-Calculus

Welcome to Pre-Calculus. Pre-Calculus is a demanding course that relies heavily upon a student's algebra and geometry skills. You are expected to have a strong background in the skills reviewed in this packet. Resource links are listed below and embedded within each section throughout the packet. This packet will be checked for completion and entered as a formative Infinite Campus grade.

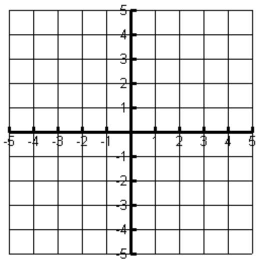
DUE: 1st week of school.

Pre-Calculus Resources In alphabetical order:

Cool Math	coolmath.com
Just Math Tutorials	patrickjmt.com
Khan Academy	khanacademy.org
Math by Fives	mathbyfives.com
Math TV	mathtv.com
Paul's Online Math Notes	tutorial.math.lamar.edu
Purple Math	purplemath.com
Wolfram Alpha	wolframalpha.com
Youtube	youtube.com

Graphs of Algebra 2: Parent Functions and key characteristics

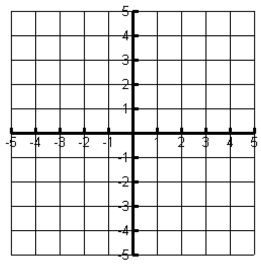
Linear: $y = x$



$y = mx + b$

$Ax + By = C$

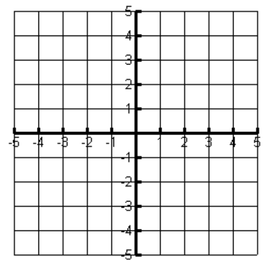
Quadratic: $y = x^2$



Translated: $y = a(x-h)^2 + k$

Vertex: (h, k)

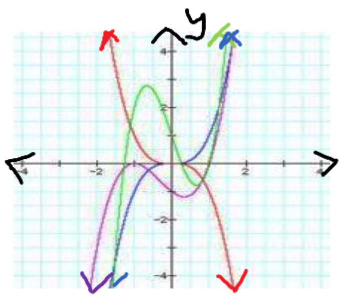
Cubic: $y = x^3$



Translated: $y = a(x-h)^3 + k$

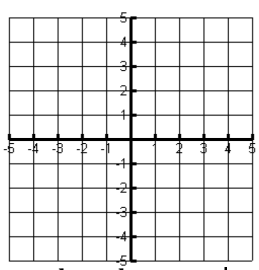
Point of Inflection: (h, k)

Polynomials: Not one 'parent' function. Here are key ideas:



Domain and Interval of Continuity: $(-\infty, \infty)$
Leading Term: Contains leading coefficient
(positive: $\lim_{x\to\infty} f(x) = \infty$ and negative: $\lim_{x\to\infty} f(x) = -\infty$
Degree: even – $\lim_{x\to\pm\infty} f(x)$ are equal, odd – $\lim_{x\to\pm\infty} f(x)$ are opposites. Also indicates the maximum number of curves (at most one less than degree)
Standard form: Identify the leading term and use factoring, synthetic division or the calculator to determine the x-intercepts/multiplicity.
Factored form: Determine the leading term, x-intercepts/multiplicity to graph.

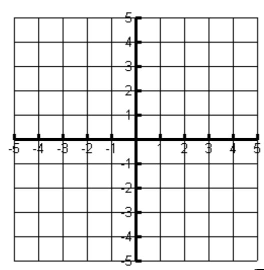
Absolute Value: $y = |x|$



Translated: $y = a|x-h| + k$

Vertex: (h, k)

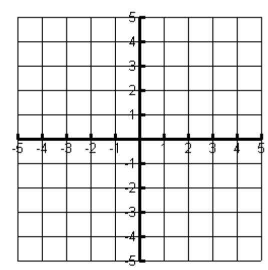
Square Root: $y = \sqrt{x}$



Translated: $y = a\sqrt{x-h} + k$

Starting point: (h, k)

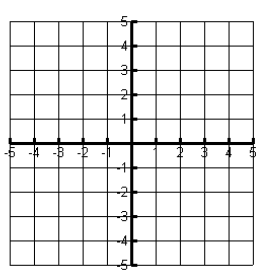
Exponential Growth: $y = b^x$



Growth: $b > 1$

Translated $y = ab^{x-h} + k$
Horizontal Asymptote: $y = k$

Logarithmic Growth: $y = \log_b x$

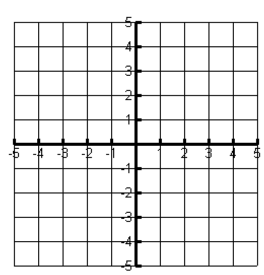


Growth: $b > 1$

Translated: $y = a\log_b(x-h) + k$

Vertical Asymptote: $x = h$

Logarithmic Decay: $y = \log_b x$

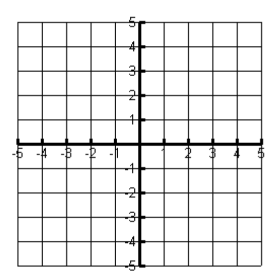


Decay: $0 < b < 1$

Translated: $y = a\log_b(x-h) + k$

Vertical Asymptote: $x = h$

Exponential Decay: $y = b^x$



Decay: $0 < b < 1$

Translated $y = ab^{x-h} + k$

Horizontal Asymptote: $y = k$

1. Simplify the following expressions completely.

a. $\frac{2}{3} \cdot \frac{4}{8} + 7\left(-\frac{3}{5}\right)$

b. $\frac{2}{9} \cdot \frac{15}{14} \cdot \frac{-3}{4} \cdot \frac{6}{5}$

c. $\dfrac{\frac{1}{4} - \frac{2}{3}}{\frac{3}{4} - \frac{5}{3}}$

2. Simplify the following expressions completely, if possible. Final answers should contain no negative exponents. Assume all variables represent positive values.

a. $\dfrac{4r^4y^5}{24r^4y^{-5}}$

b. $\dfrac{y^{11}}{4z^3} \cdot \dfrac{8z^7}{y^7}$

c. $\dfrac{3y^{-2}}{9y^{16}}$

d. $\left(2^{\frac{2}{3}}y^3\right)^{-3}$

3. Simplify the following expressions completely, if possible. Assume all variables represent positive values.

a. $\sqrt{10}$

b. $\sqrt{96}$

c. $\sqrt{75} + \sqrt{27}$

d. $3\sqrt{3} + \sqrt{3}$

e. $\frac{\sqrt{3}}{2} - 5\sqrt{3}$

f. $5 + \sqrt{5}$

4. Simplify the expressions.

a. $\sqrt[3]{64x^6y^7}$

b. $(32y^{15})^{\frac{1}{5}}$

5. Write each expression as a radical.

a. $x^{\frac{1}{3}}$

b. $x^{\frac{4}{5}}$

6. Determine the exponent that goes in the box.

a. $\frac{1}{\sqrt{x}} = x^{\square}$

b. $\sqrt[4]{x^3} = x^{\square}$

c. $\left(\dfrac{x^{\frac{2}{3}}}{x^{\frac{1}{6}}}\right)^{18} = x^{\square}$

d. $\dfrac{x^{\square}}{x^{\frac{1}{6}}} = x^{\frac{2}{3}}$

7. Rationalize the denominator of the following expressions. Then, simplify.

a. $\frac{7}{\sqrt{14}}$

b. $\frac{2}{1-\sqrt{3}}$

c. $\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}}$

8. Factor the following expressions completely, if possible.

a. $x^3 + 8$

b. $x^2 + 8$

c. $9x^6 - 4y^2$

d. $5x^3 - 3x^2 - 45x + 27$

e. $3(x+5)^3 + 2(x+5)^2$

f. $6x^6 - 13x^3 - 5$

g. $12x^5 - 17x^3 - 5x^2$

h. $(x^2+x)^2 - 14(x^2+x) + 24$

i. $18x^2 - 50y^4$

j. $x^2 - 3x - 70$

k. $3x^2 - 10x + 8$

l. $6x^2 - 11x - 7$

m. $4x^2 + 4xy - 15y^2$

n. $4x^3 + 4x^2y - xy^2 - y^3$

9. Evaluate or simplify the following expressions.

a. $(-8)^{\frac{2}{3}}$

b. $\sqrt{25^3x^4}$

c. $9^{3/2}$

d. $-16^{1/4}$

e. $32^{\frac{3}{5}}$

10. Rewrite the absolute value function as a piecewise function.

a. $f(x) = |x-3|$

b. $g(x) = 2|x+4| - 1$

c. $h(x) = -|2x+1| + 5$

11. Factor the polynomial using the given factor and synthetic division. Then, state all zeros.

a. $x^3 + 7x^2 + 4x - 12; (x + 6)$

b. $6x^3 - 2x^2 - 16x - 8; (x - 2)$

12. Express the following inequalities in proper interval notation.

a. $-1 < x < 5$

b. $-2\pi \leq x < \frac{\pi}{2}$

c. $x \leq -\frac{7}{2}$

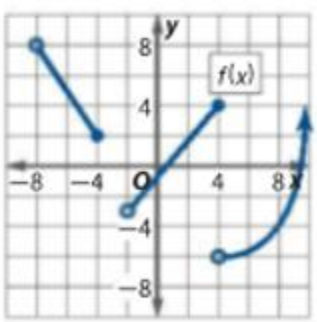
d. $x \geq -27.3$

e. $x < 5$ or $x \geq 9$

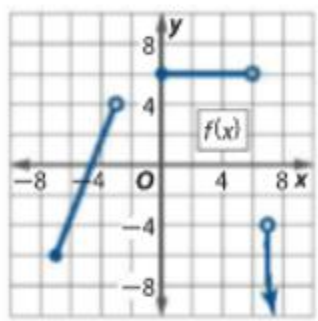
f. $x = 0$ or $2 < x \leq 10$

13. For the following functions, state the domain, range, interval of continuity, and intervals of increasing/decreasing/constant.

a.



b.



14. Solve the following equations for x. Give only exact values of all real solutions.

a. $x^2 - 28 = -3x$

b. $18x^4 - 6x^2 = -3x^3$

c. $2x^2 + 4x = 5$

d. $x(2x - 13) = -6$

e. $\frac{x^2}{3} - 1 = 2$

f. $2|x + 1| = 16$

g. $x^4 - 2x^2 - 15 = 0$

h. $2x^6 - 10 = 8x^3$

i. $(x - 3)^4 + 5 = 0$

j. $\sqrt{x - 2} = x - 2$

k. $2 + \sqrt{10 - x} = -x$

l. $-3|4 - 2x| + 1 = -8$

15. Solve by completing the square. Give only exact values.

a. $x^2 - 4x - 7 = 0$

b. $2x^2 + 6x - 10 = 0$

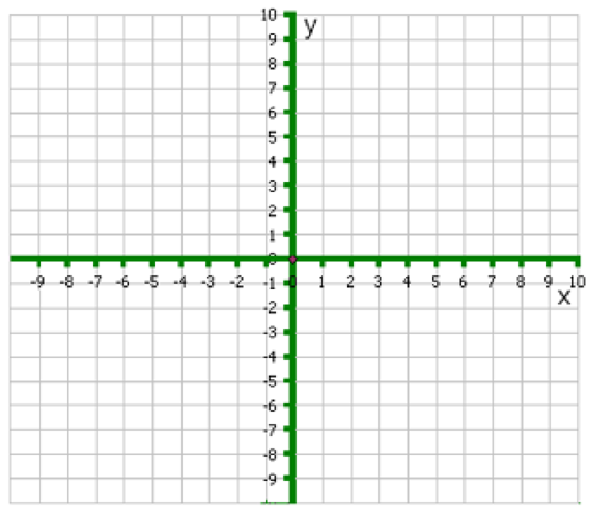
16. Rewrite following quadratic into vertex form and state the vertex by completing the square.

a. $x^2 - 4x + 7 = y$

b. $2x^2 + 4x - 3 = y$

17. Given the function $f(x) = -2\sqrt{x+4} + 2$, find:

a. The x- and y-intercepts for the function.
b. The domain and range of the function.
c. List the transformation required to graph the function, then draw a sketch of the graph.



d. Using the function, determine: The exact value of $f(-2)$, $f(-5)$, and $f(3x-4)$.

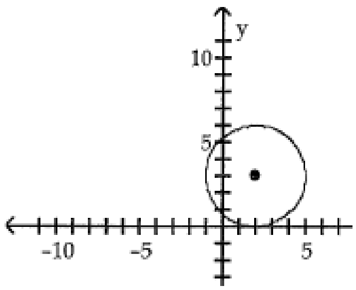
18. Simplify the following expressions completely. Leave in the form of $a + bi$.

a. $(3+2i) - (4-9i)$

b. $(4+2i)(3-5i)$

c. $\frac{2}{6+8i}$

19. Write the standard form for the equation of the circle below.

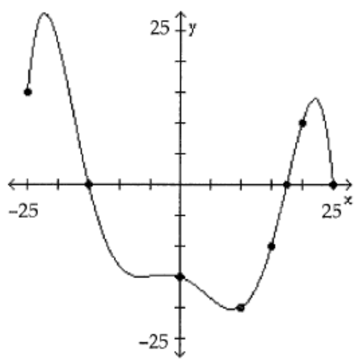


20. Find the center and radius of the circle $x^2 + y^2 + 14x + 12y + 21 = 0$ by completing the square.

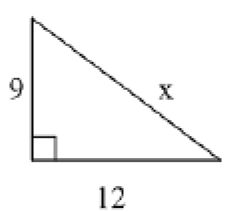
21. Given $f(x) = 4x - 5$ and $g(x) = x^2 + 1$, find:

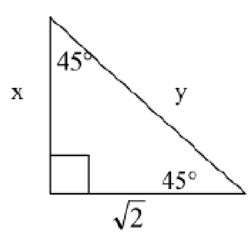
a. $(f + g)(x)$ b. $(g - f)(x)$ c. $(f \cdot g)(x)$ d. $f(g(0))$

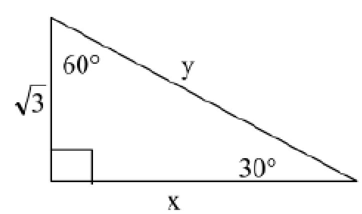
22. Given the graph of $f(x)$ below, determine the values of x where $f(x) > 0$. Use interval notation.



23. For each of the triangles, find any missing sides.







24. A college student earned $7300 during summer vacation working as a waiter in a popular restaurant. The student invested part of the money at rate of 3.2% that compounds monthly. After 3 years, how much does the student have in the bank. Use a calculator to solve.

$A = p\left(1 + \frac{r}{n}\right)^{nt}$

25. Answer the following about the graphs of $f(x)$. Then sketch the graph using the zeros, multiplicity, and end behavior. State the end behavior using limits.

a. $f(x) = 2x^4 - 7x^3 + x^2 + 16x - 12$

b. $f(x) = 3x^2(x-5)^3(x+4)^4$

x-intercepts (state multiplicity):

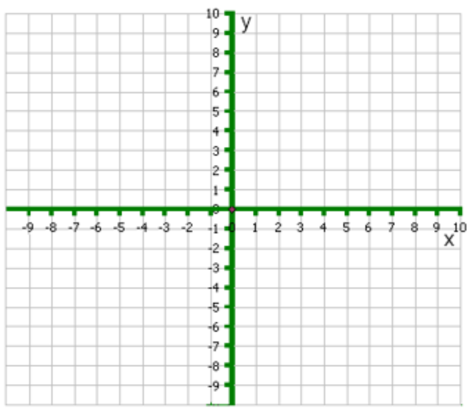
y-intercept:

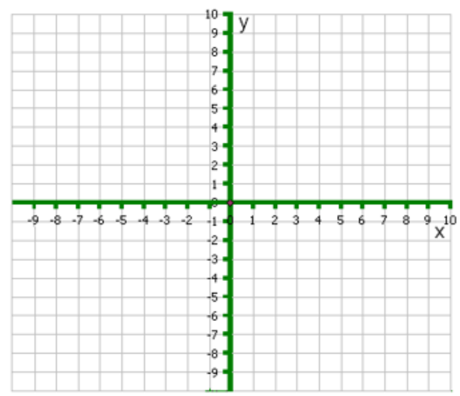
End behavior:

x-intercepts (state multiplicity):

y-intercept:

End behavior:





26. Each function below is a transformation of the function $f(x) = e^x$. After each given transformation, write the function rule.

a. The graph of g is the graph of $f(x) = e^x$ translated one unit right. $g(x) =$______________________

b. The graph of h is the graph of $f(x) = e^x$ with a horizontal asymptote of $y = -4$. $h(x) =$______________________

c. The graph of p is the graph of $f(x) = e^x$ reflected over the x-axis. $p(x) =$______________________

d. The graph of r is the graph of $f(x) = e^x$ reflected over the y-axis. $r(x) =$______________________

27. Rewrite each logarithm into exponential form.

a. $\log_2 4 = 2$ b. $\log_{\frac{1}{2}} 4 = -2$ c. $\log \frac{1}{1000} = -3$ d. $\ln x = 4$ e. $\log_b a = e$

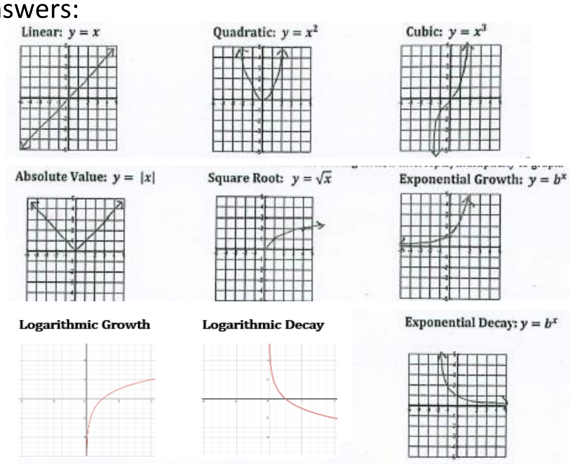
28. Rewrite each exponential as a logarithm.

a. $64^{\frac{1}{3}} = 4$ b. $\left(\frac{1}{10}\right)^{-1} = 10$ c. $x^y = z$ d. $e^y = x$ d. $4^2 = 16$

29. Evaluate each logarithm without a calculator.

a. $\log_3 9$ b. $\log_4 \left(\frac{1}{16}\right)$ c. $\log 10{,}000$ d. $\log_6 1$ e. $\log_{100} 10$

Answers:



1. a. $-\frac{58}{15}$ b. $-\frac{3}{14}$ c. $\frac{5}{11}$
2. a. $\frac{y^{10}}{6}$ b. $2y^4z^4$ c. $\frac{1}{3y^{18}}$ d. $\frac{1}{4y^9}$
3. a. $nothing$ b. $4\sqrt{6}$ b. $8\sqrt{3}$ c. $-\frac{9\sqrt{3}}{2}$
4. a. $4x^2y^2\sqrt[3]{y}$ b. $2y^3$
5. a. $\sqrt[3]{x}$ b. $\sqrt[5]{x^4}$
6. a. $-1/2$ b. $3/4$ c. 9 d. $5/6$
7. a. $\frac{\sqrt{14}}{2}$ b. $-1-\sqrt{3}$ c. $\frac{(\sqrt{a}+\sqrt{b})^2}{a-b}$
8. a. $(x-2)(x^2-2x+4)$
   b. not factorable/prime/irreducible quadratic
   c. $(3x^3-2y)(3x^3+2y)$
   d. $(5x-3)(x+3)(x-3)$
   e. $(x+5)^2(3x+17)$
   f. $(3x^3+1)(2x^3-5)$
   g. $x^2(4x+1)(3x-5)$
   h. $(x+4)(x-3)(x+2)(x-1)$
   i. $2(3x+5y^2)(3x-5y^2)$
   j. $(x-10)(x+7)$
   k. $(3x-4)(x-2)$
   l. $(2x+1)(3x-7)$
   m. $(2x+5y)(2x-3y)$
   n. $(2x+y)(2x-y)(x+y)$
9. a. 4 b. $125x^2$ c. 27 d. -2 e. 8
10. a. $f(x)=\begin{cases} x-3 & x\ge 3 \\ 3-x & x<3 \end{cases}$ b. $g(x)=\begin{cases} 2x+7 & x\ge -4 \\ -2x-9 & x<-4 \end{cases}$
    c. $h(x)=\begin{cases} -2x+4 & x\ge -1/2 \\ 2x+6 & x<-1/2 \end{cases}$
11. a. $(x+6)(x+2)(x-1)$ $x=-6,-2,1$
    b. $2(x-2)(3x+2)(x+1)$ $x=2,-\frac{2}{3},-1$
12. a. $(-1,5)$ b. $\left[-2\pi,\frac{\pi}{2}\right)$ c. $\left(-\infty,-\frac{7}{2}\right]$
    $d.$ $[-27.3,\infty)$ e. $(-\infty,5)\cup[9,\infty)$
    f. $[0]\cup(2,10]$
13. a. D: $(-8,-4]\cup(-2,\infty)$ R: $(-6,\infty)$
    Continuity: $(-8,-4)\cup(-2,4)\cup(4,\infty)$
    Inc: $(-2,4)\cup(4,\infty)$ Dec: $(-8,-4)$
    Constant: None
    b. D: $(-7,-3]\cup[0,6)\cup(7,\infty)$ R: $(-\infty,4)\cup[6]$
    Continuity: $(-7,-3)\cup(0,6)\cup(7,\infty)$
    Inc: $(-7,-3)$ Dec: $(7,\infty)$
    Constant: $(0,6)$
14. a. $x=-7,4$ b. $x=0,-\frac{2}{3},\frac{1}{2}$ c. $x=-1\pm\frac{\sqrt{14}}{2}$
    d. $x=\frac{1}{2},6$ e. $x=\pm 3$ f. $x=7,-9$
    g. $x=\pm\sqrt{5}$ h. $x=3\sqrt{5},-1$ i. $none$
    j. x = 2, 3 k. x = -6 l. $x=\frac{1}{2},\frac{7}{2}$
15. a. $x=2\pm\sqrt{11}$ b. $x=\frac{-3\pm\sqrt{29}}{2}$
16. a. $y=(x-2)^2+3$ $vertex$: $(2,-3)$
    b. $y=2(x+1)^2-5$ $vertex$: $(-1,5)$
17. a. xint: $x=-3$ or $(-3,0)$
    yint: $y=-2$ or $(0,-2)$
    b. D: $[-4,\infty)$ R: $(-\infty,2]$
    c. reflext over xaxis, vertical stretch by factor of 2, left 4, up 2
    d. $f(-2)=-2\sqrt{2}+2, f(-5)=DNE$

$$f(3x-4)=-2\sqrt{3x}+2$$

18. a. $-1+11i$ b. $22-14i$ c. $\frac{3}{25}-\frac{4}{25}i$
19. $(x-2)^2+(y-3)^2=9$
20. Center: $(-7,-6)$ radius: $r=8$
21. a. x^2+4x-4 b. x^2-4x+6
    c. $4x^3-5x^2+4x-5$ d. -1
22. $[-25,-15)\cup(-17.5,25)$
23. $a.$ $x=15$ b. $x=\sqrt{2}, y=2$ c. $x=3, y=2\sqrt{3}$
24. \$8034.51
25. a. xint: $x=-\frac{3}{2}$ $mult.1, x=2$ $mult.2, x=1$ $mult$ 1
    yint: $y=-12$ or $(0,-12)$
    EB: $\lim_{x\to+\infty} f(x)=\infty$: $\lim_{x\to-\infty} f(x)=\infty$
    b. xint: $x=0$ $mult.2, x=5$ $mult.3, x=-4$ $mult$ 4
    yint: $y=0$ or $(0,0)$
    EB: $\lim_{x\to+\infty} f(x)=\infty$, $\lim_{x\to-\infty} f(x)=-\infty$
26. a. $g(x)=e^{x-1}$ b. $h(x)=e^x-4$
    c. $p(x)=-e^x$ d. $r(x)=e^{-x}$
27. a. $2^2=4$ b. $\left(\frac{1}{2}\right)^{-2}=4$ c. $10^{-3}=\frac{1}{1000}$
    d. $e^4=x$ e. $b^e=a$
28. a. $\log_{64}4=1/3$ b. $\log_{1/10}10=-1$
    c. $\log_x z=y$ d. $\ln x=y$ e. $\log_4 16=2$
29. a. 2 b. -2 c. 4 d. 0 e. $1/2$