ST. BRENDAN HIGH SCHOOL

AP Calculus BC
Summer Math Packet

Course 1364

This summer math packet is a review of some of the concepts you learned in Pre-calculus that are needed when you begin your Calculus class in August. It will assure that all students will be on the same page as to what they are expected to know.

Instructions for completing the packet:

- ✓ Please print the packet or use loose leaf paper to complete the packet by hand showing all work when necessary. Work must be neat and legible.
- ✓ Please use your Pre-Calculus notes of the websites provided to help you if you need reminders on how to complete some practice problems.
- ✓ Take notes as you complete your work. You will be given a quiz on this material the first week of school.
- ✓ Work on the packet with your friends. Help each other. Every student is responsible for knowing the material in this packet.
- ✓ Bring your packet the first day of school. It will be collected for a grade. Only packets done with paper and pencil will be accepted.

Helpful Websites:

http://www.mathtv.com/

http://www.purplemath.com/modules/index.htm

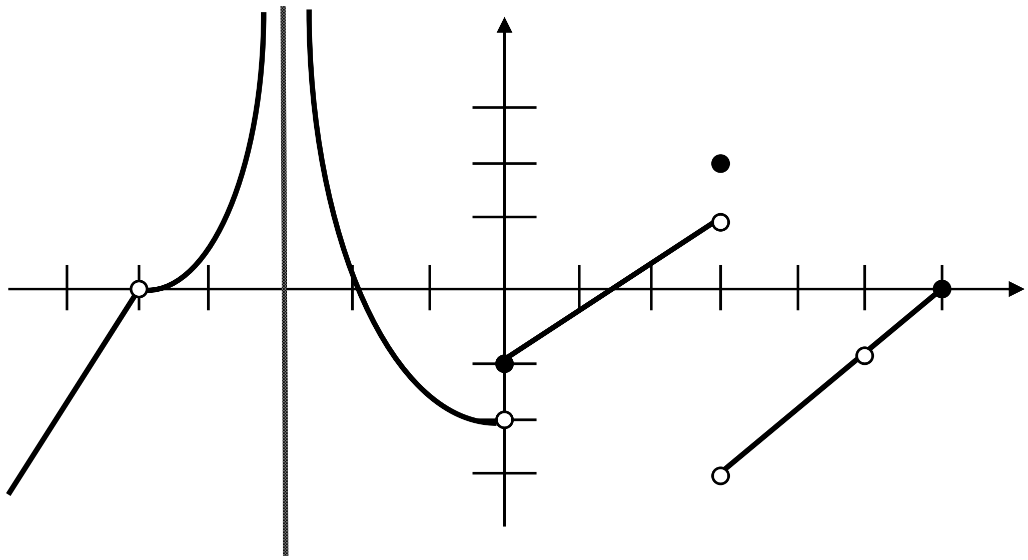
http://www.khanacademy.org

Name____________________________

AP Calculus BC Summer Review Packet (Limits & Derivatives)

Limits

1. Answer the following questions using the graph of $f(x)$ given below.



(a) Find $f(0)$

(b) Find $f(3)$

(c) Find $\lim_{x \to -5} f(x)$

(d) Find $\lim_{x \to 0^+} f(x)$

(e) Find $\lim_{x \to 3^-} f(x)$

(f) Find $\lim_{x \to -3^+} f(x)$

(g) List all x-values for which $f(x)$ has a removable discontinuity. Explain what section(s) of the definition of continuity is (are) violated at these points.

(h) List all x-values for which $f(x)$ has a nonremovable discontinuity. Explain what section(s) of the definition of continuity is (are) violated at these points.

In problems 2-10, find the limit (if it exists) using analytic methods (i.e. without using a calculator).

2. $\lim_{x \to -2} \frac{3x^2 + 21x + 30}{x^3 + 8}$

3. $\lim_{x \to \pi/6} \frac{1 - \cos^2 x}{4x}$

4. $\lim_{x \to 4} \frac{\sqrt{x-3} - 1}{x - 4}$

5. $\lim_{x \to 0} \frac{[1/(x+1)] - 1}{x}$

6. $\lim_{x \to 0} \frac{\left[1/\sqrt{1+x}\right]-1}{x}$

7. $\lim_{\theta \to 0} \frac{\sin 6\theta^3}{7\theta}$

8. $\lim_{t \to 0} \frac{\sin^2 3t^2}{t^3}$

9. $\lim_{x \to 6^-} \frac{|6x-36|}{6-x}$

10. $\lim_{\Delta x \to 0} \frac{\sin((\pi/6)+\Delta x)-(1/2)}{\Delta x}$

Hint: $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$

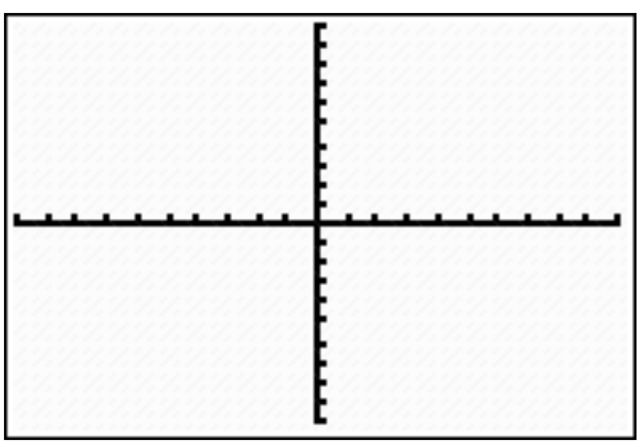
11. **Suppose** $f(x)=\begin{cases}\dfrac{\sqrt{2x+1}-\sqrt{3}}{x-1}, x\geq 0\\ 4x^2+k, x<0\end{cases}$.

(a) For what value of *k* will *f* be piecewise continuous at *x* = 0? Explain why this is true using one-sided limits. (**Hint: A function is continuous at**

***x* =*c* if (1) *f*(*c*) exists, (2)** $\lim_{x\to c} f(x)$ **exists, and (3)** $\lim_{x\to c} f(x)=f(c)$ **.)**

(b) Using the value of *k* that you found in part (a), **accurately** graph *f* below. Approximate the value of $\lim_{x\to 1} f(x)$

$\lim_{x\to 1} f(x)=$________________



(c) Rationalize the numerator to find $\lim_{x\to 1} f(x)$ analytically.

12. **Analytically** determine the values of *b* and *c* such that the function *f* is continuous on the entire real number line. **See the hint given in problem 11.**

$$f(x)=\begin{cases} x+1, 1<x<3 \\ x^2+bx+c, x<1 \text{ or } x>3 \end{cases}$$

In problem 13, circle the correct answer and explain why the answer is the correct one.

13. If $f(x)=x^3+x-3$, and if *c* is the only real number such that *f(c)* = 0, then by the Intermediate Value Theorem, *c* is necessarily between

(A) -2 and -1

(B) -1 and 0

(C) 0 and 1

(D) 1 and 2

(E) 2 and 3

Hint: The Intermediate Value Theorem states that if *f* is a continuous function on the interval [a, b] and *k* is any number between *f(a)* and *f(b)*, then there must exist at least one number $c\in[a,b]$ such that *f(c) = k*.

Derivatives

In problems 1 & 2, find the derivative of the function by using the limit definition of the derivative.

1. $f(x) = x^3 - 2x + 3$

2. $f(x) = \dfrac{x+1}{x-1}$

In problems 3-14, find the derivative of the given function using the power, product, quotient, and/or chain rules.

3. $f(x) = (3x^2 + 7)(x^2 - 2x + 3)$

4. $f(x) = \sqrt{x}\sin x$

5. $f(t) = t^3 \cos t$

6. $f(x) = \dfrac{x^2 + x - 1}{x^2 - 1}$

7. $f(x) = \dfrac{x^4 + x}{\tan^2 x}$

8. $f(x) = 3x^2 \sec^3 x$

9. $f(x) = 3x\csc x + x\cot x$

10. $f(x) = \left(\dfrac{x+5}{x^2 - 6x}\right)^2$

11. $f(x) = (x^3 - 2)^{3/2}(5x^2 + 1)^{5/2}$

12. $f(x) = x^3 \cot^4(7x)$

13. $f(x) = 5\sin^2(\sqrt{3x^4 + 1})$

Problems continue on the next page.

In problems 14 & 15, find an equation of the tangent line to the graph of *f* at the indicated point *P*.

14. $f(x) = \dfrac{1+\cos x}{1-\cos x}, P\left(\dfrac{\pi}{2}, 1\right)$

15. $f(x) = \left(x^2 - 1\right)^{2/3}, P(3,4)$

In problems 16 & 17, find the second derivative of the given function.

16. $f(x) = (4x^2 - 3x)^{3/2}$

17. $h(x) = x^3 \cos(\pi x)$

In problem 18, use the position function $s(t) = -16t^2 + v_0 t + s_0$ for free-falling objects.

18. A ball is thrown straight down from the top of a 220-foot tall building with an initial velocity of -22 feet per second.

(a) Determine the average velocity of the ball on the interval [1, 2].

(b) Determine the instantaneous velocity of the ball at $t = 3$.

(c) Determine the time t at which the average velocity on [0, 2] equals the instantaneous velocity.

(d) What is the velocity of the ball when it strikes the ground?

In problem 19-24, circle the correct answer and explain why the answer is the correct one.

19. $\lim_{h \to 0} \dfrac{\cos\left(\dfrac{\pi}{6}+h\right)-\cos\left(\dfrac{\pi}{6}\right)}{h} =$

(A) Does not exist

(B) $\dfrac{1}{2}$

(C) $-\dfrac{1}{2}$

(D) $\dfrac{\sqrt{3}}{2}$

(E) $-\dfrac{\sqrt{3}}{2}$

20. Let *f* and *g* be differentiable functions with values for *f(x), g(x), f′(x), and g′(x)* shown below for $x = 1$ and $x = 2$.

x	*f(x)*	*g(x)*	*f′(x)*	*g′(x)*
1	4	-4	12	-8
2	5	1	-6	4

Find the value of the derivative of $f(x) \bullet g(x)$ at $x = 1$.

(A) -96

(B) -80

(C) -48

(D) -32

(E) 0

21. Let $f(x) = \begin{cases} 3x^2 + 4, x < 1 \\ x^3 + 3x, x \geq 1 \end{cases}$. Which of the following is true?

I. *f(x)* is continuous at $x = 1$

II. *f(x)* is differentiable at $x = 1$

III. $\lim_{x \to 1^-} f(x) = \lim_{x \to 1^+} f(x)$

(A) I only

(B) II only

(C) III only

(D) I and III only

(E) II and III only

22. The equation of the line tangent to the curve $f(x)=\frac{kx+8}{k+x}$ at x = -2 is $y = x + 4$. What is the value of k?

(A) -3

(B) -1

(C) 1

(D) 3

(E) 4

23. An equation of the line <u>normal</u> to the curve $y=\sqrt[3]{x^2-1}$ at the point where x = 3 is

(A) $y+12x=38$

(B) $y-4x=10$

(C) $y+2x=4$

(D) $y+2x=8$

(E) $y-2x=-4$

Hint: A <u>normal line</u> to a curve at a point is perpendicular to the tangent line to the curve at the same point.

24. If the nth derivative of *y* is denoted as $y^{(n)}$ and $y = -\sin x$, then $y^{(14)}$ is the same as

(A) *y*

(B) $\frac{dy}{dx}$

(C) $\frac{d^2 y}{dx^2}$

(D) $\frac{d^3 y}{dx^3}$

(E) None of the above

Summer Assignment

Chapter 1 – Limits and Their Properties

1. $\lim_{x\to 1} \dfrac{\sqrt{x+1}}{x+1} =$

2. $\lim_{x\to \pi} \cos 2x =$

3. $\lim_{x\to 4} \dfrac{x-4}{x^2-16} =$

4. $\lim_{x\to 2} \dfrac{x^2+5x-14}{x-2} =$

5. $\lim_{x\to o} \dfrac{\frac{1}{x+5}-\frac{1}{5}}{x} =$

6. $\lim_{x\to 0} \dfrac{\sin x}{3x} =$

7. $\lim_{x\to 3^+} \|x\| =$

8. $\lim_{x\to 0^-} \dfrac{|x|}{x} =$

9. Find the x-values at which f is discontinuous. Then tell whether the discontinuity is removable or non removable.

a. $f(x) = \frac{3}{x-4}$

b. $f(x) = \frac{x-4}{x^2+2x-24}$

10. Find the constant a so that the function is continuous on the entire real line.

$$F(x) = \begin{cases} ax - 2 & x < 2 \\ 3x - 5 & x \geq 2 \end{cases}$$

Chapter 2 - Differentiation

1. Use the limit definition of the derivative to find $f'(x)$ of $f(x) = x^2 - 5x + 3$.

2. Find the derivative of the function.

   a. $y = 3x^4 + 2x^2 - 5x$

   b. $f(x) = \frac{2}{3x}^3$

   c. $g(x) = 3\text{Cos}x\text{Csc}x$

   d. $g(x) = \frac{x+4}{2x-5}$

   e. $f(x) = \sqrt{x} \cdot Sec(2x)$

   f. $y = \sqrt[3]{x}\,(\sqrt{x} + 3)$

g. $f(x)=\sqrt{\dfrac{1}{x^2-2}}$

h. $h(t)=5Sin^4(\pi x-1)$

i. $g(x)=\dfrac{1}{(x^2-3x)^2}$

3. Find an equation of the tangent line to the graph of the function at the indicated point.
   a. $F(x) = 5x^3$ (1,5)

Find $\dfrac{dy}{dx}$ by Implicit Differentiation.

4. $y^2 + x^3 = 10 - xy$

Find $\dfrac{d^2y}{dx^2}$

5. $x^2y^2 - 2x = 3$

Find the equation of the tangent line at the indicated point.

6. $x^2 + 2xy = 5x - y + 2$ (1,2)

Find the points at which the graph of the equation has a vertical or horizontal tangent line.

7. $16x^2 + 25y^2 - 160x + 200y + 400 = 0$

8. Air is being pumped into a spherical balloon at a rate of 9 cubic feet per minute. Find the rate of change of the radius when the radius is 1.5 feet.

9. As the sides of a rectangle are changed, the Area increases at a rate of 25 square feet per minute. If the length of the rectangle is decreasing at a rate of 1 foot per minute, how fast is the rate of change of the width at the time when the length is 20 feet and the width is 15 feet?

10. A ladder 13 feet long is leaning against the wall of a house. The base of the ladder is pulled away from the wall at a rate of 1.5 feet per second.

a. How fast is the top of the ladder sliding down the wall when the base of the ladder is 5 feet?

b. Find the rate at which the angle of elevation between the ladder and the ground is changing when the ladder is 5 feet from the wall.

11. A conical tank (with vertex down) is 14 feet across the top and 20 feet deep. If water is flowing into the tank at a rate 20 cubic feet per minute, find the rate of change of the depth of the water when the water is 5 feet deep.

Chapter 3 – Applications of Differentiation

1. Locate the absolute extrema of the function on the closed interval. $F(x) = 3x^4 - 4x^3$ on the interval [-1,2].

2. Apply the mean value theorem of f on the interval. Find all values of c in the interval that prove this theorem for $f(x) = 5 - \frac{3}{x}$ on the interval from [1,3].

Find all relative extrema, intervals of increasing and decreasing, points of inflection, concave up and concave down.

3. $f(x) = x + \frac{4}{x}$

4. $f(x) = -3x^5 + 5x^3$

Determine the Limits.

5. a. $\lim_{x\to\infty} \dfrac{3x+5}{x^2-4} =$

b. $\lim_{x\to\infty} \dfrac{4x^2-6}{3x^2-2x} =$

c. $\lim_{x\to\infty} \dfrac{3x^3+5}{2x^2-4} =$

Analyze the Graph.

6. $f(x) = \dfrac{x^2-1}{x^2-16}$

7. A physical fitness room consists of a rectangular region with a semicircle on each end. If the perimeter of the room is to be a 200-meter running track, find the dimensions that will make the area of the **rectangular region** as large as possible.

Chapter 4 – Integration

EVALUATE THE INDEFINITE INTEGRAL.

1. $\int (x^2 + 3x)dx =$

2. $\int \frac{1}{\sqrt[3]{x^2}} dx =$

3. $\int (Cosx + 3SecxTanx)\, \text{dx} =$

Evaluate each integral.

4. $\int (x^2 + 5)dx =$

5. $\int x(3x^2 + 5)^3 dx =$

6. $\int (2x+1)\sqrt{2x+2}dx =$

7. $\int_0^3 (x^2 + 3x - 4)dx =$

8. $\int_0^1 2x\sqrt{1-x^2}\, dx =$

9. $\int_{0}^{\frac{\pi}{3}} Sin(x)dx =$

10. Prove the Mean Value Theorem for Integrals when given the function and the interval: f(x) = $3x^2 + 3$ on the interval from [1 , 5].

11. Find the Average Value of the function over the given interval: f(x) = $\sqrt{2x+1}$ on the interval from [0 , 4].

12. Use the Second Fundamental Theorem of Calculus to find $F'(x) =$ if you are given that F(x) = $\int_{2}^{x} (t^2 + 3t - 4)dt$.

Chapter 5 – Logarithmic, Exponential, and Other Transcendental Functions

Find the Derivative.

1. $f(x) = \ln(3x^2 + 2x - 5)$

2. $f(x) = \ln\sqrt{\dfrac{x^3+1}{x+5}}$

Find the equation of the tangent line to the graph of the equation at the indicated point.

3. $y = \ln(x^2 - 8)$ $(3,0)$

Find the indicated Indefinite Integral.

4. $\int \dfrac{3}{3x+2}\,dx$

5. $\int \dfrac{x}{\sqrt{16-x^2}}\,dx$

6. $\int \dfrac{Sec^2 t}{1+Tan t}\,dt$

Find the Definite Integral.

7. $\int_{e}^{e^2} \frac{1}{x \ln x}$ dx

8. $\int_{0}^{2} \frac{x^2 - 2}{x + 1}$ dx

Use implicit differentiation to find $\frac{dy}{dx}$.

9. $\ln(xy) + 5x = 30$

Find $f^{-1'}(x)$ for the function f and the real number x.

10. $f(x) = x^3 - 5x^2 + 15$ x=3

Find the derivative of the function.

11. $f(x) = 4^{x^2+1}$

12. $g(x) = x^2 e^x$

13. $h(x) = \log_3 \dfrac{x^2}{x-1}$

14. $f(x) = arcSec(3x)$

Find the indefinite integral or evaluate the definite integral.

15. $\int \dfrac{e^{\sqrt{x}}}{\sqrt{x}} dx$

16. $\int_{-2}^{2} 4^{\frac{x}{2}} dx$

17. $\int \dfrac{e^{2x}}{1+e^{2x}} dx$

18. $\int_{-2}^{0} x^2 e^{\frac{x^3}{2}}\, dx$

Find an equation of the tangent line to the graph of the function at the given point.

19. $f(x) = xe^x - e^x$ at the point $(1, 0)$

Use implicit differentiation to find $\frac{dy}{dx}$.

20. $xe^y - 10x + 3y = 252$

Chapter 7 – Applications of Integration

1. Determine the area of the region bounded by the graphs of $y = -x^2 + 2x + 3$ and $y = 3$.

a. $\frac{4}{3}$ b. $\frac{9}{2}$ c. $\frac{22}{3}$

d. $\frac{-4}{3}$ e. None of these.

2. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^3$, $x = 2$, and $y = 1$ about the y-axis.

a. $\frac{93\pi}{5}$ b. $\frac{120\pi}{7}$ c. $\frac{47\pi}{5}$

d. $\frac{62\pi}{5}$ e. None of these.

3. Which of the following integrals represents the volume of the solid fromed by revolving the region bounded by the graphs of $y = x^3$, y = 1, and x = 2 about the line y = 10?

a. $\pi\int_1^8 (10-y)(2-\sqrt[3]{y})dy$ b. $\pi\int_1^2 [81-(10-x^3)^2]dx$

c. $2\pi\int_1^8 y(2-\sqrt[3]{y})dy$ d. $\pi\int_1^2 [1-(10-x^3)^2]dx$ e. None of these.

4. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \frac{1}{2}(x-2)^2$ and y = 2 about the y-axis.

a. $\frac{128\pi}{15}$ b. $\frac{64\pi}{3}$ c. $\frac{32\pi}{3}$

d. $\frac{20\pi}{3}$ e. None of these.

5. Find the area of the region bounded by the graphs of $y = \frac{1}{x}$ and $2x + 2y = 5$.

6. Find the volume of the solid fromed by revolving the region bounded by the graphs of $y = e^x$, y = 0, x = 0, and x = 1 about the x – axis.

7. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = \frac{1}{x}$ and $2x + 2y = 5$ about the line $y = \frac{1}{2}$.

8. Find the volume of the solid formed by revolving the region bounded by the graphs of $y = x^2$ and y = 4 about x=4.

9. Find the Volume of the Solid whose base is bounded by $f(x) = 1 - x$, $g(x) = -1 + x$, and $x = 0$, with cross sections in the shape of squares taken perpendicular to the x – axis.

10. A solid is generated by revolving the region bounded by $y = 4 - x^2$, $x = 0$, $y = 0$ about the y axis. A hole centered along the axis of revolution, is drilled through this solid so that one-half of the volume is removed. Find the diameter of the hole.