



FAIRFIELD
PUBLIC SCHOOLS

Summer Packet for students entering Pre-Calculus

Welcome to Pre-Calculus. Pre-Calculus is a demanding course that relies heavily upon a student's algebra and geometry skills. You are expected to have a strong background in the skills reviewed in this packet. Resource links are listed below and embedded within each section throughout the packet. This packet will be checked for completion and entered as a formative Infinite Campus grade.

DUE: 1st week of school.

Pre-Calculus Resources In alphabetical order:

Cool Math	coolmath.com
Just Math Tutorials	patrickjmt.com
Khan Academy	khanacademy.org
Math by Fives	mathbyfives.com
Math TV	mathtv.com
Paul's Online Math Notes	tutorial.math.lamar.edu
Purple Math	purplemath.com
Wolfram Alpha	wolframalpha.com
Youtube	youtube.com

Quadratic Formula:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

Standard Form of a Circle:

$$(x - h)^2 + (y - k)^2 = r^2$$

Continuously Compounded

$$y = Pe^{rt}$$

Distance Formula:

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

Midpoint Formula:

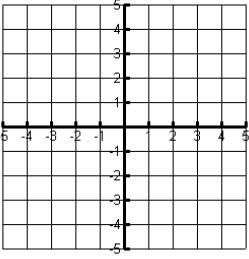
$$m = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2} \right)$$

Compounded:

$$y = P \left(1 + \frac{r}{n} \right)^{nt}$$

Graphs of Algebra 2: Parent Functions and key characteristics

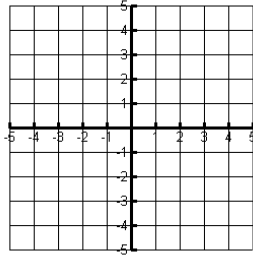
Linear: $y = x$



$$y = mx + b$$

$$Ax + By = C$$

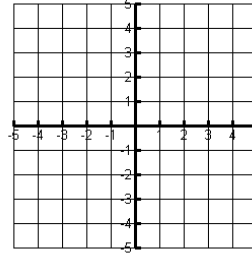
Quadratic: $y = x^2$



$$\text{Translated: } y = a(x - h)^2 + k$$

Vertex: (h, k)

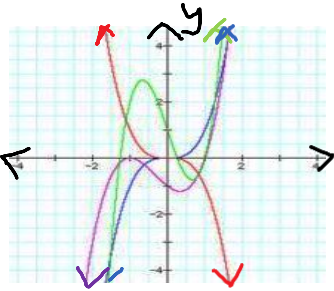
Cubic: $y = x^3$



$$\text{Translated: } y = a(x - h)^3 + k$$

Point of Inflection: (h, k)

Polynomials: Not one 'parent' function. Here are key ideas:



Domain and Interval of Continuity: $(-\infty, \infty)$

Leading Term: Contains leading coefficient

(positive: $\lim_{x \rightarrow \infty} f(x) = \infty$ and negative: $\lim_{x \rightarrow \infty} f(x) = -\infty$)

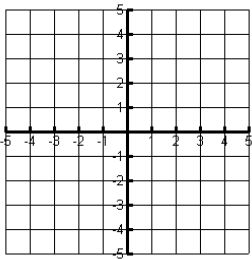
Degree: even - $\lim_{x \rightarrow \pm\infty} f(x)$ are equal, odd - $\lim_{x \rightarrow \pm\infty} f(x)$ are opposites. Also indicates

the maximum number of curves (at most one less than degree)

Standard form: Identify the leading term and use factoring, synthetic division or the calculator to determine the x-intercepts/multiplicity.

Factored form: Determine the leading term, x-intercepts/multiplicity to graph.

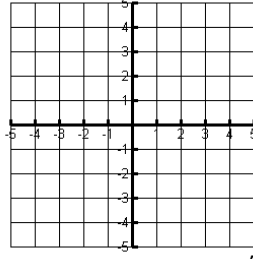
Absolute Value: $y = |x|$



$$\text{Translated: } y = a|x - h| + k$$

Vertex: (h, k)

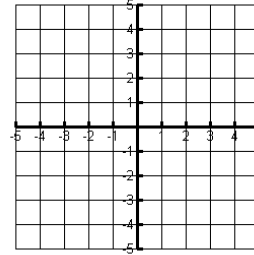
Square Root: $y = \sqrt{x}$



$$\text{Translated: } y = a\sqrt{x - h} + k$$

Starting point: (h, k)

Exponential Growth: $y = b^x$



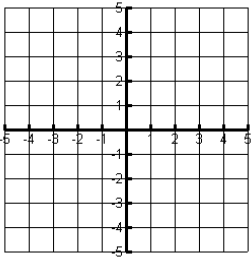
Growth: $b > 1$

Translated $y = ab^{x-h} + k$
Horizontal Asymptote: $y = k$

Logarithmic Growth: $y = \log_b x$

Logarithmic Decay: $y = \log_b x$

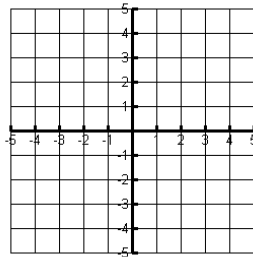
Exponential Decay: $y = b^x$



Growth: $b > 1$

$$\text{Translated: } y = a \log_b(x - h) + k$$

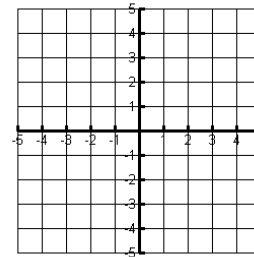
Vertical Asymptote: $x = h$



Decay: $0 < b < 1$

$$\text{Translated: } y = a \log_b(x - h) + k$$

Vertical Asymptote: $x = h$



Decay: $0 < b < 1$

$$\text{Translated } y = ab^{x-h} + k$$

Horizontal Asymptote: $y = k$

1. Simplify the following expressions completely, final expressions should contain no negative exponents.

a. $2\left(\frac{2}{3} \cdot \frac{4}{8}\right)$

b. $\frac{5}{3} \div \frac{10}{7}$

c. $\frac{\frac{1}{4} - \frac{2}{3}}{\frac{3}{4} - \frac{5}{3}}$

d. $9^{\frac{3}{2}}$

e. $\left(\frac{x^3y^5}{x^4y^9}\right)^2$

f. $(x^2y^{1/3}z^{-2})^3$

g. $\sqrt{32}$

h. $\sqrt{2} + 3\sqrt{2} + 4\sqrt{2}$

i. $\sqrt[4]{16x^8y^5}$

j. $8^{\frac{5}{3}}$

k. $2\sqrt{5}(3\sqrt{15})$

2. Factor the following expressions completely, if possible.

a. $x^2 - 100$

b. $x^2 + 8$

c. $9x^6 - 4$

d. $5x^3 - 3x^2 - 45x + 27$

e. $3x^2 - 10x + 8$

f. $6x^2 - 13x - 5$

g. $x^6 - 3x^3 - 70$

h. $6x^2 - 11x - 7$

i. $18x^2 - 50$

j. $4xy^2 - 2xy$

k. $x^2 + 8xy + 7y^2$

3. Factor the polynomial using the given factor and synthetic division. Then, state all zeros.

a. $x^3 + 7x^2 + 4x - 12; x + 6$

b. $x^4 - x^3 - 7x^2 + 5x + 10; (x + 1)$

4. Solve the following equations for x . Give only exact values of all real solutions.

a. $x - 3\left(\frac{1}{3}x - (2 - x)\right) = 7$

b. $x^2 - 28 = -3x$

c. $18x^4 - 6x^2 = -3x^3$

d. $x^4 - 2x^2 - 15 = 0$

e. $x(2x - 13) = -6$

f. $\sqrt{x - 2} = x - 2$

g. $2|x + 1| = 16$

h. Use Calculator. Round to nearest thousandth

$$x^4 - 2x^3 - 5x^2 - x - 7 = 0$$

5. Solve by completing the square. Give only exact values.

a. $x^2 - 4x - 7 = 0$

b. $2x^2 + 8x - 16 = 0$

6. Given the points (2,3) and (1,5), find:

a. The distance between them.

b. The midpoint of the segment formed by them.

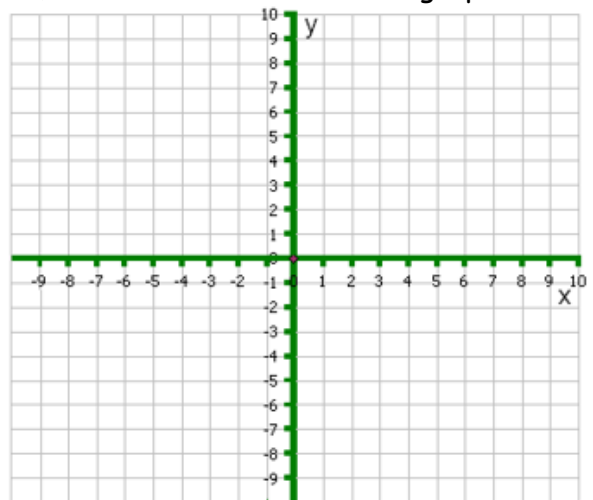
7. Given the function $f(x) = -\sqrt{x+4} + 2$, find:

a. The x- and y-intercepts for the function.

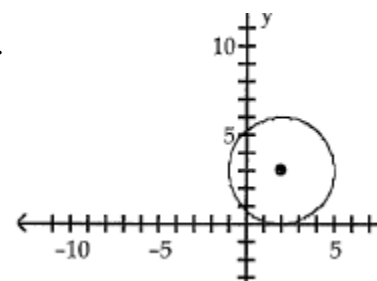
b. The domain and range of the function.

c. The exact value of $f(-2)$ and $f(3x - 4)$.

d. List the transformation required to graph the function, then draw a sketch of the graph.



8. Write the standard form for the equation of the circle below.



9. Find the center and radius of the circle $x^2 + y^2 + 14x + 12y + 21 = 0$ by completing the square.

10. Given $f(x) = 4x - 5$ and $g(x) = x^2 + 1$, find:

a. $(f + g)(x)$

b. $(g - f)(x)$

c. $(f \cdot g)(x)$

d. $f(g(0))$

11. Express the following inequalities in proper interval notation.

a. $-1 < x < 5$

b. $-2\pi \leq x < \frac{\pi}{2}$

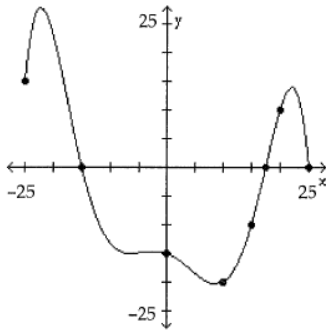
c. $x \leq -\frac{7}{2}$

d. $x \geq -27.3$

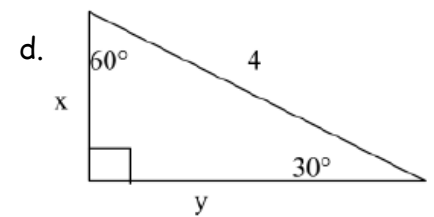
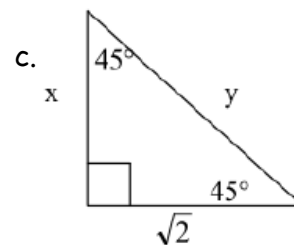
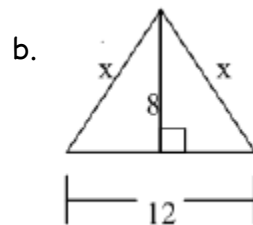
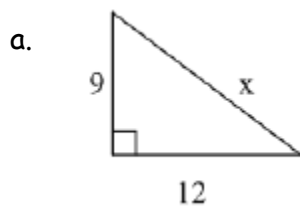
e. $x < 5$ or $x \geq 9$

f. $x = 0$ or $2 < x \leq 10$

12. Given the graph of $f(x)$ below, determine what values of x $f(x) > 0$. Give your answer in interval notation.



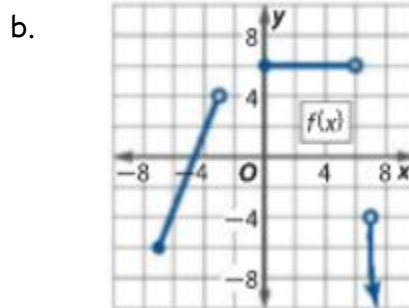
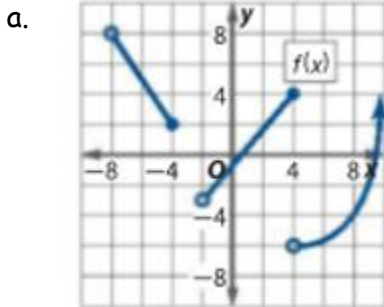
13. For each of the triangles, find the exact value of the missing variables.



14. **CALCULATOR:** If Angela has \$100 to invest at 10% compounded monthly, how long will it take her to have \$175?

15. **CALCULATOR:** Maggie deposited \$2000 in an Individual retirement account that will pay interest of 3% per annum compounded continuously. What will the IRA be worth in 5 years?

16. For the following functions, state the domain, range, interval of continuity and intervals of increasing, decreasing, and constant.



17. Answer the following about the graphs of $f(x)$. Then sketch the graph using the zeros, multiplicity, and end behavior.

a. $f(x) = 2x^3 + x^2 - 18x - 9$

b. $f(x) = 3x^2(x - 2)(x + 2)$

x-intercepts (state multiplicity):

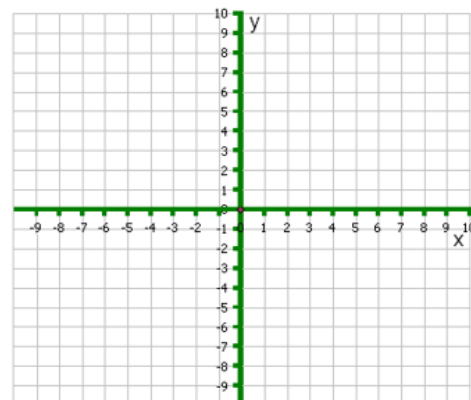
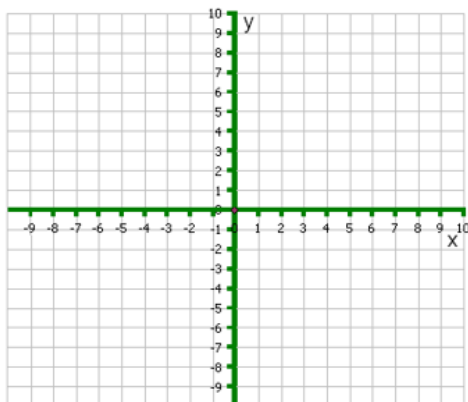
x-intercepts (state multiplicity):

y-intercept:

y-intercept:

End behavior:

End behavior:



Answers:

Linear: $y = x$



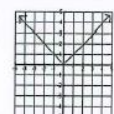
Quadratic: $y = x^2$



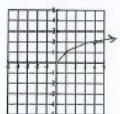
Cubic: $y = x^3$



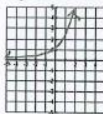
Absolute Value: $y = |x|$



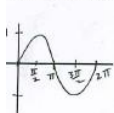
Square Root: $y = \sqrt{x}$



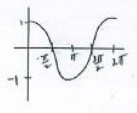
Exponential Growth: $y = b^x$



Sine: $y = \sin(x)$



Cosine: $y = \cos(x)$



Exponential Decay: $y = b^{-x}$



1. a. $\frac{2}{3}$ b. $\frac{7}{6}$ c. $\frac{5}{11}$ d. 27 e. $\frac{1}{x^2y^8}$

f. $\frac{x^6y}{z^6}$ g. $4\sqrt{2}$ h. $8\sqrt{2}$ i. $2x^2y^4\sqrt{y}$

j. 32 k. $30\sqrt{3}$

2. a. $(x + 10)(x - 10)$
b. not factorable (prime/irreducible quadratic)

c. $(3x^3 - 2)(3x^3 + 2)$

d. $(5x - 3)(x + 3)(x - 3)$

e. $(3x - 4)(x - 2)$

f. $(3x + 1)(2x - 5)$

g. $(x^3 - 10)(x^3 + 7)$

h. $(3x - 7)(2x + 1)$

i. $2(3x + 5)(3x - 5)$

j. $2xy(2y - 1)$

k. $(x + 7y)(x + y)$

3. a. $(x + 6)(x + 2)(x - 1)$ $x = -6, -2, 1$

b. $(x + 1)(x - 2)(x - \sqrt{5})(x + \sqrt{5})$
 $x = -1, 2, \pm\sqrt{5}$

5. a. $x = -\frac{1}{3}$ b. $x = -7, 4$ c. $x = 0, -\frac{2}{3}, \frac{1}{2}$

d. $x = \pm\sqrt{5}$ e. $x = \frac{1}{2}, 6$ f. $x = 3, 2$

g. $x = 7, -9$ h. $x = -1.770, 3.610$

6. a. $x = 2 \pm \sqrt{11}$ b. $x = -2 \pm 2\sqrt{3}$

7. a. $d = \sqrt{5}$ b. $(\frac{3}{2}, 4)$

8. a. x-int: $x = 0$ or $(0, 0)$

y-int: $y = 0$ or $(0, 0)$

b. D: $[-4, \infty)$ R: $(-\infty, 2]$

c. $f(-2) = -\sqrt{2} + 2, f(3x - 4) = -\sqrt{3x} + 2$

d. reflect over x-axis, left 4, up 2

9. $(x - 2)^2 + (y - 3)^2 = 9$

10. Center: $(-7, -6)$ radius: $r = 8$

11. a. $x^2 + 4x - 4$ b. $x^2 - 4x + 6$

c. $4x^3 - 5x^2 + 4x - 5$ d. -1

12. a. $(-1, 5)$ b. $[-2\pi, \frac{\pi}{2})$ c. $(-\infty, -\frac{7}{2}]$
d. $[-27.3, \infty)$ e. $(-\infty, 5) \cup [9, \infty)$ f. $[0] \cup (2, 10]$

13. $[-25, -15) \cup (-17.5, 25)$

14. a. $x = 15$ b. $x = 10$ c. $x = \sqrt{2}, y = 2$
d. $x = 2, y = 2\sqrt{3}$

15. 5.6 years

16. \$2323.67

17. a. D: $(-8, -4] \cup (-2, \infty)$ R: $(-6, \infty)$

Continuity: $(-8, -4) \cup (-2, 4) \cup (4, \infty)$

Inc: $(-2, 4) \cup (4, \infty)$ Dec: $(-8, -4)$

Constant: None

b. D: $[-7, -3) \cup [0, 6) \cup (7, \infty)$ R: $(-\infty, 4) \cup [6]$

Continuity: $(-7, -3) \cup (0, 6) \cup (7, \infty)$

Inc: $(-7, -3)$ Dec: $(7, \infty)$

Constant: $[0, 6)$

18. a. x-int: $x = 3$ mult. 1, $x = -3$ mult. 1, $x = -\frac{1}{2}$ mult. 1

y-int: $y = -9$

EB: $\lim_{x \rightarrow +\infty} f(x) = \infty, \lim_{x \rightarrow -\infty} f(x) = -\infty$

b. x-int: $x = 0$ mult. 2, $x = 2$ mult. 1, $x = -2$ mult. 1

y-int: $y = 0$

EB: $\lim_{x \rightarrow +\infty} f(x) = \infty, \lim_{x \rightarrow -\infty} f(x) = \infty$

