Letter to incoming AP Calculus AB students

Congratulations on enrolling in AP Calculus AB! This fast-paced, rigorous course requires dedication and active participation. My goal is to help you build a strong calculus foundation and achieve a great AP score. I expect your best effort, and in return, I will provide the instruction, support, and encouragement needed for success. Your achievement is a team effort between you, your parents, and me. Remember, it's not student vs. teacher—it's student and teacher vs. the AP test. Stay focused, work hard, and let's make this a great year!

Mathematics is more than just numbers—it builds problem-solving skills and discipline that will benefit you far beyond this class. To succeed in AP Calculus AB, you'll need a solid grasp of key concepts from Algebra, Trigonometry, and Geometry. This packet is here to help you review and refresh those skills, so you feel confident going into the course. Take the time to identify any areas that need extra attention and strengthen them now. While not required, I **highly recommend** completing the packet before Wednesday, August 6, 2025—I want you to start the year feeling prepared and ready to succeed!

Clear communication of your work and thought process is essential in this class, especially on the AP exam. To help you develop this skill, here are the expectations for how your work should be shown on both practice assignments and assessments. Start applying these standards in your summer packet to better prepare for AP Calculus AB next year.

Work Submission Expectations

A. Complete all problems on separate sheets of paper.
B. Clearly number and label each problem.
C. Solve all problems without a calculator unless otherwise stated.
D. Show all steps leading to your solution in a neat and organized manner:

- Work should be presented vertically.
- Start with the given problem (copy the question, except for word problems).

Explanation and Justification

E. When asked to explain or justify, write in full sentences and be specific. Avoid vague terms like "it," "the graph," or "the function." If function notation is not provided, you may begin with "The function…"

Resources for Extra Support

If you encounter unfamiliar concepts, use these resources to reinforce your understanding:

- Khan Academy
- SOS Math
- Purple Math
- Lamar University – Algebra
- Lamar University – Algebra & Trig Review
- Math Is Power 4U

Focus on Process Over Answers

While arriving at the correct answer is important, demonstrating a clear and logical thought process is even more valuable. On tests, proper communication of your reasoning will earn you more points than simply writing down the right answer. Strive for a deep understanding of the process—it will serve you well in AP Calculus AB and beyond!

You will need a graphing calculator for this class. If you don't have one, rentals will be available at the start of the semester. If you're considering purchasing one over the summer, please note that I can only provide support for TI-83 or TI-84 models. If you choose a different brand or model, you will need to learn its functions on your own. For a list of approved calculators, visit the College Board site: Calculator Policy.

During the first week of school, you'll take a two-day diagnostic exam—one non-calculator and one calculator portion—that covers key concepts needed for AP Calculus AB. This exam **won't count** toward your grade, but it will be a valuable tool for you to identify areas of strength and areas where you might need more practice. While the questions won't be identical to those in the summer packet, they will focus on the same fundamental concepts. I strongly encourage you to review the material thoroughly over the summer so you can approach the exam with confidence and clarity. To ensure everyone is on track, we will start the year with a 3.5 to 4-week review unit to go over these topics in greater detail. Taking the time to practice now will set you up for a strong start and a successful year in AP Calculus AB!

I look forward to working with you this fall—have a great summer! Take your time completing this packet, ensuring that your work is clear, your notations are precise, and your answers are correct. I recommend starting after July 4th, giving you at least three and a half weeks to complete it while keeping the material fresh for the fall. Finishing too early may cause you to forget key concepts by the time school starts.

Here are some key tips to help you succeed in AP Calculus AB:

- **Plan ahead.** Procrastination hurts your understanding and leaves you less prepared for tests and quizzes.
- **Memorize important formulas, theorems, and concepts.** Regular review will strengthen your foundation.
- **Work efficiently.** Solve the problems you know first, then spend time on the more challenging ones.
- **Use available help.** Success doesn't mean struggling through everything alone. Ask questions and seek support when needed.
- **Take homework seriously.** Doing assignments regularly and effectively is key to your success. I only assign work that I believe is important and meaningful—every problem is genuinely worth doing. (*Homework time is expected to be 1 hour to 1.5 hours per day for the class).

Stay consistent, stay engaged, and you'll be well-prepared for a great year in AP Calculus AB!

Review Topics for AP Calculus AB

Functions & Their Properties

1. Graph of Functions: Linear, Quadratic, Polynomial, Radical, Rational, Exponential, Logarithmic, Trigonometric, Inverse Trigonometric, Piecewise, Greatest Integer
2. Characteristics of Functions:
    - Domain and Range
    - Intercepts
    - Zeros
    - Discontinuities
    - End Behavior
    - Transformations
    - Increasing/Decreasing Intervals
    - Extrema (Local/Global Max & Min)
3. Even and Odd Functions
4. Symmetry: Y-axis, X-axis, and Origin
5. Transformation of Functions

Algebra & Equation Solving (For all function families)

6. Factorization Techniques (Difference of Squares, Sum/Difference of Cubes, Grouping)
7. Solving Equations
8. Solving Inequalities (Using Critical Values)
9. Properties of Exponents
10. Properties of Logarithms
11. Inverses & Inverse Functions
12. Systems of Equations
13. Rate of Change
14. Complex Fractions
15. Difference Quotient
16. Composite Functions

Trigonometry

17. Trigonometric Ratios
18. Trigonometric Identities

Other Key Concepts

19. Arithmetic Skills
20. Limits (Introductory Concepts)
21. Number Sense
22. Algebraic Manipulation & Simplification
23. Application questions (Polynomial, Exponential and Trig)
24. Geometry: Areas & Volumes of Various Shapes

AP Calculus Summer Packet

Please read the cover letter before starting the packet.

Before you get overwhelmed by the length of this packet, remember we will be going over the whole packet as part of the Review Unit at the beginning of the semester. Below is the tentative schedule of the pacing of the review unit.

AP Calculus AB Review Unit Pacing:

Week 1:
Day 1: Parent functions and characteristics of functions
Day 2: Transformation of functions
Day 3: Even and Odd Functions, Operation with functions and Inverse Function
Day 4: Solving Polynomial Function and inequalities by factoring
Day 5: Sketch polynomial functions and Finding zeros using p and q

Week 2:
Day 1: Sketching Rational Functions Day 1
Day 2: Sketching Rational Functions Day 3
Day 3: Solving Rational and polynomial inequalities
Day 4: Radical Function
Day 5: Graphing Exponential and log functions

Week 3:
Day 1: Solving Exponential and Log equations and application
Day 2: Angle in x-y coordinate plane, application of trig ratios and special right triangle
Day 3: Trig Function in x-y coordinate plane, and evaluate trig ratios
Day 4: Graphing Trig Functions
Day 5: Trig Identities (With double angle formula)

Week 4:
Day 1: Solving Trig equation
Day 2: Last minute topics needed for Calculus
Day 3: Catch up day
Day 4: Unit test -Calculator
Day 5: Unit test – Non Calculator

The purpose of making this packet available to you during the summer is to give you an opportunity to refresh your previous years of learning, so feel free to start after July. ***Just do the best you can*** with all the topics in this packet. If there are concepts that you have mastered already, then you can complete that section, which means less work for you once the semester starts. If there are concepts that you are struggling in, then please mark those, and make sure you spend time paying attention in class, and practice with all the problems once we have reviewed those.

Topic: Do you know your function family and their characteristics?

<u>*Characteristics of functions*</u>:
Domain: The set of input values (x-values) where the function is defined.
Range: The set of output values (y-values) the function can produce.
***(You need to know how to write both interval and set notations)**

Intercepts: The point(s) where the graph intersects one of the axes.

Zeros: The input (x) value(s) that produce an output or function value of zero.

Discontinuity: The point where the function or graph is not continuous.
There are three types of discontinuity.

- Nonremovable discontinuity:
    - VA
    - Jump
- Removable Discontinuity:
    - Hole

End-behaviors: The behavior of the graph or function as the input value approaches positive or negative infinity. (*Write only using limit notation)

Increasing/decreasing interval: The interval of input (x) values where the function value is increasing or decreasing.

Extrema:

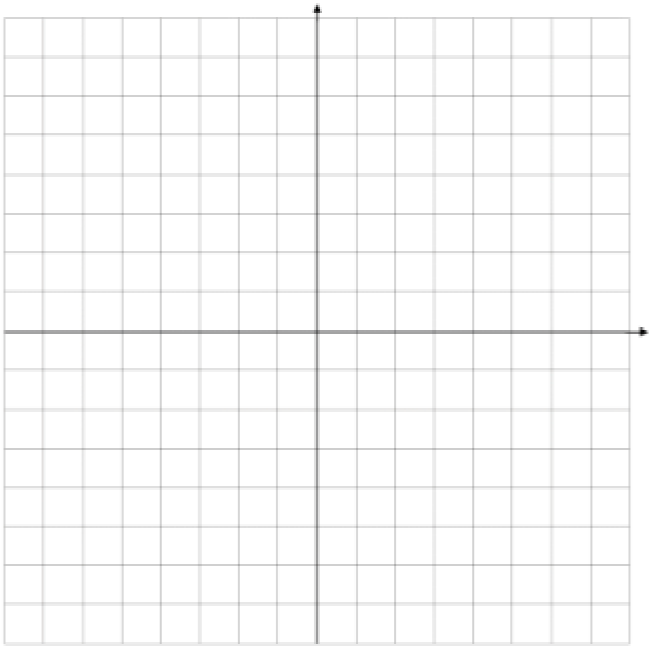
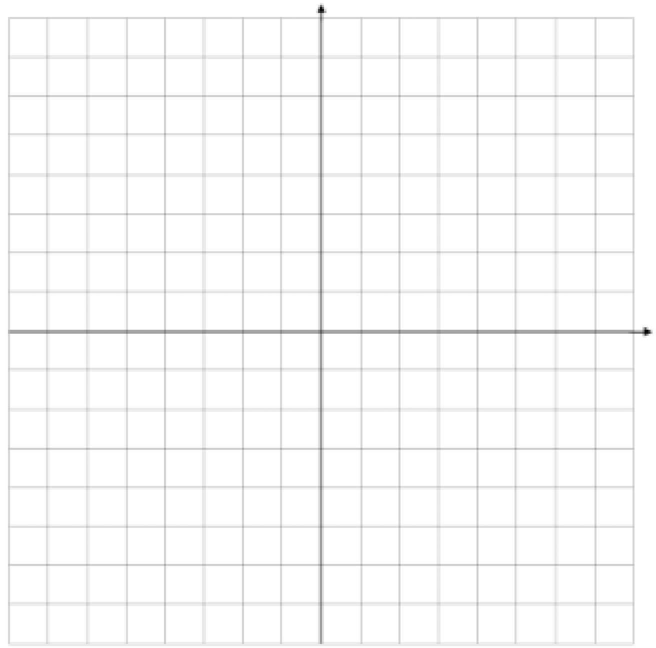
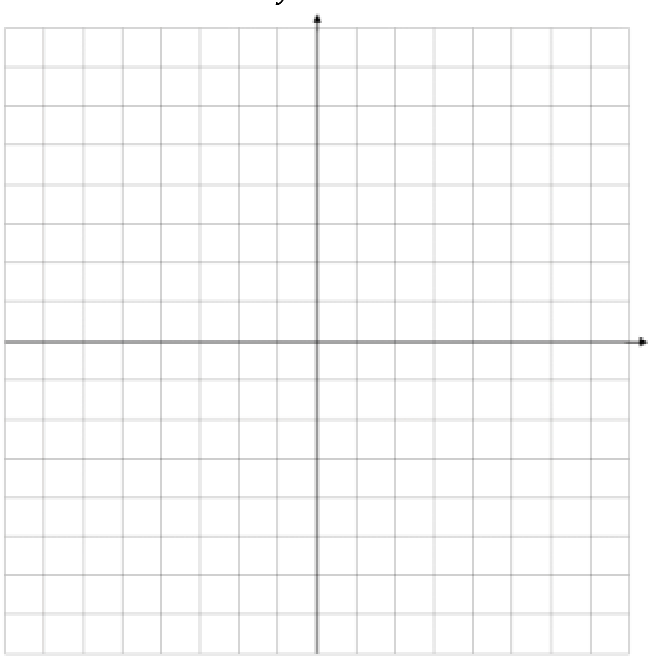
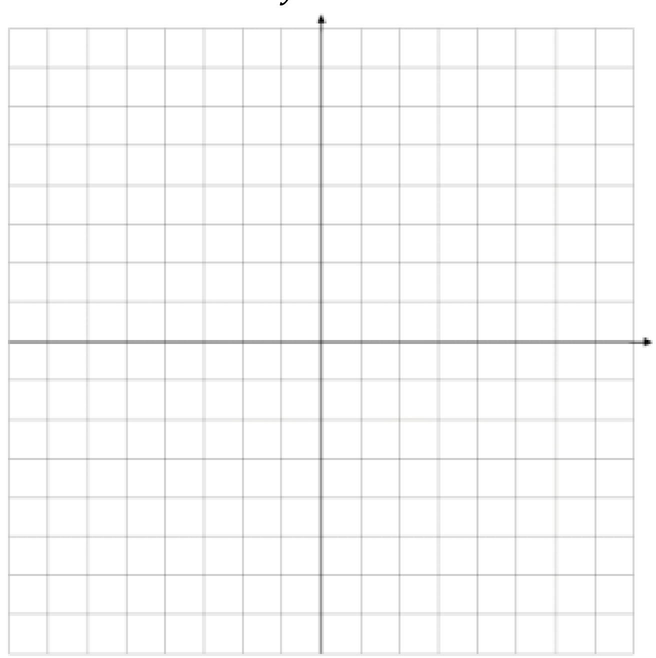
- Local/Relative: the highest or lowest output/function value comparing to it's immediate neighboring values. (Typically you will see a change of direction on the graphs with these)
- Global/Absolute: the highest or lowest output/function value within the domain of the graph.

Symmetry:

- Symmetric to the x-axis
- Symmetric to the y-axis (even function); $f(-x) = f(x)$
- Symmetric to the origin (odd function): $f(-x) = -f(x)$

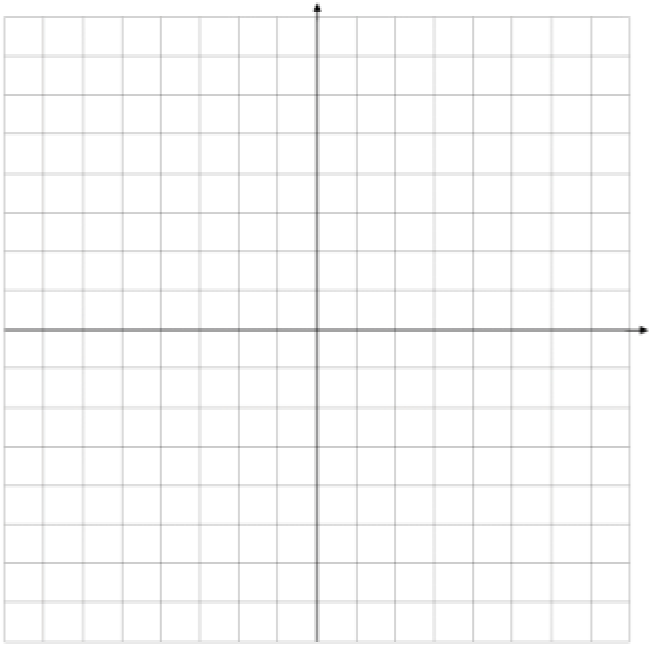
For the given parent functions: sketch a graph, and then write the characteristics of the functions.

Polynomial Function family:

Constant Function $y = k, k \in R$	**Linear Function** $y = x$
	
Quadratic Function $y = x^2$	**Cubic Function** $y = x^3$
	

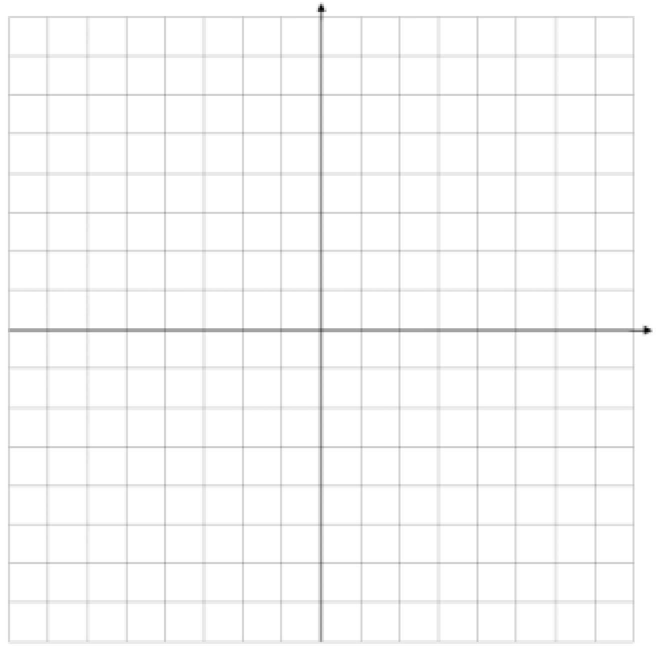
Rational Function

$y = \frac{1}{x}$



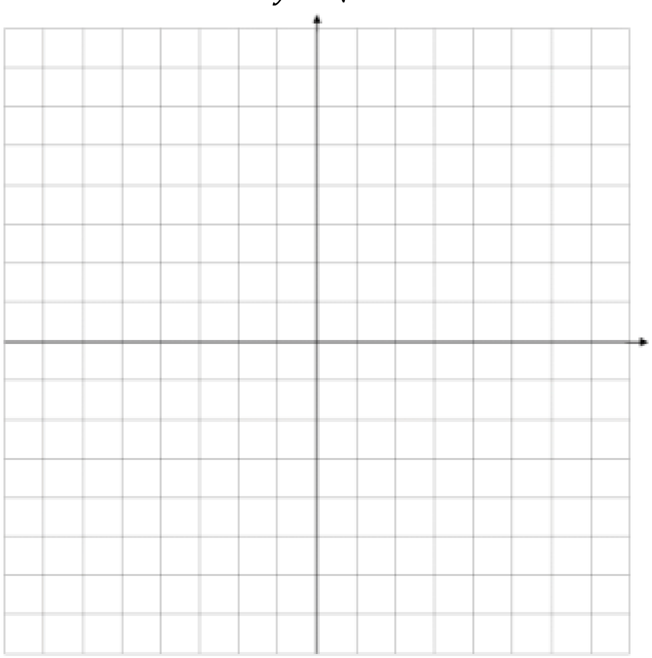
Rational Function

$y = \frac{1}{x^2}$



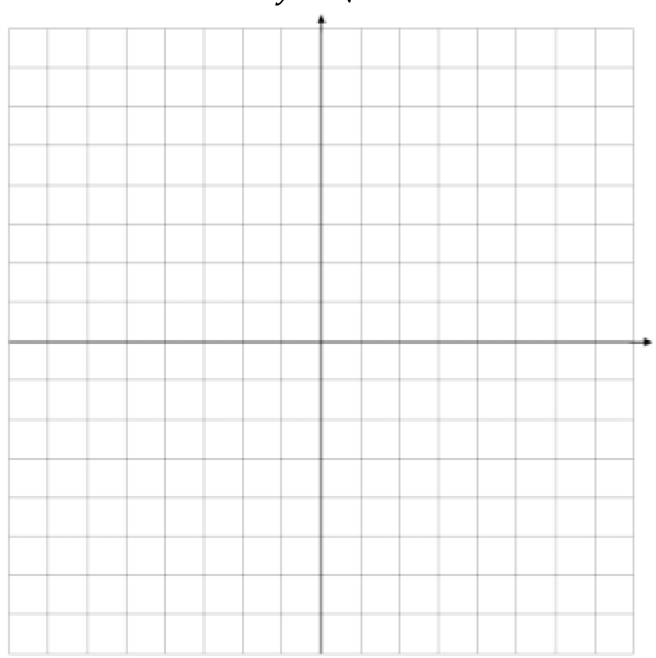
Radical Function family:
Square root Function

$y = \sqrt{x}$



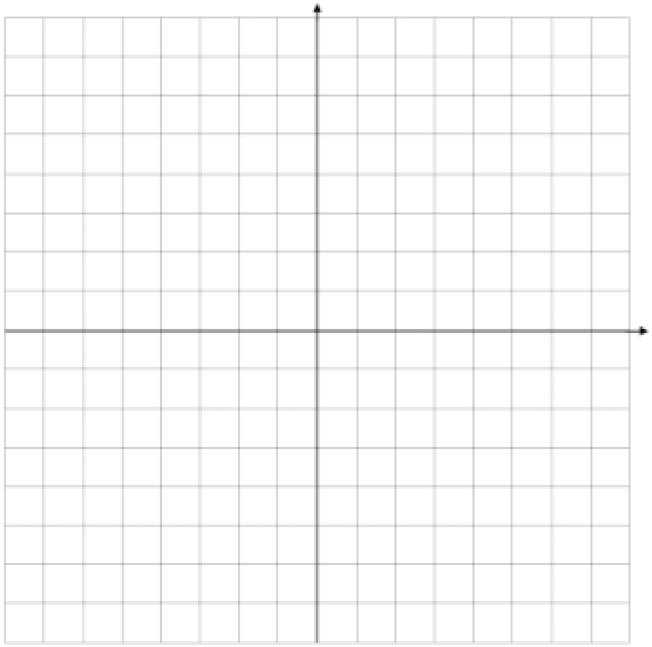
Radical Function family:
Cube root Function

$y = \sqrt[3]{x}$



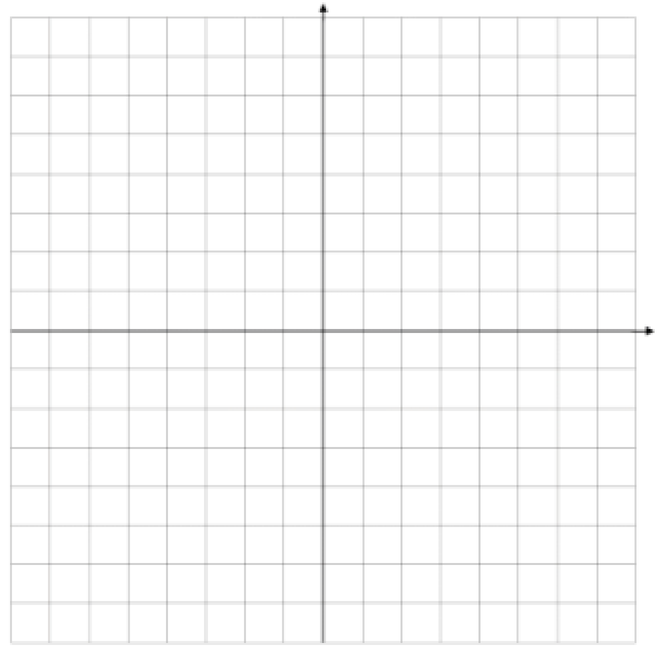
Exponential Function:

$y = a^x, a > 1$



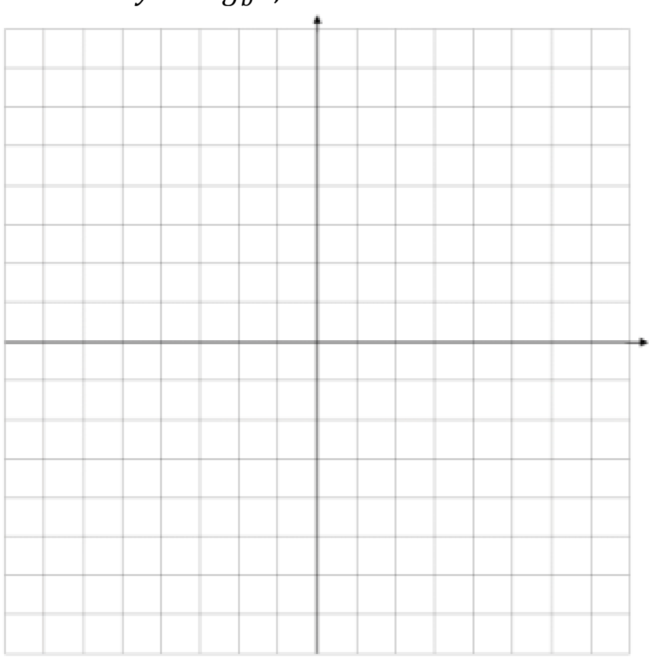
Exponential Function

$y = a^x, 0 < a < 1$



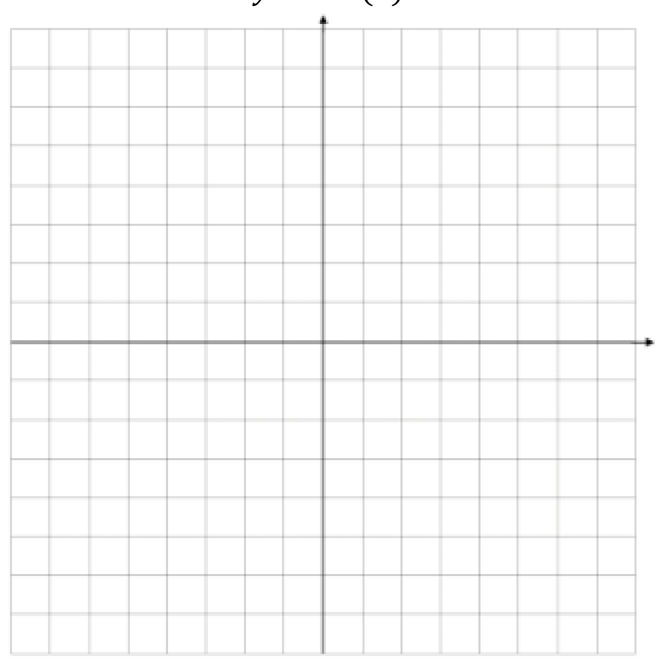
Log Function

$y = log_b x, b > 0 \ and \ b \neq 1$

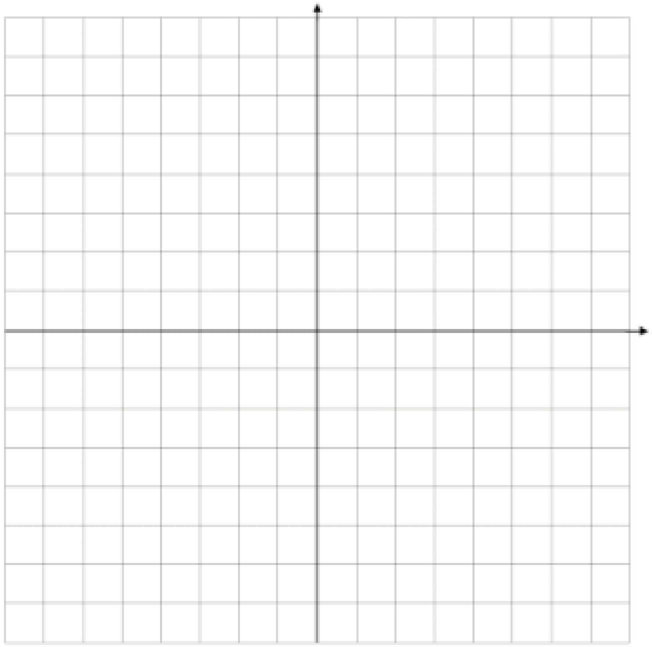


Trig Function family:

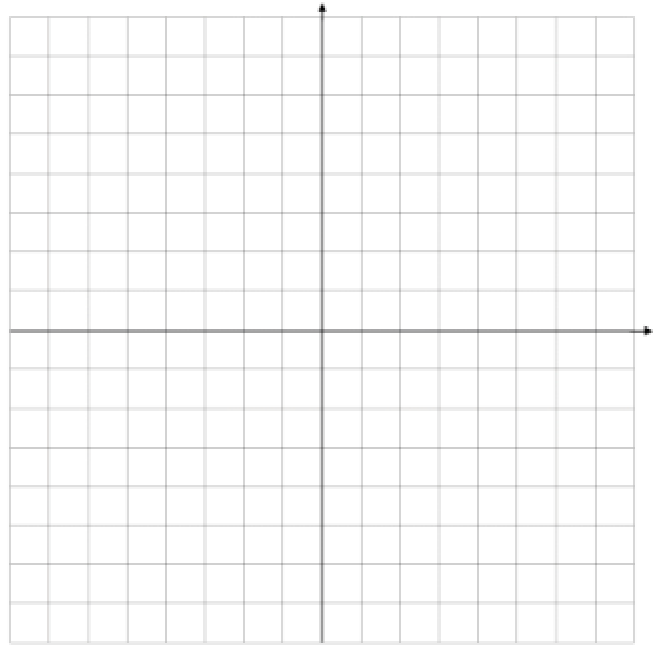
$y = \sin(x)$



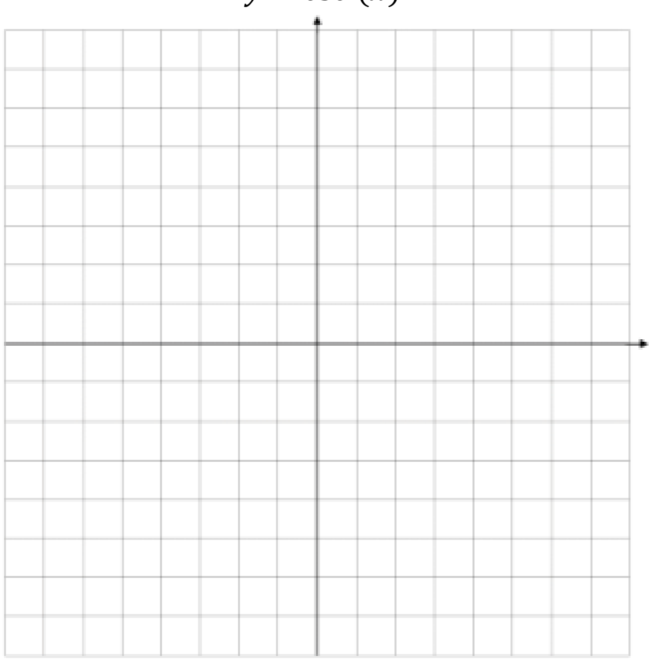
Trig Function family:
$y = cos\ (x)$



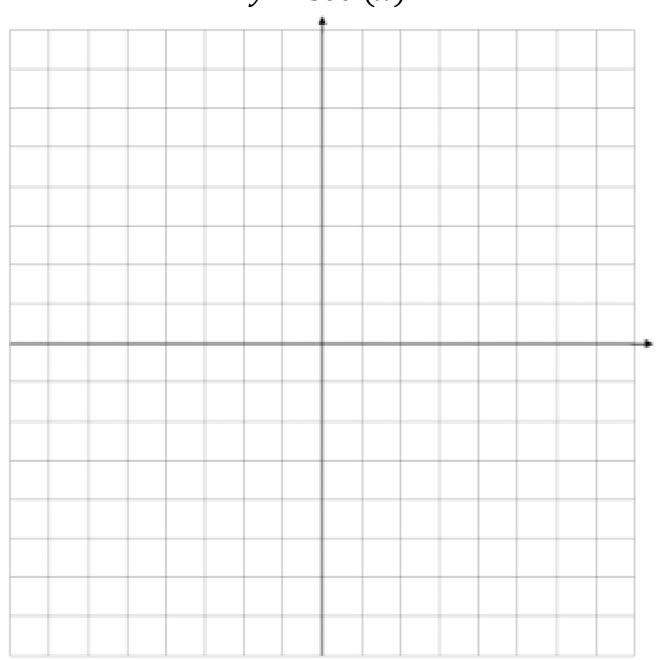
Trig Function family:
$y = tan\ (x)$



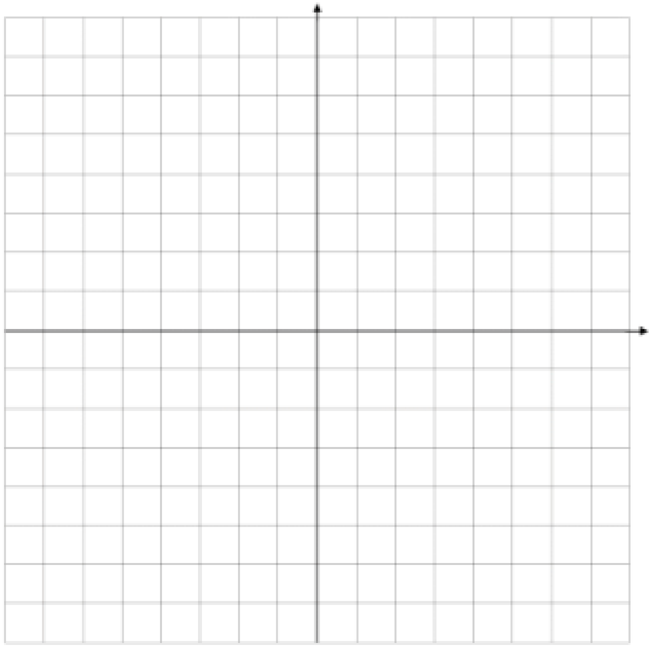
Trig Function family:
$y = csc\ (x)$



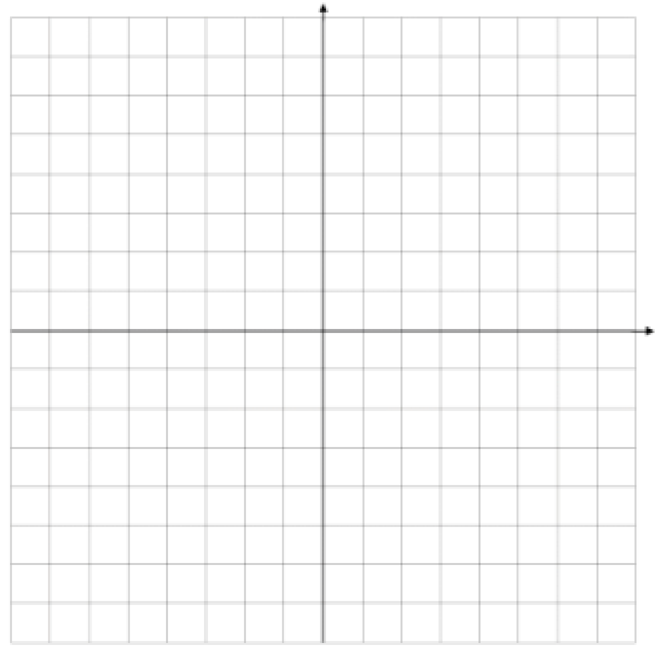
Trig Function family:
$y = sec\ (x)$



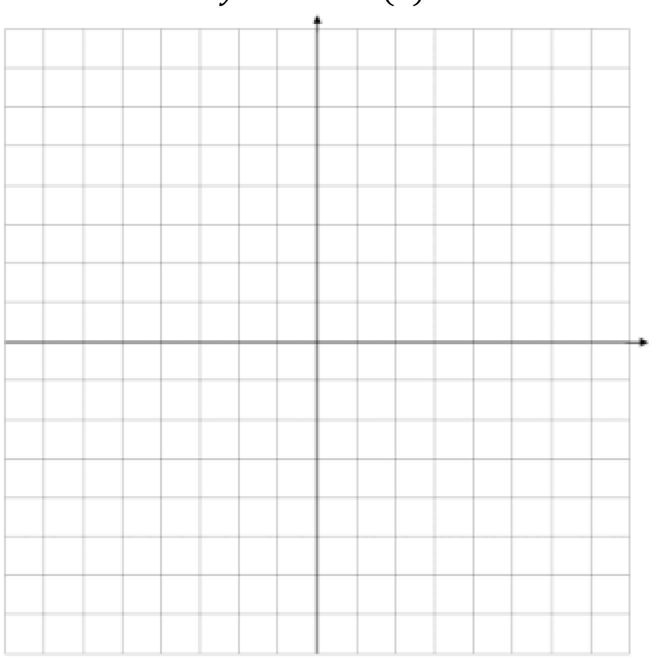
Trig Function family:
$y = cot\ (x)$



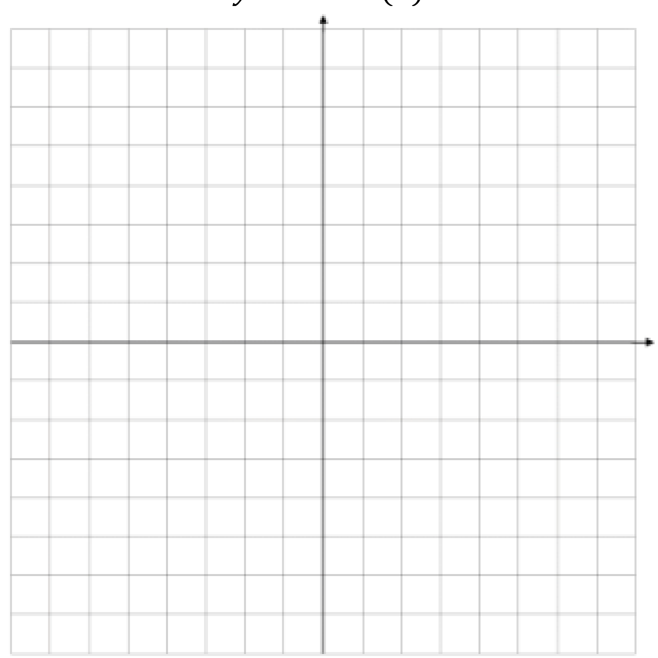
Inverse Trig Function family:
$y = arcsin\ (x)$



Inverse Trig Function family:
$y = arccos\ (x)$

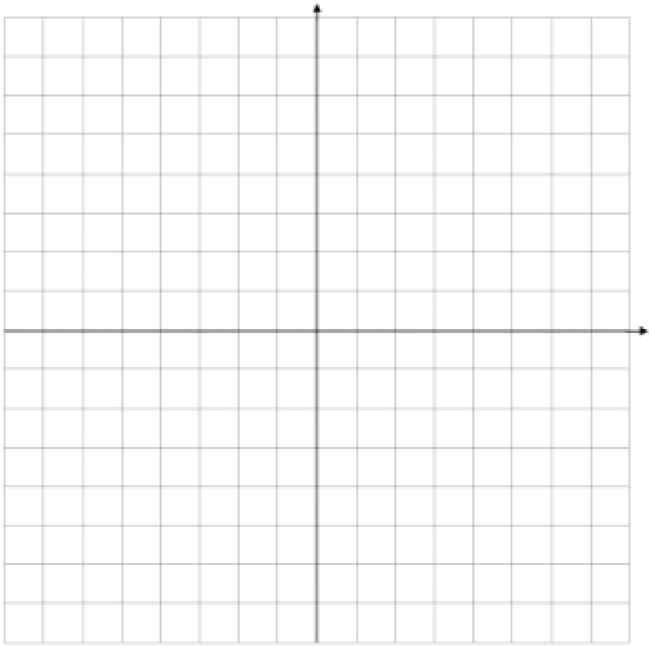


Inverse Trig Function family:
$y = tan^{-1}\ (x)$



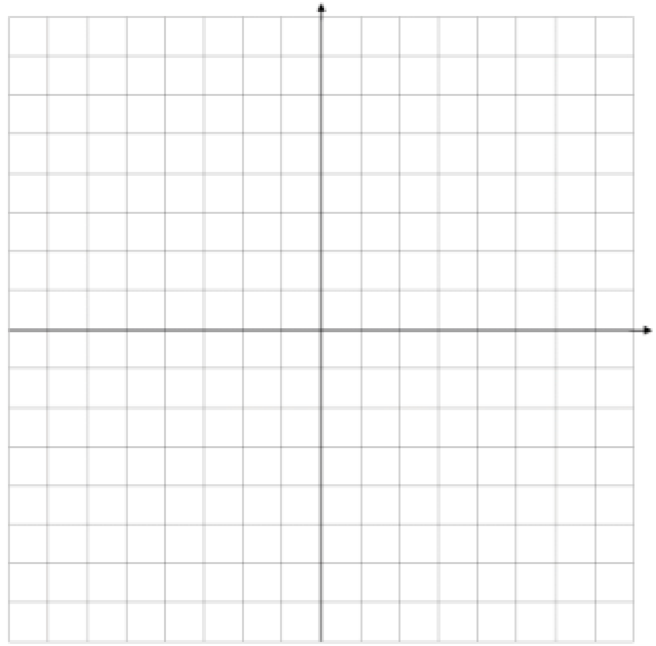
Piecewise Function Family:

$y = |x|$

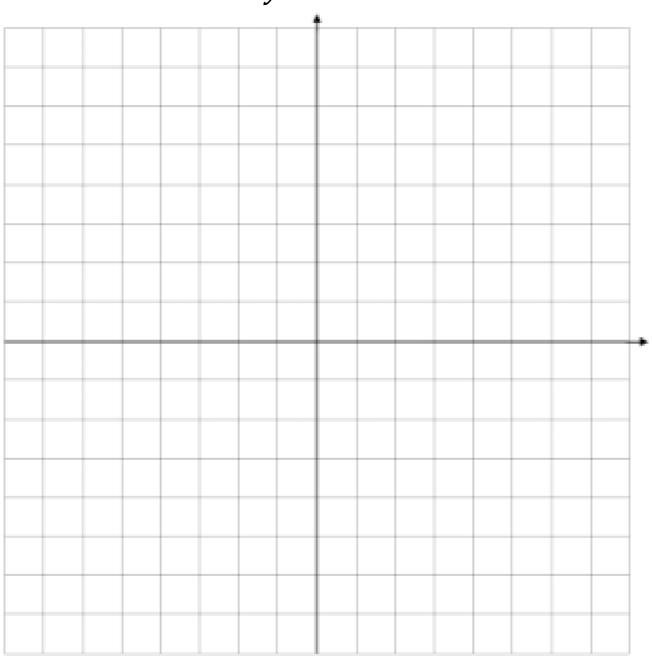


Piecewise Function Family:
Greatest integer function

$y = [\![x]\!]$



$y = \sqrt[3]{x^2}$



Topic: Transformation of functions:

- Reflection (Vertical and horizontal)
- Stretch or shrink (Vertical and horizontal)
- Shift (Vertical and horizontal)

Resources:

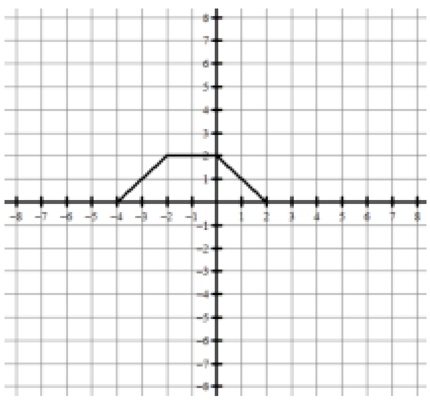
https://precalculus.flippedmath.com/112a-translations-of-functions.html

https://precalculus.flippedmath.com/112b-dilations-of-functions.html

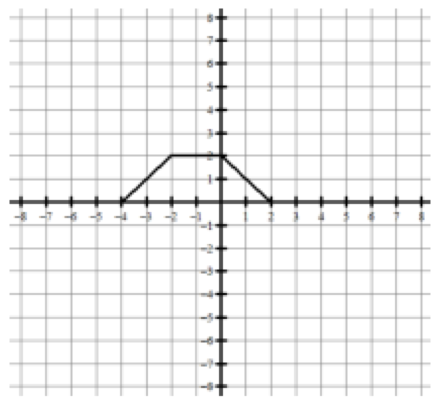
Transformation set 1

GRAPHICAL TRANSFORMATION. Use the graph of f to graph $g(x)$.

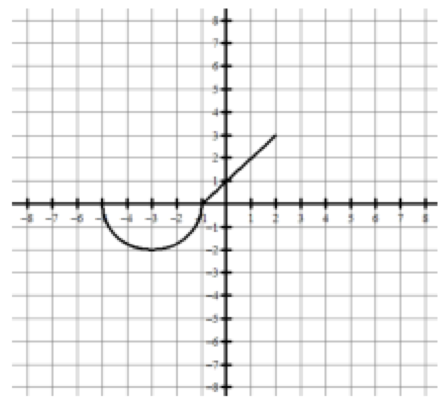
1. $g(x) = f(x-2) + 4$



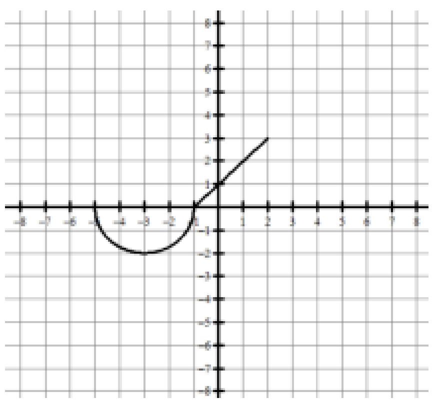
2. $g(x) = -f(x+3)$



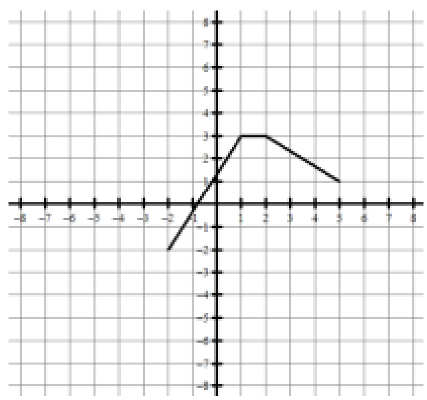
3. $g(x) = -f(x) + 5$



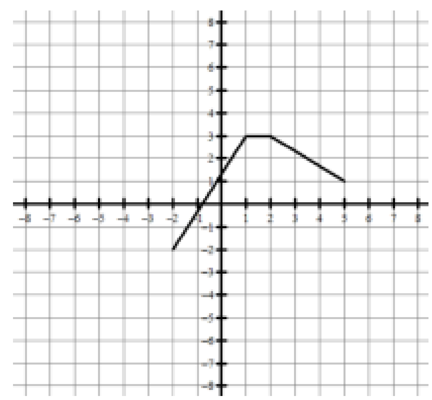
4. $g(x) = f(x-5) - 3$



5. $g(x) = f(x) - 4$



6. $g(x) = -f(x-3) + 1$



9. $f(x) = x^3 + 2x^2$

   $g(x) = -f(x) + 5$, find $g(x)$.

10. $f(x) = 2x^2 - 3x + 1$

    $g(x) = f(x-2) + 5$, find $g(x)$.

NUMERIC TRANSFORMATION. Use the table of values to answer the following.

11. Given the table of values for f.

x	$f(x)$
-6	2
-3	8
2	15
5	-2
8	-13

Let $g(x) = f(x) + 2$, find $g(5)$.

12. Given the table of values for f.

x	$f(x)$
0	0
1	2
2	4
3	8
4	16

Let $g(x) = f(x+2) - 3$, find $g(1)$.

13. Given the table of values for f.

x	$f(x)$
-4	-32
-2	6
0	-8
2	21
4	14

Let $g(x) = -f(x-2)$, find $g(4)$.

DOMAIN AND RANGE TRANSFORMATION. Find the domain and range of the transformed function.

14.
Given the graph for f has a domain of $(-5,3]$ and range of $[-4,8]$.

Let $g(x) = f(x+5)$.

Find the domain and range of $g(x)$.

15.
Given the graph for f has a domain of $(0,5)$ and range of $[-10,4]$.

Let $g(x) = f(x-2) + 4$.

Find the domain and range of $g(x)$.

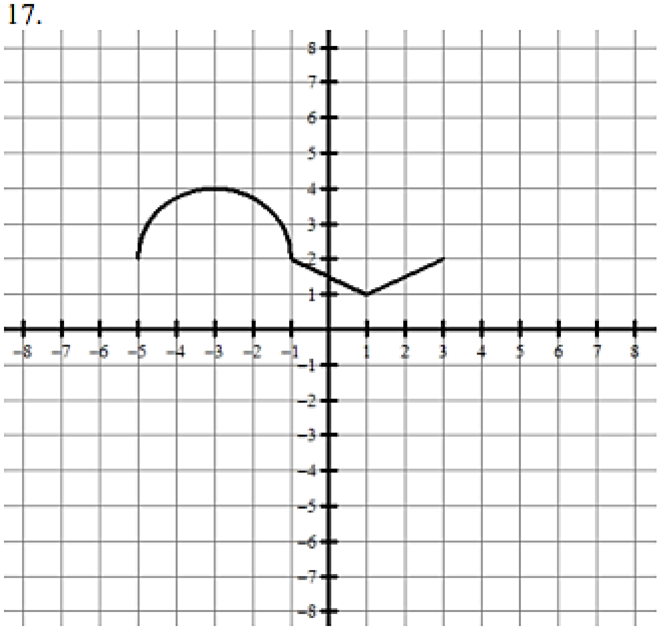
16.
Given the graph for f has a domain of $[-2,4]$ and range of $(-1,8)$.

Let $g(x) = -f(x+3) + 5$.

Find the domain and range of $g(x)$.

Use the graph f to answer the following.

17.



Let the $g(x) = -f(x+3) + 2$

a. Graph the $g(x)$.

b. State the domain of $g(x)$.

c. State the range of $g(x)$.

d. Find $g(-2)$.

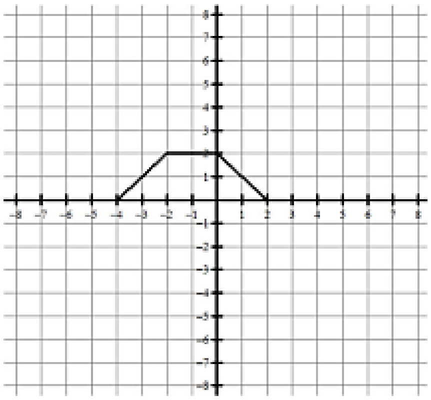
e. Find the zeroes of $g(x)$.

f. Find the y-intercept of $g(x)$.

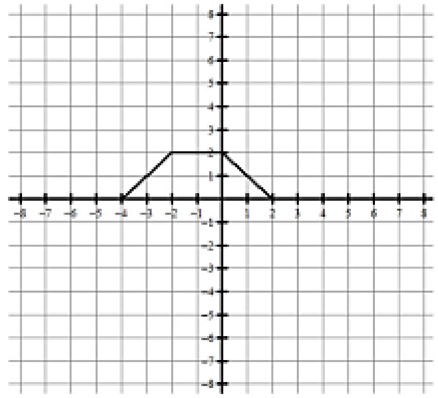
Transformation set 2:

GRAPHICAL TRANSFORMATION. Use the graph of f to graph $g(x)$.

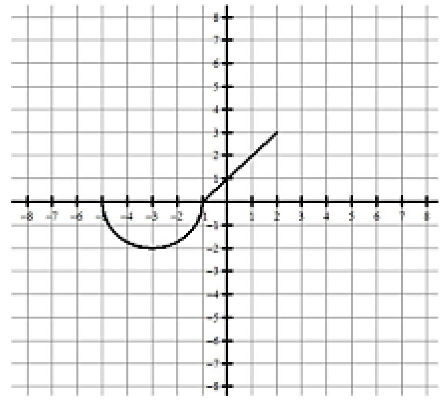
1. $g(x) = 3f(x) - 5$



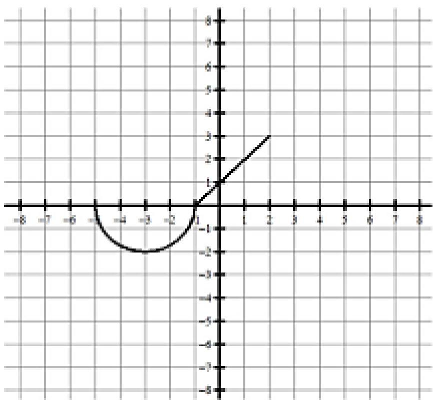
2. $g(x) = f\left(\frac{1}{2}x\right)$



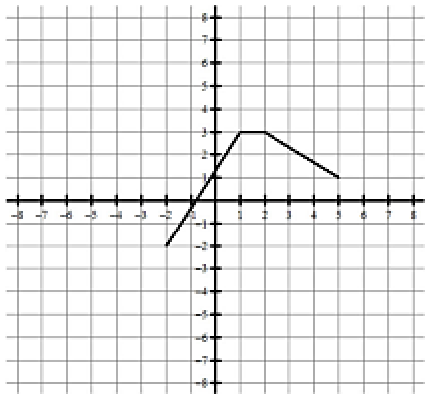
3. $g(x) = -f(x-2)$



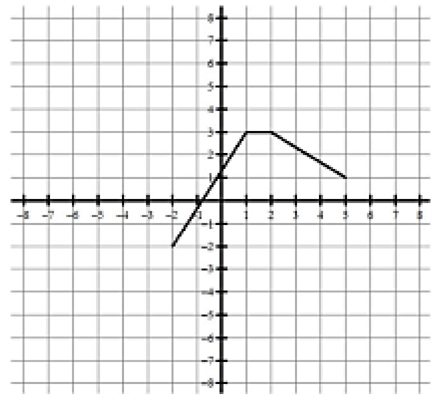
4. $g(x) = \frac{1}{2}f(x)$



5. $g(x) = f(-x) - 4$



6. $g(x) = f(2(x+1))$



ALGEBRAIC TRANSFORMATION. Express the $g(x)$ in terms of x.

7. $f(x) = 4x + 3$

   $g(x) = 3f(x) + 5$, find $g(x)$.

8. $f(x) = 2x + 6$

   $g(x) = f\left(\frac{1}{2}(x+2)\right) - 1$, find $g(x)$.

9. $f(x) = 4x - 5$

   $g(x) = -2f(x+1) + 5$, find $g(x)$.

10. $f(x) = 2x^2 - 3x + 1$

    $g(x) = f(2x) + 3$, find $g(x)$.

NUMERIC TRANSFORMATION. Use the table of values to answer the following.

11. Given the table of values for f.

x	$f(x)$
-9	2
-3	8
2	15
6	-2
8	-13

Let $g(x) = f(3x) + 1$, find $g(2)$.

12. Given the table of values for f.

x	$f(x)$
0	0
1	2
2	4
3	8
4	16

Let $g(x) = 2f(x+2) - 3$, find $g(1)$.

13. Given the table of values for f.

x	$f(x)$
-8	-32
-2	6
0	-8
2	21
8	14

Let $g(x) = 4f(-x)$, find $g(2)$.

DOMAIN AND RANGE TRANSFORMATION. Find the domain and range of the transformed function.

14.
Given the graph for f has a domain of $(-6,8]$ and range of $[-4,8]$.

Let $g(x) = -3f(2x)$.

Find the domain and range of $g(x)$.

15.
Given the graph for f has a domain of $(0,5)$ and range of $[-10,4]$.

Let $g(x) = 2f(x-3) + 4$.

Find the domain and range of $g(x)$.

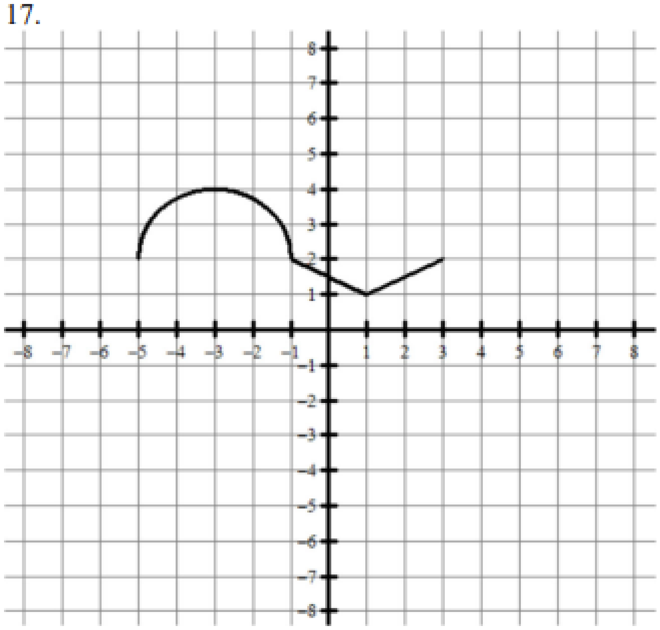
16.
Given the graph for f has a domain of $[-2,4]$ and range of $(-6,8)$.

Let $g(x) = f\left(\frac{1}{2}x\right) + 5$.

Find the domain and range of $g(x)$.

Use the graph f to answer the following.

17.



Let the $g(x) = 2f(x-3) - 1$

a. Graph the $g(x)$.

b. State the domain of $g(x)$.

c. State the range of $g(x)$.

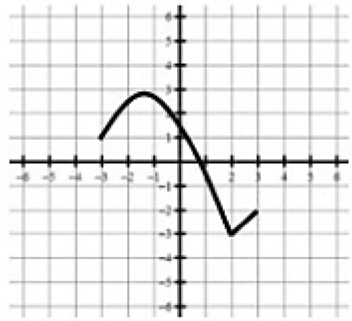
d. Find $g(-2)$.

e. Find the zeroes of $g(x)$.

f. Find the y-intercept of $g(x)$.

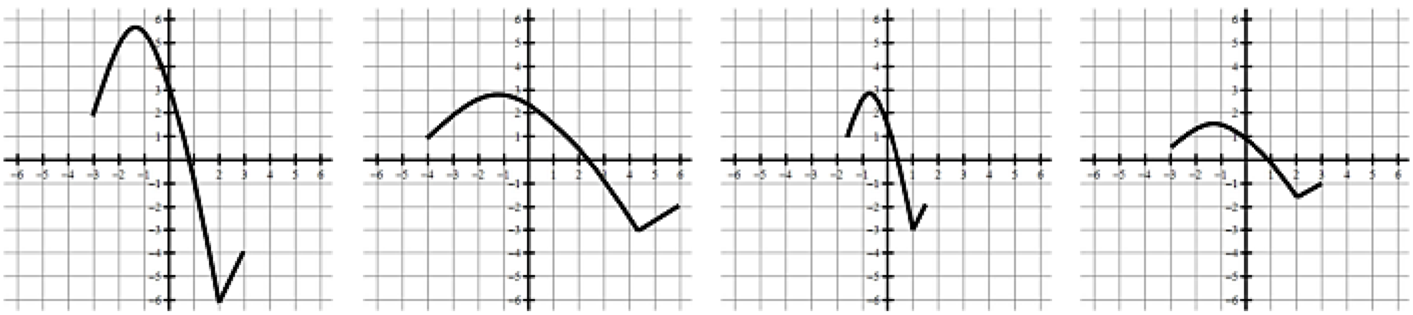
Multiple Choice

18. The graph of $y = f(x)$ is shown for $-3 \leq x \leq 4$.



Which of the following is the transformed graph for $y = f(2x)$?

(A) (B) (C) (D)



19. The table gives values for a polynomial function f at selected values of x.

x	-2	0	2	4	8	16
$f(x)$	38	12	-24	-32	-12	-48

In the *xy*-plane, the graph of g is constructed by applying three transformations to the graph of f in this order: a horizontal dilation of $\frac{1}{2}$, a vertical dilation of 4 and vertical translation by 6 units. What is the value of $g(4)$?

(A) 3

(B) -42

(C) -90

(D) 0

20. The function g is defined by f such that $g(x) = 3f(2x) - b$. If $f(2) = a, f(4) = 8, g(1) = 7$ and $g(2) = 19$ what is the value of a ?

(A) 6

(B) 4

(C) 2

(D) -2

Topic: Symmetry and Operation with Functions

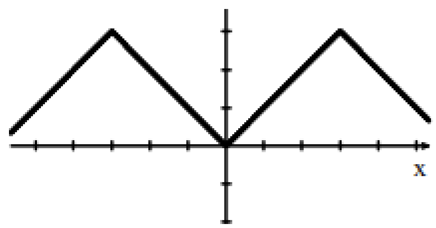
Symmetry:

- Symmetric to the x-axis
- Symmetric to the y-axis (even function); $f(-x) = f(x)$
- Symmetric to the origin (odd function): $f(-x) = -f(x)$

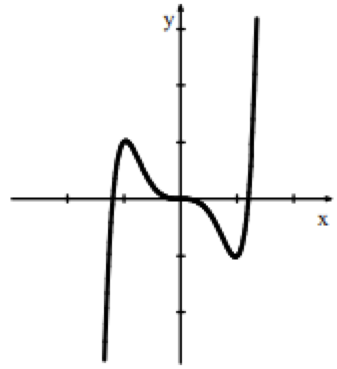
Determine algebraically whether each function is even, odd, or neither. SHOW WORK!	
1. $y = \frac{1}{2}x^2 + 5$	2. $y = x^2 + 3x - 5$
3. $y = x^3 + 3x$	4. $f(x) = \frac{x^2-4}{x^4+2}$
5. $f(x) = 1 - 2x^5$	6. $f(x) = \frac{1+x^2}{x}$

State whether the following graphs represent functions that are even, odd, or neither.

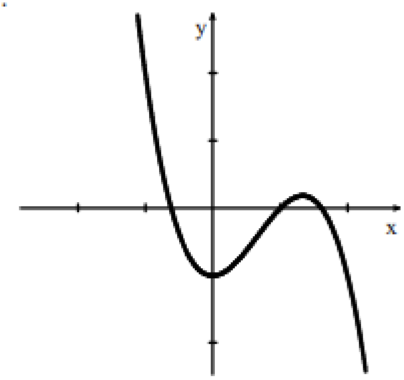
1.



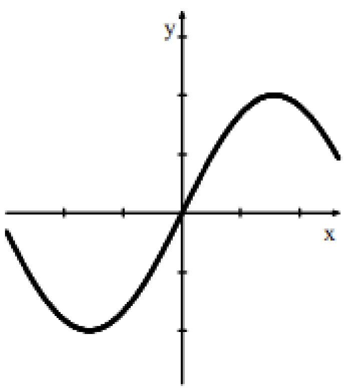
2.



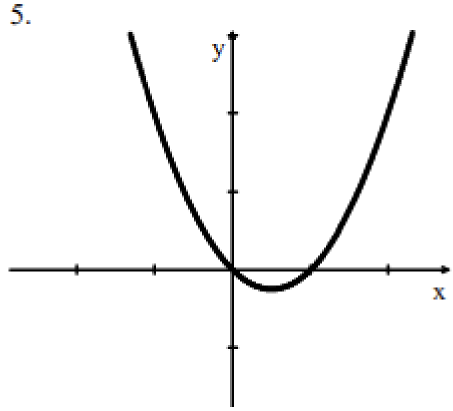
3.



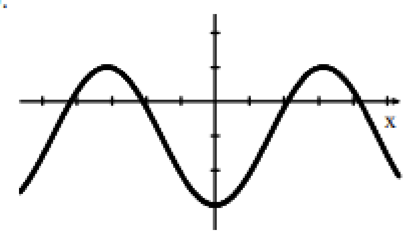
4.



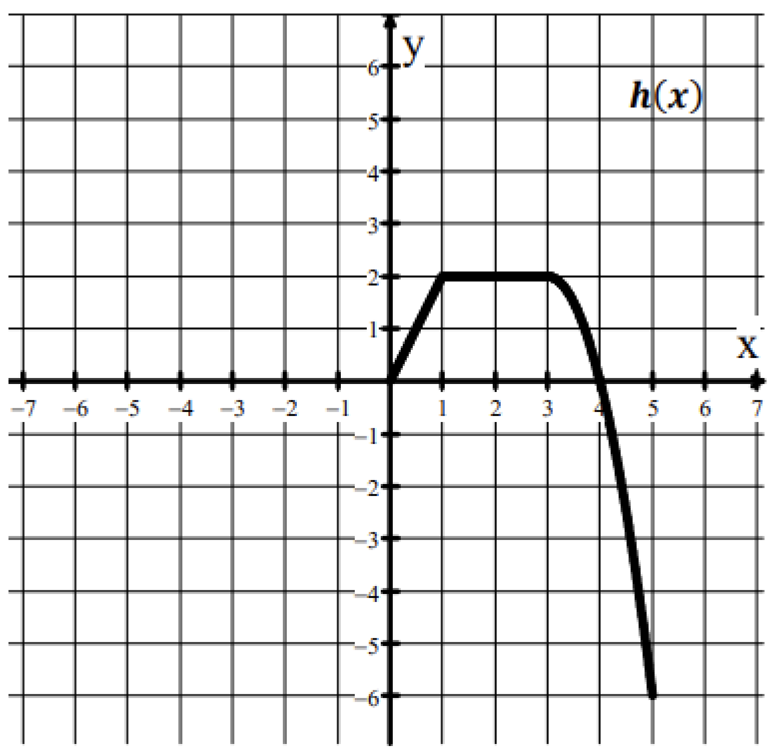
5.



6.



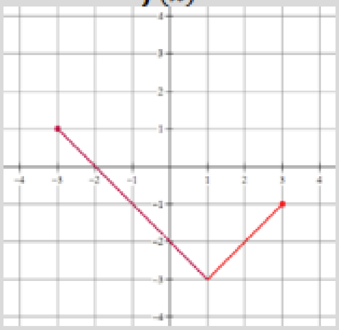
7. Given that $h(x)$ is continuous on $-5 \leq x \leq 5$ and odd, draw the graph $h(x)$ from $-5 \leq x \leq 0$.



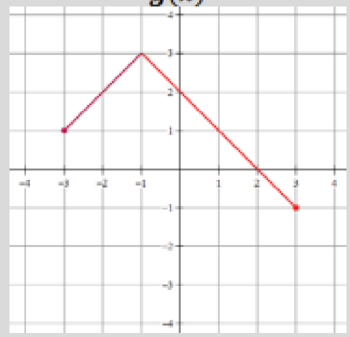
Topic: Operation with Function

For 1-4, use the graphs of the functions f and g.

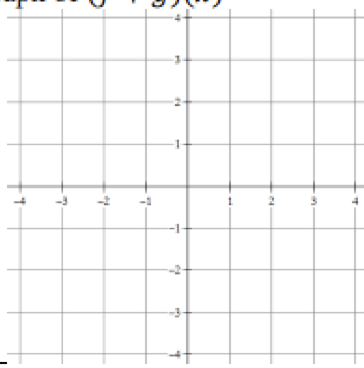
$f(x)$



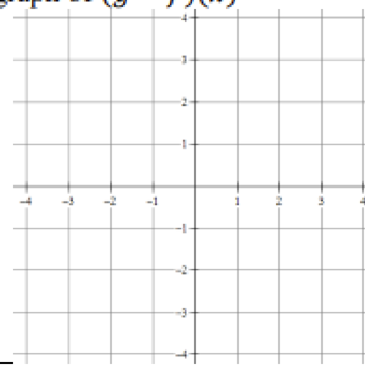
$g(x)$



1. Sketch a graph of $(f + g)(x)$



2. Sketch a graph of $(g - f)(x)$



3. Find $g(f(2))$

4. Find $(f \circ g)(2)$

Find the functions of $f + g, f - g, fg$, and $\frac{f}{g}$, and find their domains.

5. $f(x) = 3x + 5$ and $g(x) = x^2 - 1$

$f + g =$ **Domain:**

$f - g =$ **Domain:**

$fg =$ **Domain:**

$\frac{f}{g} =$ **Domain:**

6. $f(x) = 3x$ and $g(x) = x - 2$

$f + g =$ **Domain:**

$f - g =$ **Domain:**

$fg =$ **Domain:**

$\frac{f}{g} =$ **Domain:**

Find the following compositions and find their domains when necessary.	
7. $f(x) = 2x$ and $g(x) = 8 - x^3$	8. $f(x) = \sqrt{x-1}$ and $g(x) = x^2$
$(f \circ g)(x) =$ **Domain:**	$(f \circ g)(x) =$ **Domain:**
$(g \circ f)(x) =$ **Domain:**	$(g \circ f)(x) =$ **Domain:**
$f(g(-2)) =$	$g(f(3)) =$

Given $f(x) = x^2 - 2x + 5$, find the following.		
1. $f(-2) =$	2. $f(x+2) =$	3. $f(x+h) =$

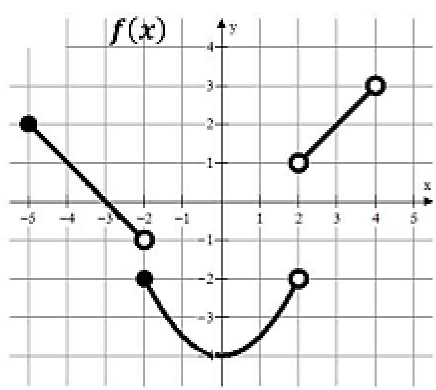
Use the graph $f(x)$ to answer the following.

4. $f(0) =$ $f(4) =$

$f(-1) =$ $f(-2) =$

$f(2) =$ $f(3) =$

$f(x) = 2$ when $x = ?$ $f(x) = -3$ when $x = ?$



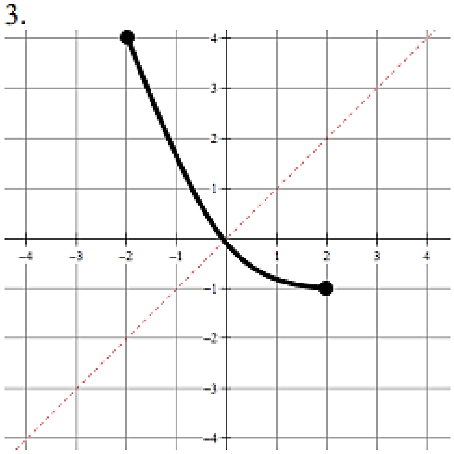
Topic: Inverse Functions

One to one function: All monotonic functions are one to one function.

Determine if g is the inverse of f.	
1. $f(x) = 2x - 4$ and $g(x) = \frac{1}{2}x - 2$	2. $f(x) = (x-3)^3 + 4$ and $g(x) = \sqrt[3]{x-4} + 3$

Find the domain and range of f, sketch the graph of f^{-1}, and find the domain and range of f^{-1}. The graph of $y = x$ is provided.

3.



f

D:

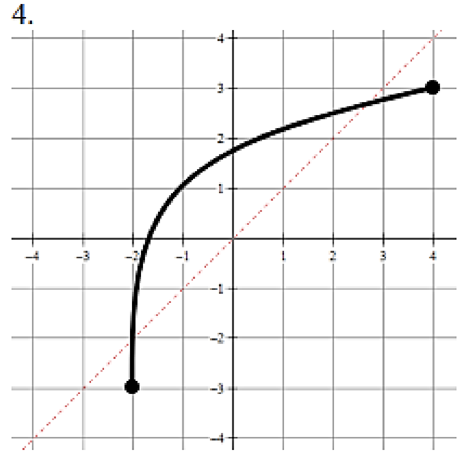
R:

f^{-1}

D:

R:

4.



f

D:

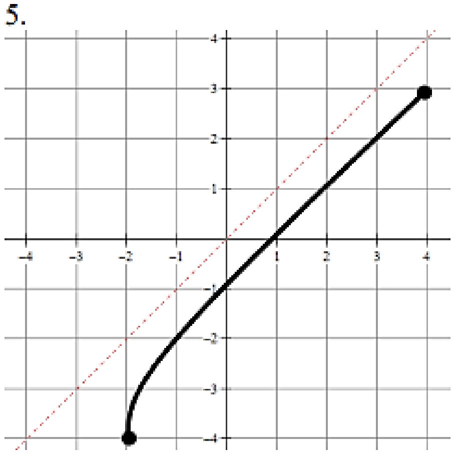
R:

f^{-1}

D:

R:

5.



f

D:

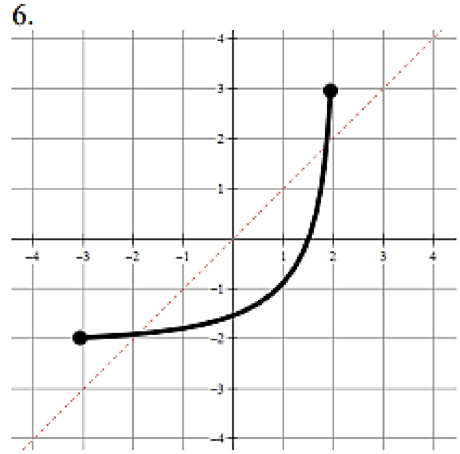
R:

f^{-1}

D:

R:

6.



f

D:

R:

f^{-1}

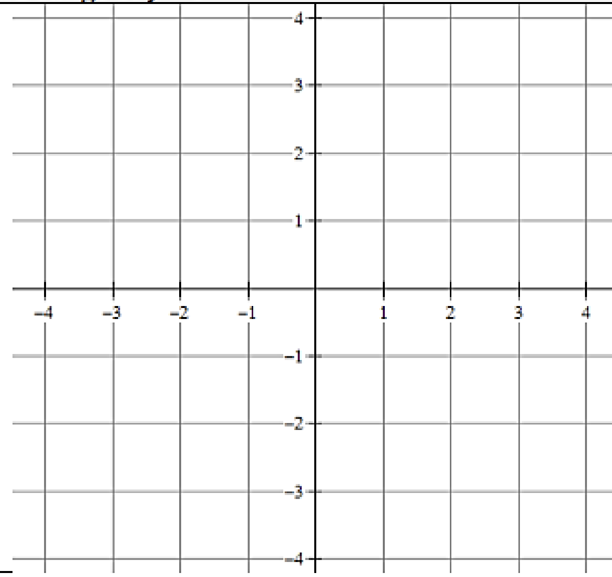
D:

R:

Graph *f* and verify that *f* is one-to-one function. Find f^{-1} and add the graph of f^{-1} and the line $y = x$ to the graph *f*. State the domain and range of *f* and the domain and range of f^{-1}.

7. $f(x) = \sqrt{x+2} - 3$

f	f^{-1}
D:	**D:**
R:	**R:**



If $f(x) = \{(3,5),(2,4),(1,7)\}$, $h(x) = \{(3,2),(4,3),(1,6)\}$, $g(x) = \sqrt{x-3}$, $k(x) = x^2 + 5$, **then determine each of the following.**

81. $(f+h)(1)$

82. $(k-g)(5)$

83. $f(h(3))$

84. $g(k(7))$

85. $h(3)$

86. $g(g(9))$

87. $f^{-1}(4)$

88. $k^{-1}(x)$

89. $k(g(x))$

90. $g(f(2))$

Topic: Solve Polynomial Function/Equations

Factor each expression completely.

Practice 7.3

1. $x^3 + x^2 - 6x$
2. $2x^4 - 12x^3 + 18x^2$
3. $10x^4 - 90x^2$
4. $x^3 - 7x^2 + 12x$

Factor each expression using the sum of cubes formula.

$$a^3 + b^3 = (a+b)(a^2 - ab + b^2)$$

5. $27x^3 + 125$
6. $8x^3 + 27$
7. $8x^3 - 1$
8. $64 - x^3$

Factor each expression by grouping.

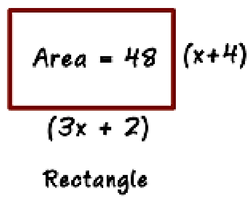
9. $x^3 + 5x^2 - 6x - 30$
10. $7r^3 - 42r^2 - 3r + 18$
11. $5n^3 + 40n^2 - n - 8$
12. $6x^3 - x^2 + 42x - 7$

Factor each quadratic form.

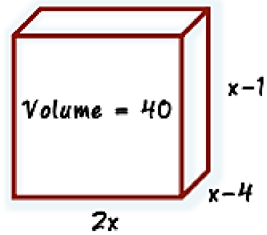
13. $x^4 + 6x^2 - 16$
14. $m^4 - 1$
15. $5a^5 + 55a^3 + 150a$
16. $4x^5 - 16x^3 + 12x$

4. Find the possible value(s) of x.

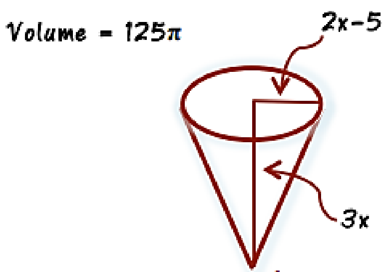
a.



b.



c.

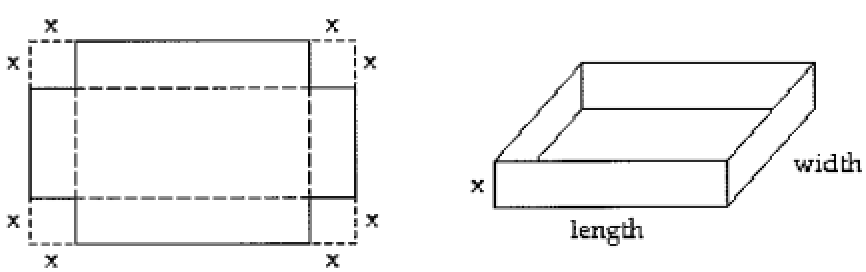


$$V_{cone} = \frac{1}{3}\pi r^2 h$$

5. The longer leg of a right triangle is one unit shorter than twice the length of the shorter leg. The hypotenuse is one unit longer than twice the length of the shorter leg. Use the Pythagorean Theorem to find the lengths of each side of the right triangle.

Solve the problem.

11) A box with an open top is formed by cutting squares out of the corners of a rectangular piece of cardboard and then folding up the sides. If x represents the length of the side of the square cut from each corner, and if the original piece of cardboard is 16 inches by 9 inches, what size square must be cut if the volume of the box is to be 120 cubic inches?



Topic: Rational Zero theorem and Synthetic substitution

Use the Rational Zero Theorem to list all possible rational zeros for the given function.

1) $f(x) = x^5 - 4x^2 + 6x + 5$

2) $f(x) = x^5 - 2x^2 + 2x + 14$

3) $f(x) = 6x^4 + 3x^3 - 3x^2 + 3x - 5$

Find a rational zero of the polynomial function and use it to find all the zeros of the function.

4) $f(x) = x^3 + 2x^2 - 5x - 6$

5) $f(x) = x^3 - 8x^2 + 16x - 8$

6) $f(x) = x^4 - 9x^3 + 48x^2 - 78x - 136$

7) $f(x) = 2x^4 + 19x^3 + 71x^2 + 109x + 39$

Topic: Polynomial Function Curve Sketching

*Consider multiplicity of zeros and the end behaviors of the graph.

Find the end behavior (both left and right), the zeros and their behavior:

1. $F(x) = -x(x-2)^2$

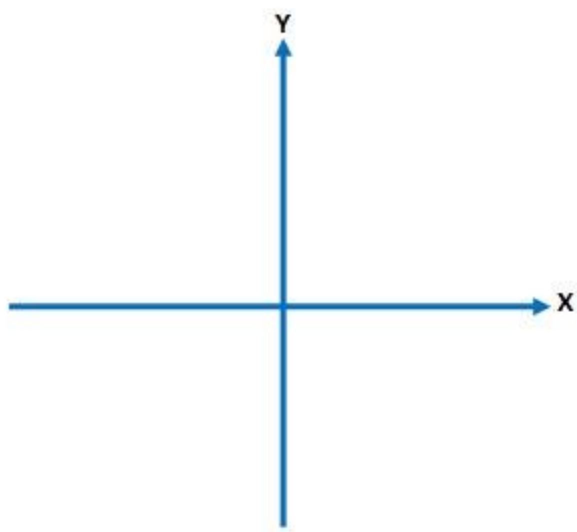
2. $G(x) = 2x^3 - 5x^2 - 2x + 5$

3. $G(x) = 3(x+2)^4(x-3)$

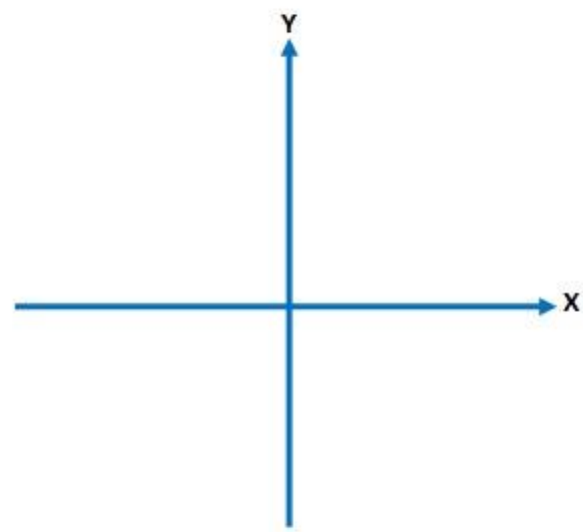
4. $R(x) = 5x^2 - 5x^4$

Sketch the following functions without graphing technology.

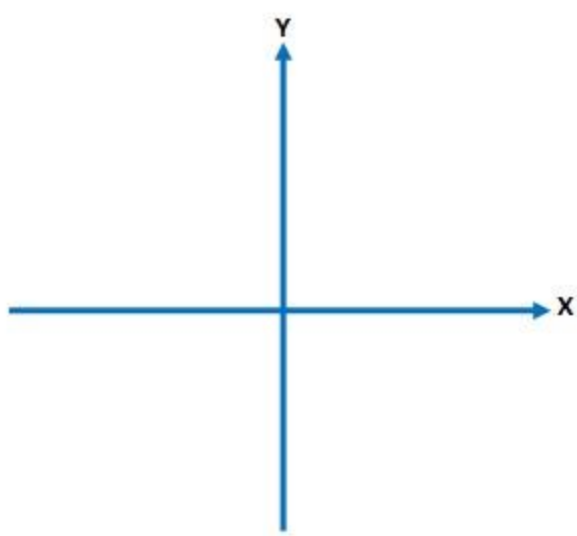
10. $J(x) = x(x-5)^4(x-3)^{11}$



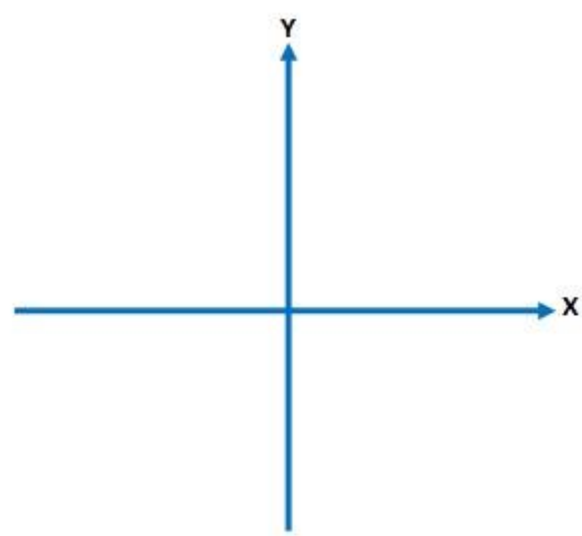
11. $F(x) = x^3 - x^2 - 9x + 9$



15. $T(x) = 2x(2x+1)(x-5)^2$



16. $D(x) = -3x^3 + 27x$



Topic: Sketching Rational Functions (VA)

Find the domain and vertical asymptote(s) of the following rational function if one exists.

1. $f(x) = \frac{x(x+2)}{x^2-4}$

Domain:

Vertical Asymptote(s):

2. $d(t) = \frac{t^2+4t-12}{(t-2)^2}$

Domain:

Vertical Asymptote(s):

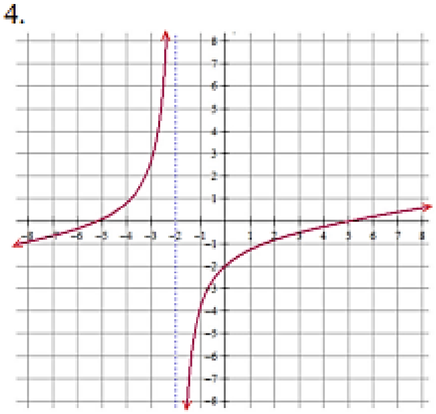
3. $h(x) = \frac{x^3-3x^2}{x^2+8x+15}$

Domain:

Vertical Asymptote(s):

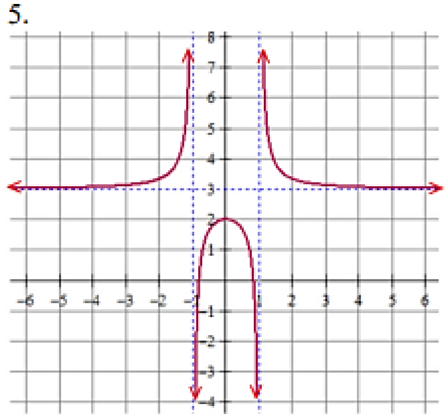
State the vertical asymptote(s) of the following rational functions. Use limit notation.

4.



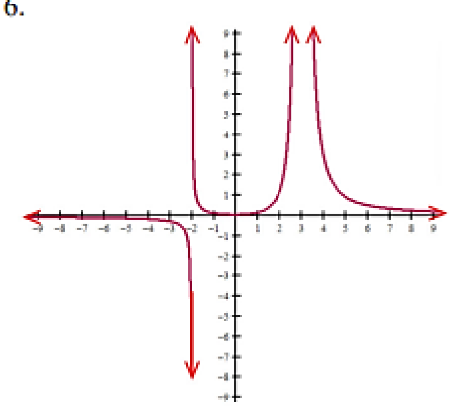
Limit Notation Vertical Asymptote(s):

5.



Limit Notation Vertical Asymptote(s):

6.



Limit Notation Vertical Asymptote(s):

CALCULATOR ACTIVE Complete the table to answer the following.

7. $f(x) = \frac{x^2-1}{x-4}$

x	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$							

Vertical Asymptote:

Limit Notation of Vertical Asymptote:

CALCULATOR ACTIVE Complete the table to answer the following.

8. $f(x) = \frac{x^2-2x}{x+2}$

x	-2.1	-2.01	-2.001	-2	-1.999	-1.99	-1.9
$f(x)$							

Vertical Asymptote:

Limit Notation of Vertical Asymptote:

Use the table of the rational function *h* to find the following.

9.

t	$d(t)$
-0.1	5,589
-0.01	37,231
-0.001	96,543
-0.0001	148,234
0	undefined
0.0001	128,341
0.001	89,437
0.01	18,235
0.1	1,455

a. Find $d(0) =$

b. Find the *y*-intercept.

c. Find $\lim_{t\to 0^-} d(t) =$

d. Find $\lim_{t\to 0^+} d(t) =$

e. As *t* approaches zero from the left the $d(t)$…

f. As *t* approaches zero from the right the $d(t)$…

Make a sketch of the rational function with the following characteristics.

10. The graph of f has…

a. $f(-4) = 0$

b. $f(6) = 0$

c. $\lim_{x\to -3^-} f(x) = -\infty$

d. $\lim_{x\to -3^+} f(x) = \infty$

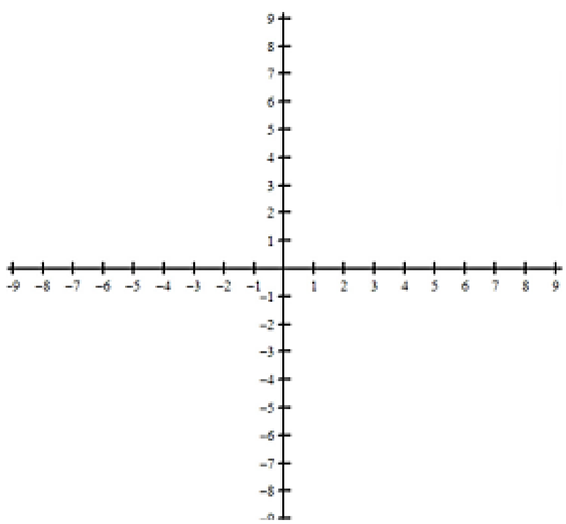
e. $\lim_{x\to 4^-} f(x) = \infty$

f. $\lim_{x\to 4^+} f(x) = -\infty$

g. $\lim_{x\to -\infty} f(x) = 2$

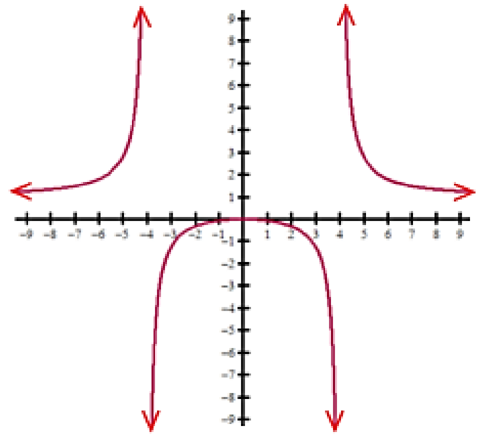
h. $\lim_{x\to \infty} f(x) = 2$

i. $f(0) = 5$



Multiple Choice

11. Given the graph of f. Which of the following describes the function f?



(A) $\lim_{x \to -4^-} f(x) = -\infty$ and $\lim_{x \to -4^+} f(x) = -\infty$

(B) $\lim_{x \to -4^-} f(x) = \infty$ and $\lim_{x \to -4^+} f(x) = -\infty$

(C) $\lim_{x \to -4^-} f(x) = -\infty$ and $\lim_{x \to -4^+} f(x) = \infty$

(D) $\lim_{x \to -4^-} f(x) = \infty$ and $\lim_{x \to -4^+} f(x) = \infty$

(E) $\lim_{x \to -4} f(x) = f(0)$

Free Response

12. The function f is a rational function graphed in the xy-plane. The polynomial in the numerator of f has exactly one real zero at $x = 3$. The polynomial of the denominator of f has exactly two real zeros at both $x = 3$ and $x = 6$. The multiplicities of the zeros at $x = 3$ in the numerator and in the denominator are equal.

a. Find the domain for the graph of f.

b. Describe any holes and/or vertical asymptotes for the graph of f.

c. Explain how your answer from part b would change if the multiplicities of the zeros at $x = 3$ in the numerator and denominator were not equal?

Topic: Sketching Rational Functions (hole)

Find the hole(s) of the following rational function if one exists.

1. $f(x) = \frac{x^2+3x}{x^2-9}$

Hole(s):

2. $d(t) = \frac{t^2+t-20}{t+5}$

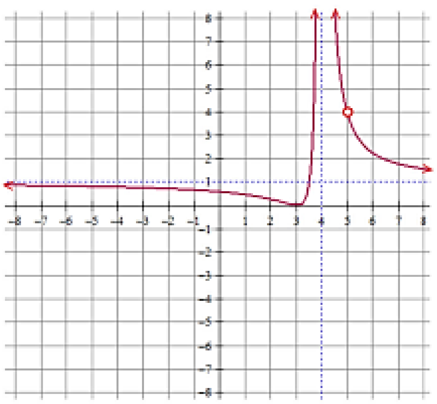
Hole(s):

3. $h(x) = \frac{(x+5)^2}{x^2+8x+15}$

Hole(s):

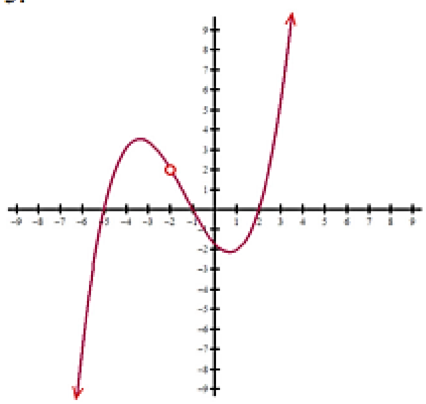
State the hole(s) of the following rational functions. Use limit notation.

4.



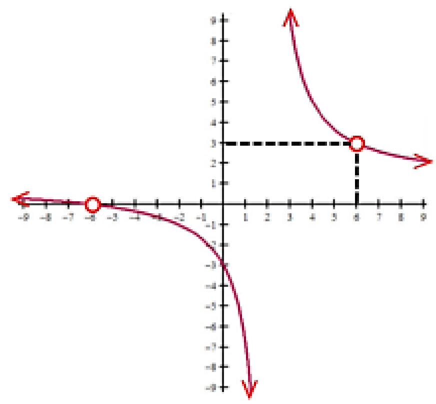
Limit Notation Hole(s):

5.



Limit Notation Hole(s):

6.



Limit Notation Hole(s):

CALCULATOR ACTIVE Complete the table to answer the following.

7. $f(x) = \frac{x^2-16}{x-4}$

x	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$							

Hole: Limit Notation of Hole:

CALCULATOR ACTIVE Complete the table to answer the following.

8. $f(x) = \frac{x^2-2x-8}{4x+8}$

x	-2.1	-2.01	-2.001	-2	-1.999	-1.99	-1.9
$f(x)$							

Hole:

Limit Notation of Hole:

Use the table of the rational function *d* to find the following.

9.

t	$d(t)$
-3.1	4.134
-3.01	4.15
-3.001	4.1893
-3.0001	4.1998
-3	undefined
-2.9999	4.2014
-2.999	4.231
-2.99	4.305
-2.9	4.37

a. Find $\lim_{t \to -3^-} d(t) =$

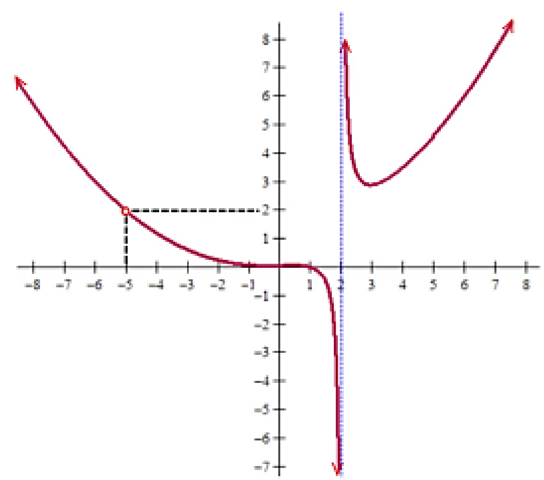
b. Find $\lim_{t \to -3^+} d(t) =$

c. As *t* approaches negative three from the left the $d(t)$…

d. As *t* approaches negative three from the right the $d(t)$…

Use the graph of the rational function *f* to find the following.

10.



a. $f(6) =$

b. $f(2) =$

c. $\lim_{x \to -5^-} f(x) =$

d. $\lim_{x \to -5^+} f(x) =$

e. $\lim_{x \to 2^-} f(x) =$

f. $\lim_{x \to 2^+} f(x) =$

g. $\lim_{x \to -\infty} f(x) =$

h. $\lim_{x \to \infty} f(x) =$

i. Domain =

Multiple Choice

11. The function f is given by $f(x) = \frac{x^2-9}{x^2+8x+15}$. Which of the following describes the function f ?

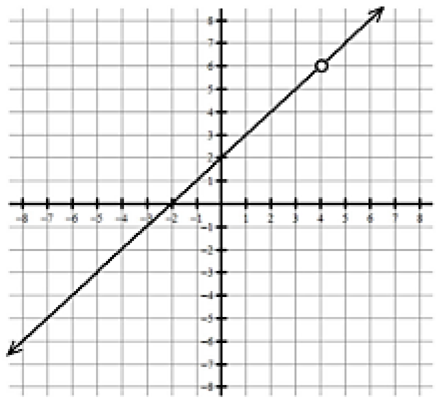
(A) There is a hole at $x = 5$.

(B) There is a hole at $x = -5$.

(C) There is a hole at $x = 3$.

(D) There is a hole at $x = -3$.

For questions 12-13 use the graph of *f*.



graph of *f*

12. The figure shows the graph of a function f. Which of the following could be an expression for the $f(x)$?

(A) $\frac{(x+2)(x-4)}{(x+2)}$

(B) $\frac{(x-2)(x+4)}{(x-2)}$

(C) $\frac{(x+2)(x-4)}{(x-4)}$

(D) $\frac{(x-2)(x+4)}{(x+4)}$

13. The figure shows the graph of a function f. Which of the following must be true?

(A) $f(2) = 0$

(B) $\lim_{x \to 4^+} f(x) = \infty$

(C) $\lim_{x \to 4^+} f(x) = 3f(0)$

(D) $f(-2) + f(0) = 0$

Topic: Sketching Rational Function Set 3

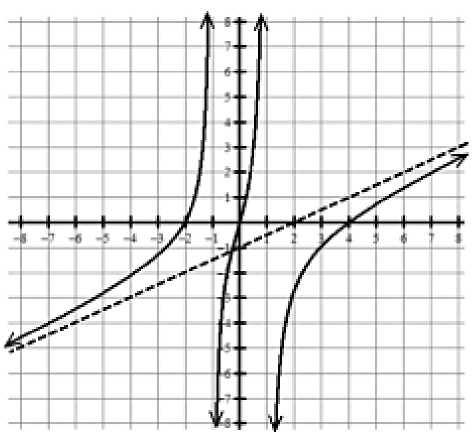
Divide the following using long division or synthetic division.

1. $\frac{3x^3-4x^2-3}{x^2+5x+1}$

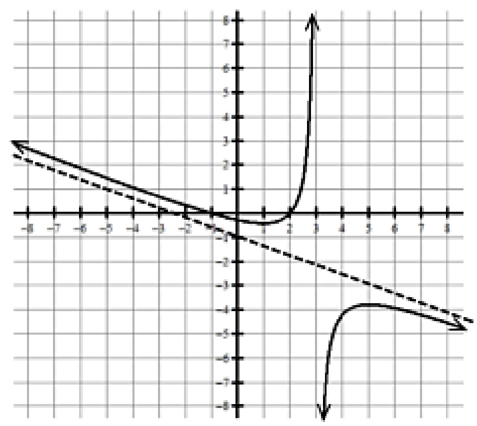
2. $\frac{x^3-4x^2+6x-4}{x-2}$

Use the graph of f to write the equation of the slant asymptote.

3.



4.



Determine if the following functions have a horizontal asymptote, slant asymptote, or neither.

5. $f(x)=\frac{4x^5-3x^3+4x+1}{5x^3-2x^2+1}$

Circle one:

The graph of f has a horizontal asymptote.

The graph of f has a slant asymptote.

The graph of f does not have a horziontal or slant asymptote.

6. $f(x)=\frac{2x^4+x^2+1}{3x^4-2x^2+5x}$

Circle one:

The graph of f has a horizontal asymptote.

The graph of f has a slant asymptote.

The graph of f does not have a horziontal or slant asymptote.

7. $f(x)=\frac{x^3+5x^2+x+2}{3x^4-2x^3+2x^2-3}$

Circle one:

The graph of f has a horizontal asymptote.

The graph of f has a slant asymptote.

The graph of f does not have a horziontal or slant asymptote.

Write the equation for the slant asymptote for the following functions.

8. $f(x) = \frac{x^3-2x^2-4x+1}{x^2-2x+1}$

9. $f(x) = \frac{x^2-9x+4}{x+6}$

10. $f(x) = \frac{4x^2+12x-6}{2x+1}$

11. $f(x) = \frac{9x^4-5x^2+3x-6}{3x^3-4x^2}$

Use the rational function to answer the following.

12.

$$f(x) = \frac{3x^3 - 12x}{x^2 - 2x - 8}$$

a. Domain:

b. Zero(s):

c. Hole(s):

d. Vertical Asymptote(s):

e. Horizontal Asymptote:

f. Slant Asymptote:

g. *y*-intercept:

h. *x*-intercept(s):

i. End Behavior:

Multiple Choice

13. The function f is a rational function. The quotient and remainder form of f is given by $f(x) = -2x + 1 + \frac{3x+4}{x^2-4x-12}$. Which describes the end behavior of f?

(A) $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = -\infty$

(B) $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = \infty$

(C) $\lim_{x \to -\infty} f(x) = -\infty$ and $\lim_{x \to \infty} f(x) = \infty$

(D) $\lim_{x \to -\infty} f(x) = \infty$ and $\lim_{x \to \infty} f(x) = -\infty$

14. Which of the following is equivalent to $\frac{x^2+5x+2}{x+5}$?

(A) $x + 1$

(B) $x + 2$

(C) $x + \frac{2}{x+5}$

(D) $x + 1 - \frac{4}{x+5}$

15. The function f is given by $f(x) = \frac{6x^2+ax+2}{x+3}$ and has a slant asymptote of $y = 6x + 3$. What is the value of a?

(A) -4

(B) 12

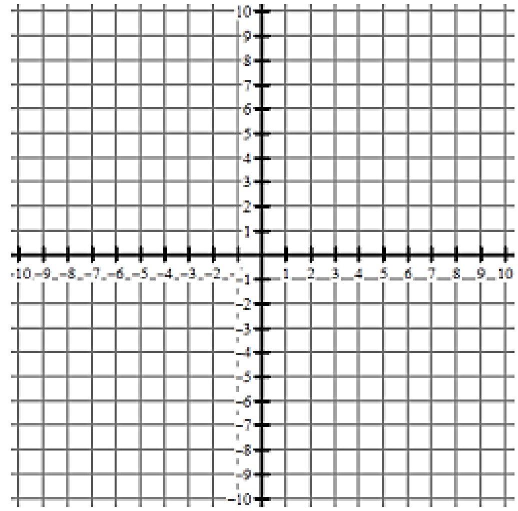
(C) 15

(D) 21

Topic: Sketching Rational Function Set 4

Directions: Find the information need and sketch. Include all relevant information on your graph.

1) $y = \frac{x^2-3x-4}{x-2}$



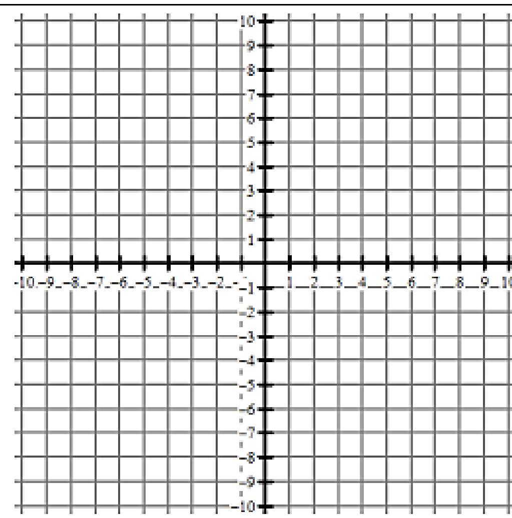
Hole/Vertical Asymptotes:

Y-Int:

X-int:

Horizontal/Slant Asymptote:

2) $y = \frac{3x^2+15x+18}{x^2+4x-5}$



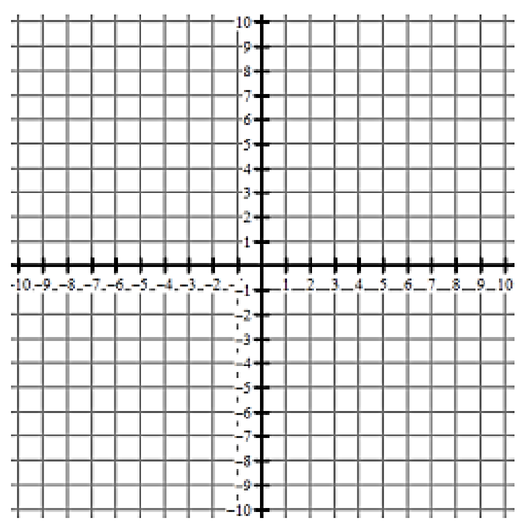
Hole/Vertical Asymptotes:

Y-Int:

X-int:

Horizontal/Slant Asymptote:

3) $y = \frac{3}{x^2-1}$



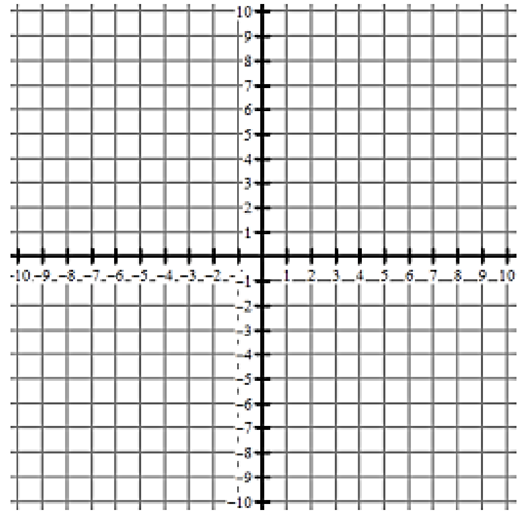
Hole/Vertical Asymptotes:

Y-Int:

X-int:

Horizontal/Slant Asymptote:

4) $y = \frac{x-4}{2-x}$

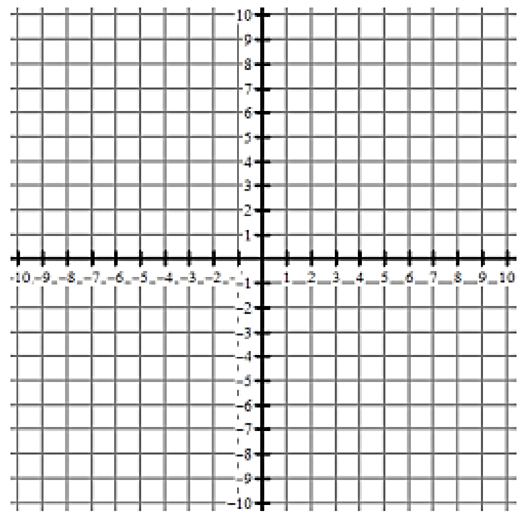


Hole/Vertical Asymptotes:

Y-Int: X-int:

Horizontal/Slant Asymptote:

5) $y = \frac{2x^3+7x^2+3x}{x^2+4x+4}$

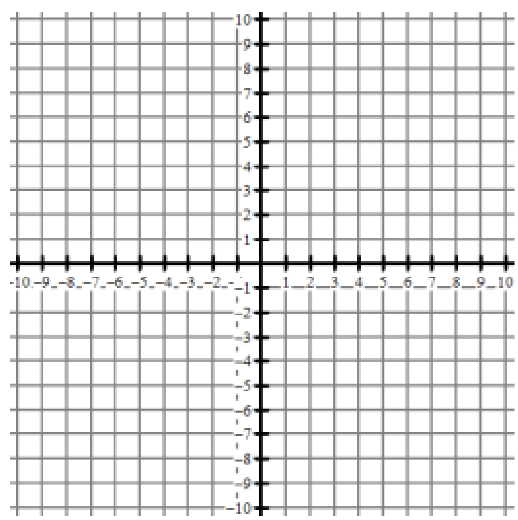


Hole/Vertical Asymptotes:

Y-Int: X-int:

Horizontal/Slant Asymptote:

6) $y = \frac{x-3}{x^3+x^2-12x}$



Hole/Vertical Asymptotes:

Y-Int: X-int:

Horizontal/Slant Asymptote:

Topic: Solving Rational and Polynomial Inequalities

Critical number: the input value that that causes the expression on one side of the inequality to equal zero or become undefined. (Corresponds to zeros and discontinuities on the graph)

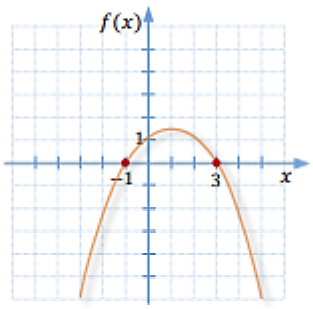
Do Questions 7-9, and all multiple of 3 after.

Given the graph of function f, state the solution set for each inequality

a. $f(x) \geq 0$

b. $f(x) < 0$

7.



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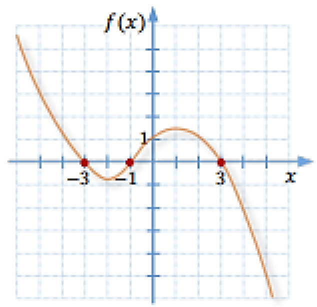

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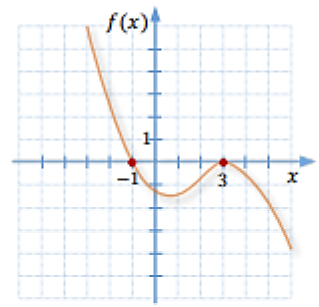


***Solve** each inequality by sketching an approximate graph for the related equation.*

10. $(x+4)(x-2) > 0$

11. $(x+1)(x-2) < 0$

12. $x^2 - 4x + 3 \geq 0$

13. $\frac{1}{2}(x^2 - 3x - 10) \leq 0$

14. $4 - 9x^2 > 0$

15. $-x^2 - 2x < 0$

***Solve** each inequality using **sign analysis**.*

16. $(x-3)(x+2) > 0$

17. $(x+4)(x-5) < 0$

18. $x^2 + 2x - 7 \leq 8$

19. $x^2 - x - 2 \geq 10$

20. $3x^2 + 10x > 8$

21. $2x^2 + 5x < -2$

22. $x^2 + 9 > -6x$

23. $x^2 + 4 \leq 4x$

24. $6 + x - x^2 \leq 0$

25. $20 - x - x^2 < 0$

26. $(x-1)(x+2)(x-3) \geq 0$

27. $(x+3)(x-2)(x-5) \leq 0$

28. $x(x+3)(2x-1) > 0$

29. $x^2(x-2)(2x-1) \geq 0$

30. $x^4 - 13x^2 + 36 \leq 0$

Solve each inequality using sign analysis.

31. $\frac{x}{x+1} > 0$

32. $\frac{x+1}{x-2} < 0$

33. $\frac{2x-1}{x+3} \leq 0$

34. $\frac{2x-3}{x+1} \geq 0$

35. $\frac{3}{y+5} > 1$

36. $\frac{5}{t-1} \leq 2$

37. $\frac{x-1}{x+2} \leq 3$

38. $\frac{a+4}{a+3} \geq 2$

39. $\frac{2t-3}{t+3} < 4$

40. $\frac{3y+9}{2y-3} < 3$

41. $\frac{1-2x}{2x+5} \leq 2$

42. $\frac{2x+3}{1-x} \leq 1$

43. $\frac{4x}{2x-1} \leq x$

44. $\frac{-x}{x+2} > 2x$

45. $\frac{2x-3}{(x+1)^2} \leq 0$

46. $\frac{2x-3}{(x-2)^2} \geq 0$

47. $\frac{x^2+1}{5-x^2} > 0$

48. $x < \frac{3x-8}{5-x}$

49. $\frac{1}{x+2} \geq \frac{1}{x-3}$

50. $\frac{2}{x+3} \leq \frac{1}{x-1}$

51. $\frac{(x-3)(x+1)}{4-x} \geq 0$

52. $\frac{(x+2)(x-1)}{(x+4)^2} \geq 0$

53. $\frac{x^2-2x-8}{x^2+10x+25} > 0$

54. $\frac{x^2-4x}{x^2-x-6} \leq 0$

Solve each inequality.

55. $(4-3x)^2 \geq -2$

56. $(5+2x)^2 < -1$

57. $\frac{(1-2x)^2}{2x^4} \leq 0$

58. $\frac{(1-2x)^2}{(x+2)^2} > -3$

59. $\frac{-2x^2}{(x+2)^2} \geq 0$

60. $\frac{-x^2}{(x-3)^2} < 0$

Topic: Radical Functions

Find the domain of each of the following radical functions in interval notation.

A) $f(x) = \sqrt{x+4} - 2$

B) $f(x) = 2\sqrt{4-x} + 1$

C) $f(x) = \sqrt{2x+3} + 1$

D) $f(x) = \sqrt{x^2 - 4}$

E) $f(x) = \sqrt{x^2}$

F) $f(x) = \frac{1}{2}\sqrt{6-x} - 3$

Find the range for each of the following.

A) $f(x) = \sqrt{x+5} - 3$

B) $f(x) = -\sqrt{x-3} + 2$

C) $f(x) = 2\sqrt{x-4} + 3$

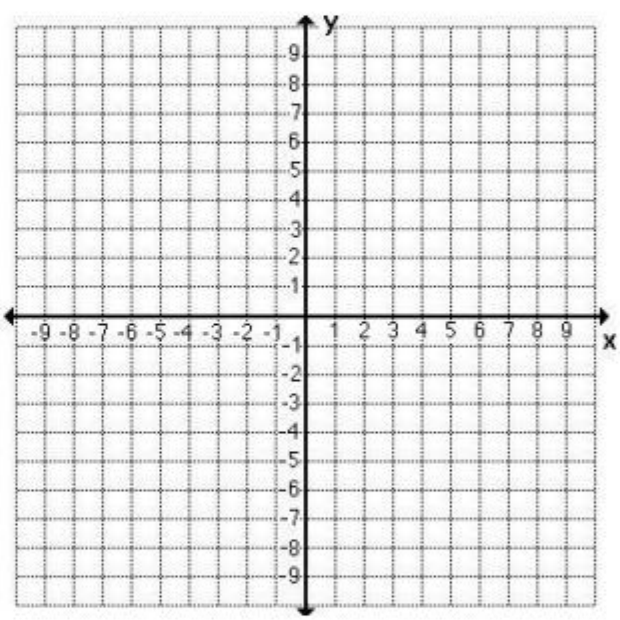
D) $f(x) = -3\sqrt{5-x} + 6$

E) $f(x) = \sqrt{4-x} - 3$

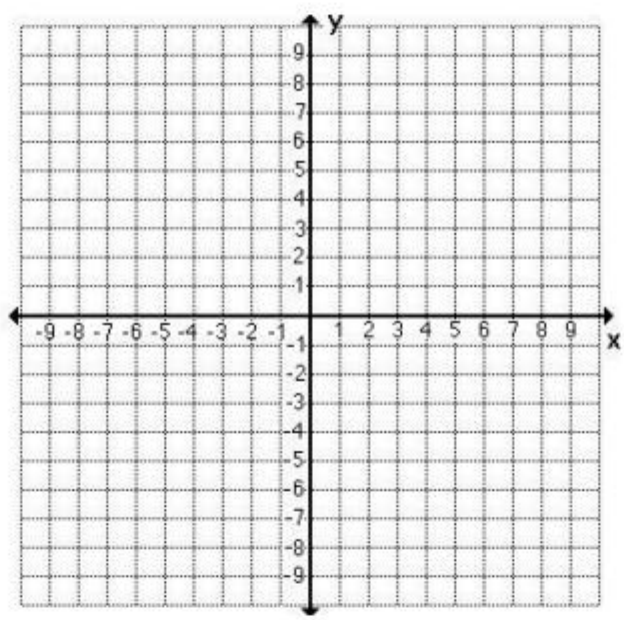
F) $f(x) = \sqrt{x-7} + 5$

Graph the given radical functions, state characteristics of functions.

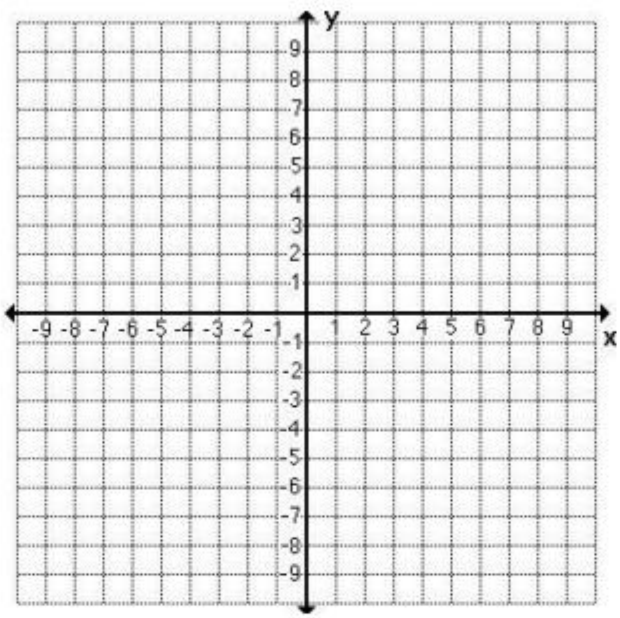
B) $f_{(x)} = -\sqrt{x-3} + 1$



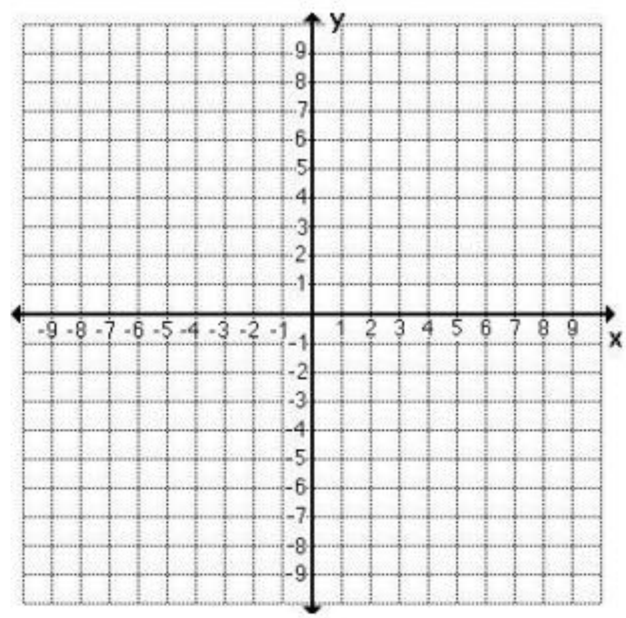
C) $f_{(x)} = \sqrt{3-x} + 1$



F) $f_{(x)} = \sqrt[3]{x+2} + 3$



G) $f_{(x)} = -\sqrt[3]{x-3} - 2$



Solve the given equations.

11) $5 = \sqrt{50x} - 5$

12) $-15 = -3\sqrt{11 - 2n}$

13) $\sqrt{-56 + 15k} = k$

14) $x = \sqrt{54 - 3x}$

15) $\sqrt{k - 1} = k - 3$

16) $n + 1 = \sqrt{2n + 50}$

Solve each of the following equations. Don't forget to check for extraneous roots.

A) $(2x-3)^{2/3}=16$

B) $x^{-1/2}=6$

C) $4x^{1/2}-3=9$

D) $(4x-1)^{3/4}=-8$

E) $(3x-1)^{-2/3}=\frac{1}{4}$

F) $(5x+6)^{2/5}=4$

G) $(9x)^{-2/3}=4$

H) $\sqrt[3]{(x+1)^2}=9$

I) $9x^{-2/3}=4$

J) $\sqrt[3]{x^2-1}=3$

K) $x^{2/3}-2x^{1/3}-15=0$

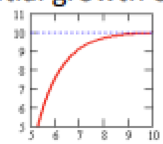
L) $\sqrt{(3x-1)^3}=27$

Topic: Graphing Exponential and Log Functions

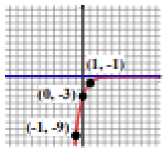
Tell whether the equation or graph represents an exponential growth or exponential decay function.

1) $y = 5(0.4)^x$

2) $y = -3\left(\frac{7}{2}\right)^x$

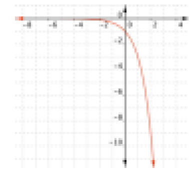
3) 

4) $y = 9(1.5)^x$

5) 

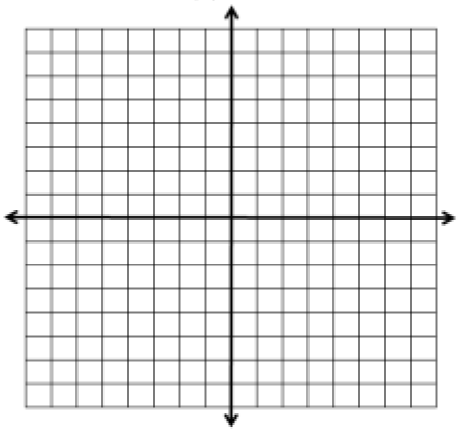
6) $y = 0.2(0.3)^{-x}$

7) $y = -3(6)^x$

8) 

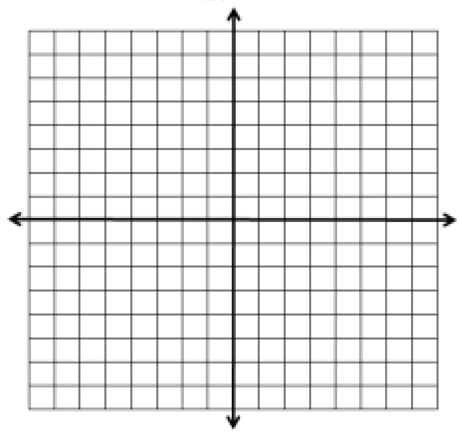
Sketch the graph of each exponential function by doing the following: Sketch the asymptote, label at least **<u>three distinct coordinate points</u>** on each graph, and write the domain and range of each function.

9. $y = 4\left(\frac{1}{2}\right)^x$



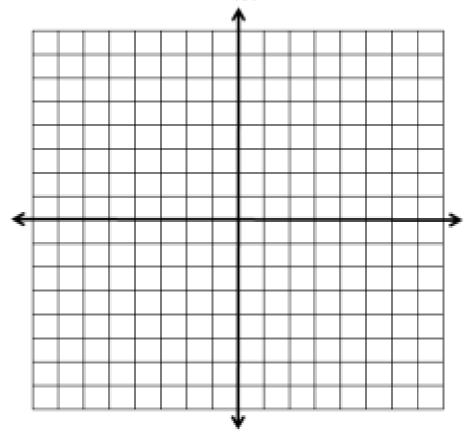
Domain: Range:

10. $y = -3\left(\frac{1}{3}\right)^x$



Domain: Range:

11. $y = -2\left(\frac{1}{5}\right)^{x+2} + 3$



Domain: Range:

Let $f(x)$ be a function on which a transformation occurs. Let $g(x)$ be a transformation of f. For each problem, name the transformation(s) of f.

1. $f(x) = 3^x$ and $g(x) = f(x) \cdot 27$	2. $f(x) = 9^x$ and $g(x) = 3^x$	3. $f(x) = 7^x$ and $g(x) = \frac{1}{7^x}$
4. $f(x) = 4^x$ and $g(x) = f(x) \cdot 16$	5. $f(x) = 4^x$ and $g(x) = \frac{f(x)}{16}$.	6. $f(x) = 5^x$ and $g(x) = \left(f(x)\right)^{-3}$
7. $f(x) = 6^x$ and $g(x) = \frac{f(x)}{36}$	8. $f(x) = 3^x$ and $g(x) = 27^x$	9. $f(x) = 4^x$ and $g(x) = -(16)^x$

13. $f(x) = 3^x$ and $g(x) = -3f(x)$

14. $f(x) = 2^x$ and $g(x) = f(x) \cdot 32$

Evaluate the function at the given input values.

15. Let $h(x) = 2 \cdot 3^{x/2}$. Find $h(1)$

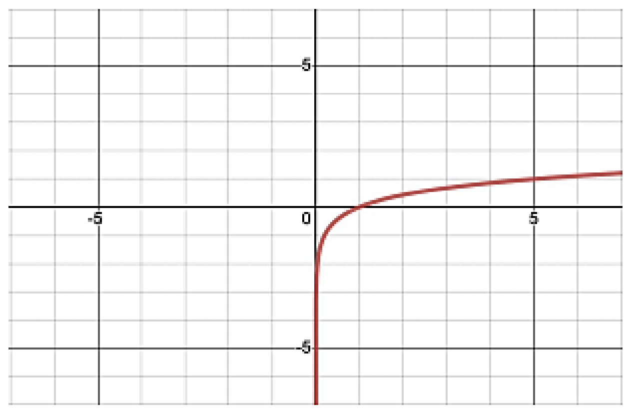
16. Let $h(x) = 4 \cdot 4^{x/5}$. Find $h(2)$

Sketch a graph of the transformation of $f(x) = \log_5 x$ onto the graph. Label each graph.

1. $g(x) = 3\log_5(x+2) - 4$

2. $h(x) = 3\log_5(3-x) + 1$

3. $j(x) = -\log_5(x-3) - 2$



Write a logarithmic function with the given information.

10. End Behavior

$$\lim_{x\to\infty} f(x) = \infty$$
$$\lim_{x\to -6^+} f(x) = -\infty$$

11. Domain

$$(-\infty, 8)$$

CALCULATOR ACTIVE: Find all relevant information.

12. $f(x) = \log(|x+4|) - 10$

Asymptote:

Domain: Range:

End Behavior:

13. $f(x) = -3\ln\left(\frac{x+7}{x}\right)$

Asymptote:

Domain: Range:

End Behavior:

Find all relevant information from the given function. Sketch a graph. No calculator.

4. $f(x) = \ln(x-3) + 5$

Asymptote:

Domain: Range:

End Behavior:

Graph:

5. $f(x) = -2\log_2(x+3) - 6$

Asymptote:

Domain: Range:

End Behavior:

Graph:

6. $f(x) = \log_6(8-x) + 1.5$

Asymptote:

Domain: Range:

End Behavior:

Graph:

7. $f(x) = -\log(2x-5)$

Asymptote:

Domain: Range:

End Behavior:

Graph:

Properties of Logarithmic Functions

	Property	But also…
Product Property:	$\log_b(xy) = \log_b x + \log_b y$	
Quotient Property:	$\log_b\left(\frac{x}{y}\right) = \log_b x - \log_b y$	
Power Property:	$\log_b x^k = k\log_b x$	

Instructions: Let x and y be positive constants. Write each as a single logarithm.	
1. $2log_4x - 6log_4y$	2. $6\ln x + \frac{1}{2}\ln y$
3. $\frac{1}{3}(\log x + 2\log y)$	4. $\frac{1}{2}log_5x^3 - 2log_5y$

Instructions: Let x and y be positive constants. Write each expression as a sum or difference of logarithms.	
5. $\ln x^4y$	6. $log_3\sqrt{\frac{x^2}{y}}$
7. $log_6\sqrt{x^3y^5}$	8. $\log(10x - 30)$

Instructions: Find the domain/range and describe any transformations on the function.

9. $log_2(8x)$

Domain:

Transformations:

Range:

10. $log_3(27 - 27x)$

Domain:

Transformations:

Range:

CALCULATOR ACTIVE. Use the change of base to change each to a logarithm with base 10 or base *e*. Then use a calculator and find the value of the logarithm to the nearest thousandth.

11. $\log_4 123$	12. $\log_9 578$

Topic: Solving Exponential and Log Equations and application

Solve, rounding to three significant digits.

1. $8^{-x} = 0.654$

2. $10^{x-4} = 92$

3. $e^{x} = 8.88$

4. $15e^{2x-1} = 272$

5. $log_8 x = 2$

6. $e^{0.045x} + 105 = 240$

7. $log\ x + log\ 10 = 2$

8. $log\ (x+4) + log\ (x) = 1$

9. $\log(15 - 2x) = \log(3x - 20)$

10. $5 - \log(x+2) = 7$

11. $5^{x+1} = 17$

Solve. Show all steps. Give decimal answers correct to **three** decimal places.

14. $e^{2x} - e^{x} - 12 = 0$

15. $\log_3(x+4) - \log_3(x-1) = 2$

16. $\log_2(\log_3(\log_5 x)) = 0$

Calculator Active. Use the information given to answer the questions. Round/truncate to three decimal places.

4. People can use the formula below to determine future population $N(t)$ of cities. N_0 represents the initial population, r is the rate of population growth, and t is the time in years.

$$N(t) = N_0e^{rt}$$

   a. What is the population of Cleveland, Ohio in 10 years if there are currently 275,000 people, with a population growth rate of 2.5%.

   b. What growth rate would Cleveland, Ohio need to achieve a population of 400,000 people in 20 years?

---

5. Forensics often use Newton's Law of Cooling to determine the elapsed time since a person has died. The formula is $t = -10\,ln\left(\frac{T-R}{98.6-R}\right)$, where T stands for the body's temperature in degrees Fahrenheit, R is the temperature of the room and t is the elapsed time since death in hours.

   a. How many hours elapsed if the temperature of the room was 75° and the body's temp was 85°?

   b. Suppose a body was found 5 hours after death (neighbor heard thud) in a room that was 65°. What was the temperature of the body when it was found?

Instructions: Use the information given to answer the questions. Round to nearest thousandth.

1) People can use the formula below to determine future populations (*N(t)*) of cities. N_0 represents the initial population, *r* is the rate of population growth, and *t* is the time in years.

$$N(t) = N_0e^{rt}$$

a) What would the population be of Rochester, NY be in 10 years if there are currently 210,000 people, with a population growth rate of 1.2%.

b) What growth rate would Rochester, NY need to achieve a population of 250,000 people in 30 years?

Show all work. All answers must be given as either simplified, exact answers. No calculator is permitted unless otherwise stated.

Multiple Choice

1. $\log 12 =$
   (A) $3\log 4$ (B) $\log 3 + \log 4$ (C) $4\log 3$ (D) $\log 3 \cdot \log 4$ (E) $2\log 6$

2. $\log_9 64 =$
   (A) $5\log_3 2$ (B) $(\log_3 8)^2$ (C) $\dfrac{\ln 64}{\ln 9}$ (D) $2\log_9 32$ (E) $\dfrac{\log 64}{9}$

3. $2^{-1} \cdot (-3\ln 2 - 1) =$
   (A) $-\dfrac{1}{2}\ln(8e)$ (B) $-\ln(8e)$ (C) $-\dfrac{3}{2}\ln 2$ (D) $-\dfrac{1}{2}$ (E) $\dfrac{1}{8}$

4. $\log_{1/2} x^2 =$
   (A) $-2\log_2 x$ (B) $2\log_2 x$ (C) $-0.5\log_2 x$ (D) $0.5\log_2 x$ (E) $-2\log_2 |x|$

5. $\ln x^5 =$
   (A) $\dfrac{5\log_7 x}{\log_7 e}$ (B) $\dfrac{2\log x^3}{\log e}$ (C) $\dfrac{x\log_{1/2} 5}{\log_{1/2} e}$ (D) $3\ln x^2$ (E) $\ln x^2 \cdot \ln x^3$

Short Answer

6. Evaluate each of the following expressions using the properties of logs (and no calculator).
   (a) $\log_3 \sqrt[3]{81}$ (b) $\log 4 + \log 25$ (c) $\log_2 6 - \log_2 15 + \log_2 20$ (d) $\ln\left(\ln e^{e^{200}}\right)$

7. Use the properties of logs to expand the following expressions.
   (a) $\log_5 \sqrt[4]{x^3\left(x^2+1\right)}$ (b) $\log_6 \sqrt{\dfrac{5x^2y^3}{x^2+y^3}}$

8. Use the properties of logs to condense the following expressions.

(a) $4\ln x - \frac{1}{3}\ln\left(x^2+1\right) + 2\ln(x-1)$

(b) $\frac{1}{3}\ln(2x+1) + \frac{1}{2}\left[\log(x-4) - \log\left(x^4 - x^2 - 1\right)\right]$

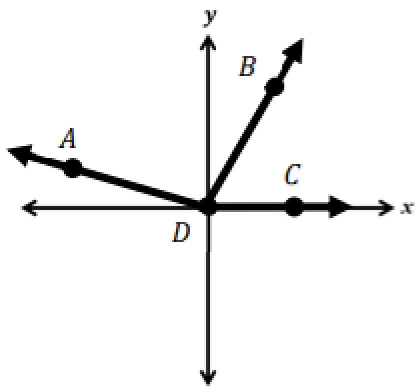
(c) $\log\left(x^2-1\right) - \ln(x-1)$ (use the change of base formula on this one first to get both in terms of base *e*)

9. If $\log_7 x = A\log_{2/3} x$, use the change of base formula to find the value of A,

10. Simplify the following to a single log expression of the form $\log_b a$: $\left(\log_7 3\right)\left(\log_2 5\right)\left(\log_5 7\right)$

Topic: Definition of Trig ratio in xy-coordinate plane and special triangle

1. Name all the angles that are in standard position. Give the initial ray and terminal ray of each angle.



Angle: __________

Initial ray: ______

Terminal ray: ______

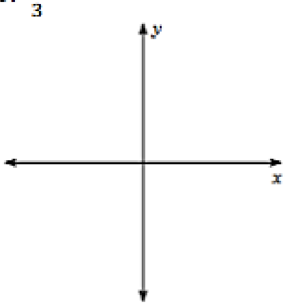
Angle: ___________

Initial ray: ______

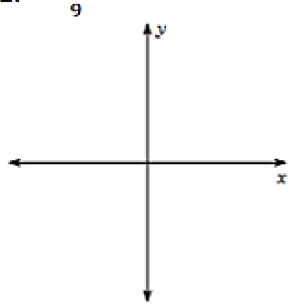
Terminal ray: ______

Draw an angle with the given measure in standard position.

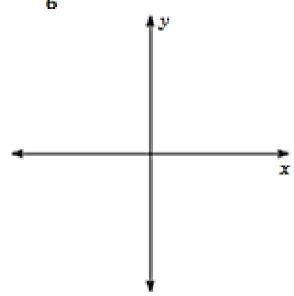
1. $\frac{4\pi}{3}$



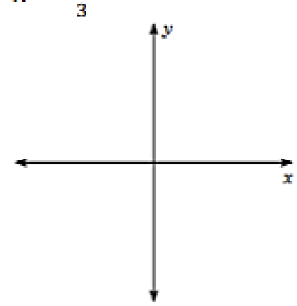
2. $-\frac{7\pi}{9}$



3. $\frac{5\pi}{6}$

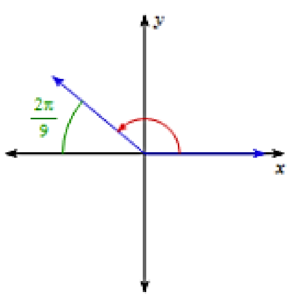


4. $-\frac{2\pi}{3}$

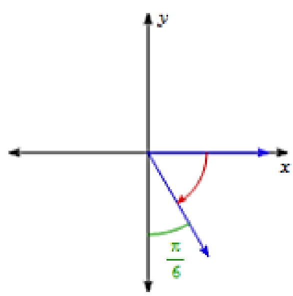


Find the measure of each angle.

5.



6.



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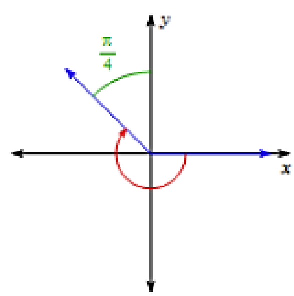

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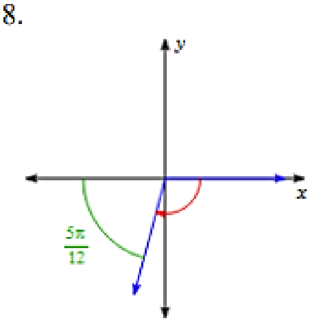


State the quadrant in which the terminal side of each angle lies.

9. $\frac{2\pi}{3}$

10. $-\frac{15\pi}{4}$

11. $\frac{4\pi}{3}$

12. $-\frac{17\pi}{9}$

Find one positive and one negative coterminal angle the angle given.

13. $-\frac{13\pi}{4}$

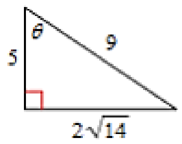
14. $\frac{7\pi}{12}$

15. $\frac{11\pi}{6}$

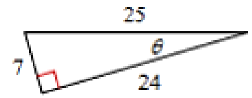
16. $-\frac{13\pi}{18}$

Find the value of the trig functions indicated.

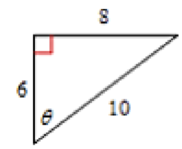
1. $\tan\theta$



2. $\cos\theta$

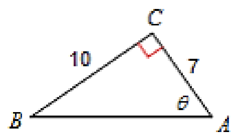


3. $\sin\theta$

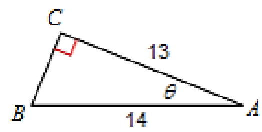


Find the measure of the indicated angle. Round to the nearest hundredth.

4.



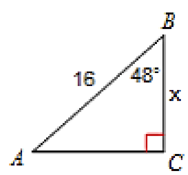
5.



6. Given ΔABC where $\angle C$ is a right angle. Find $m\angle A$ if $a = 7$ and $c = 12$. (Draw a picture!)

Find the measure of the indicated side. Round to the nearest hundredth.

7.



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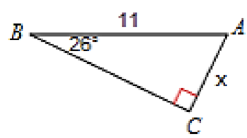
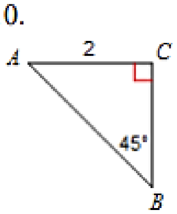


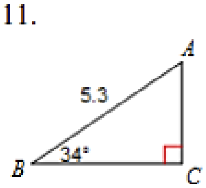
9. Given ΔABC where $\angle C$ is a right angle. Find b if $m\angle B = 64°$ and $c = 9$. (Draw a picture!)

Solve each triangle. Round to the nearest hundredth.

10.

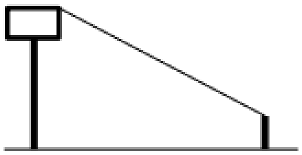


11.



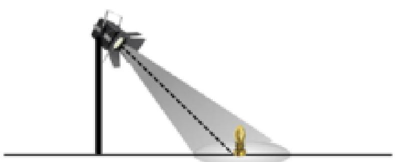
12. Given ΔABC where $\angle C$ is a right angle where $m\angle A = 67°$ and $b = 12$. (Draw a picture!)

13. One end of a zip-line is attached to a platform on top of a 150 foot pole. The other end of the zip-line is attached to the top of a 5 foot stake. The angle of elevation from the top of the stake to the top of the platform is 23°. How long is the zip-line?



14. Standing on top of a 235 foot tall building, you spot your friend on the ground who is 94 feet away from the building. What is the angle of depression you had to look to spot your friend?

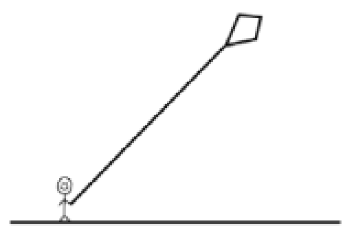
15. Mr. Brust displays his fantasy football trophy in his front yard with a spotlight illuminating the trophy 24/7. The spotlight is mounted on a 8.5 meter pole. The angle of depression formed by the spot light is 54°. How far the does the spotlight shine?



16. Driving on a flat road, you spot a mountain 28 miles away. The angle of elevation from the car to the top of the mountain is 34°. How tall is the mountain?

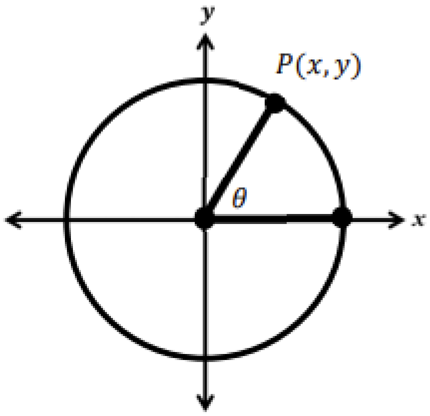
17. A 25 foot ladder leans against a building. The ladder's base is 13.5 feet from the building. Find the angle which the ladder makes with the ground.

18. A 4 foot boy flying a kite lets out 300 feet of string which makes an angle of elevation of 38°. Assuming that the string is straight, how high above the ground is the kite?



Write the trig ratio for the special right angles: $\frac{\pi}{6}, \frac{\pi}{3}, \frac{\pi}{4}$

Topic: Trig Function in x-y coordinate plane, and evaluate trig ratios



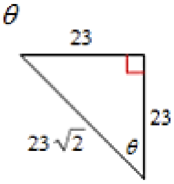
$\sin(\theta) =$ | $\csc(\theta) =$

$\cos(\theta) =$ | $\sec(\theta) =$

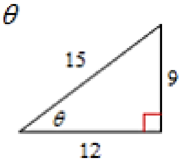
$\tan(\theta) =$ | $\cot(\theta) =$

Find the RATIO of the trig function indicated. Do NOT find the actual measure of the angle!

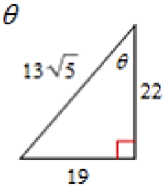
1. csc θ



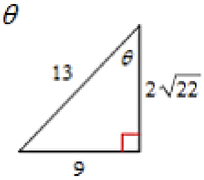
2. cot θ



3. cos θ

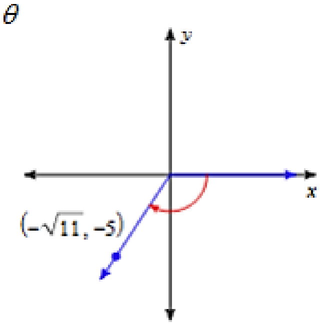


4. sec θ

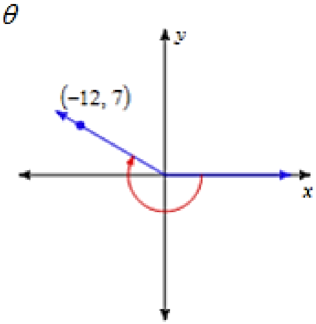


Use the given point on the terminal side of the angle θ to find the trigonometric function indicated.

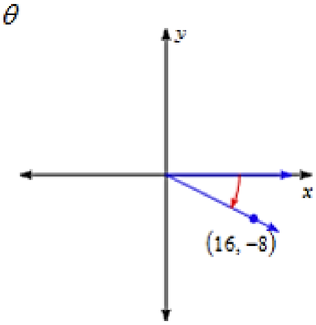
5. cos θ



6. tan θ



7. cot θ



Draw the reference triangle. Find the EXACT value of the trig ratio for θ.

8. $\sin\theta$ for $(2, \sqrt{5})$

9. $\csc\theta$ for $(-4, 3)$

10. $\sec\theta$ for $(2\sqrt{3}, -2)$

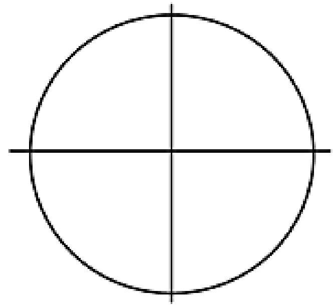
Let θ be an angle in standard position. In which quadrant or quadrants can θ lie under the given conditions?

11. $\csc\theta$ is negative

12. $\sin\theta > 0$

13. $\cos\theta$ and $\sin\theta$ have the same sign

14. $\cos\theta$ is negative and $\tan\theta$ is positive



Draw the reference triangle. Find the EXACT value of the trig ratio for θ.

15. Given $\cot\theta = -\frac{12}{5}$ in quadrant II.
Find $\sin\theta$.

16. Given $\csc\theta = \frac{5}{4}$ where $\frac{\pi}{2} < \theta < \pi$.
Find $\tan\theta$.

17. Given $\cos\theta = \frac{\sqrt{3}}{4}$ where $\frac{3\pi}{2} < \theta < 2\pi$.
Find $\sin\theta$.

18. Given $\sec\theta = -\frac{15}{9}$ where $\pi < \theta < \frac{3\pi}{2}$.
Find $\tan\theta$.

Find the value of each expression. Try not to look back at the Unit Circle for help.

1. $\sin\frac{3\pi}{2}$	2. $\cos\frac{\pi}{2}$	3. $\sin\frac{\pi}{6}$	4. $\cos\frac{\pi}{4}$	5. $\sin\frac{5\pi}{6}$
6. $\cos\frac{7\pi}{6}$	7. $\cos\frac{4\pi}{3}$	8. $\sin\frac{5\pi}{3}$	9. $\cos\left(-\frac{7\pi}{6}\right)$	10. $\cos\left(-\frac{\pi}{3}\right)$
11. $\sin\left(-\frac{3\pi}{4}\right)$	12. $\cos\frac{\pi}{3}$	13. $\sin\frac{\pi}{2}$	14. $\sin\frac{2\pi}{3}$	15. $\cos 2\pi$
16. $\cos\frac{5\pi}{6}$	17. $\sin\frac{\pi}{3}$	18. $\cos\left(-\frac{2\pi}{3}\right)$	19. $\sin\frac{\pi}{4}$	20. $\cos\frac{11\pi}{6}$

21. $\sin\left(-\frac{\pi}{6}\right)$	22. $\cos\frac{\pi}{6}$	23. $\sin 0$	24. $\sin\left(-\frac{5\pi}{4}\right)$	25. $\sin\frac{4\pi}{3}$	26. $\sin\pi$

Find the value of each expression. Try not to look back at the Unit Circle for help.

1. $\sin 2\pi$	2. $\cos\frac{3\pi}{2}$	3. $\sin\frac{\pi}{4}$	4. $\cos\frac{\pi}{3}$	5. $\sin\frac{5\pi}{3}$
6. $\cos\frac{4\pi}{3}$	7. $\sin\frac{5\pi}{6}$	8. $\cos\frac{7\pi}{6}$	9. $\cos\left(-\frac{7\pi}{6}\right)$	10. $\sin\left(-\frac{3\pi}{4}\right)$
11. $\sin\frac{\pi}{3}$	12. $\cos\frac{2\pi}{3}$	13. $\sin\left(-\frac{5\pi}{6}\right)$	14. $\cos\frac{5\pi}{6}$	15. $\sin\frac{\pi}{2}$
16. $\cos\left(-\frac{5\pi}{4}\right)$	17. $\sin\frac{5\pi}{3}$	18. $\cos\frac{\pi}{6}$	19. $\sin\frac{11\pi}{6}$	20. $\cos\pi$

Topic: Graph Trig Functions

Characteristics unique to trig functions:

Amplitude: represents the height of the function's wave from its midline to its maximum or minimum value

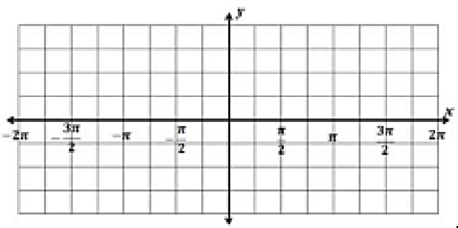
Period: the horizontal distance over which the function's graph completes one full cycle

Phase shift: describes the horizontal displacement of the graph from its standard position.

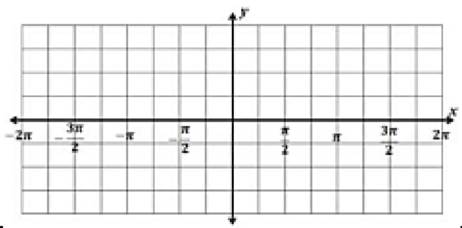
$$y = af\big(b(x - h)\big) + k$$

For 1-9, graph the given function.

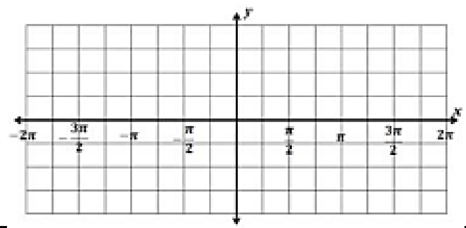
1) $y = 3\cos\left(x - \frac{\pi}{2}\right)$



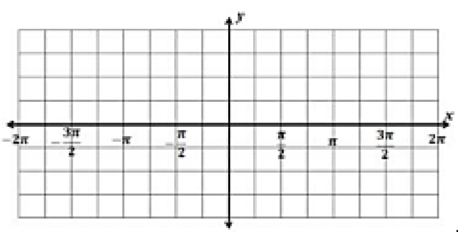
2) $y = 4\sin(x)$



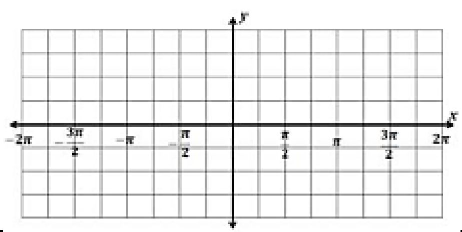
3) $y = \sin\left(\frac{x}{2} + \frac{\pi}{4}\right) + 2$



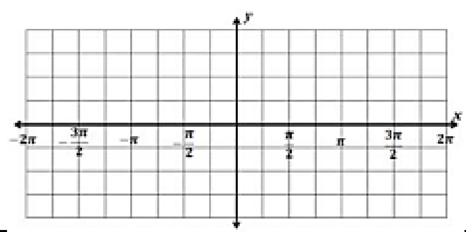
4) $y = -3\cos\left(2x - \frac{\pi}{4}\right) + 1$



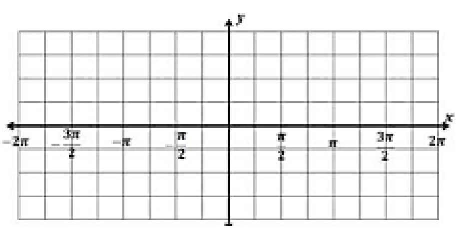
5) $y = 4\cos\left(2x - \frac{3\pi}{2}\right)$



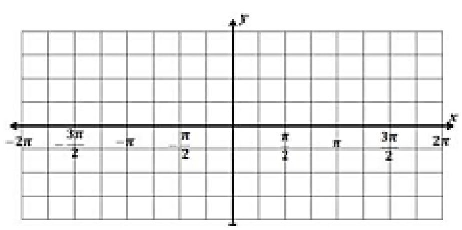
6) $y = \tan(x) - 2$



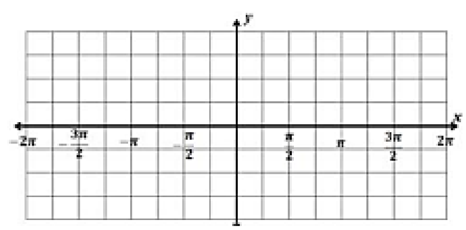
7) $y = -\sin(2x + \pi)$



8) $y = 1 + 4\tan\left(\frac{x}{2}\right)$



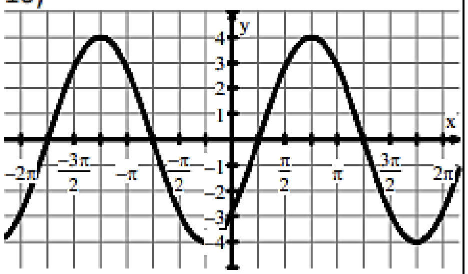
9) $y = \tan\left(x + \frac{\pi}{2}\right) - 2$



Write both sine and cosine function for the given graphs.

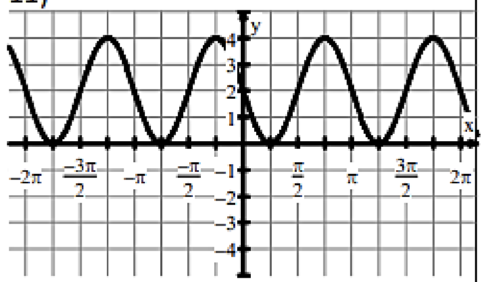
For 10-12, write the equation of the following ***sine*** curves. Use a positive leading coefficient a and the closest phase shift possible (left or right). For some problems, it may be equal to move left or right.

10)



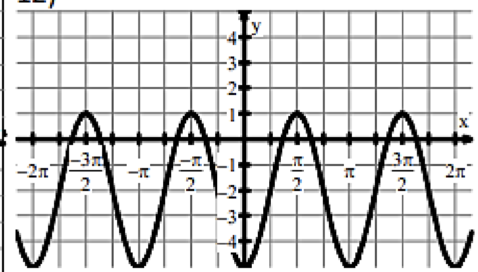
y = ____________________________

11)



y = ____________________________

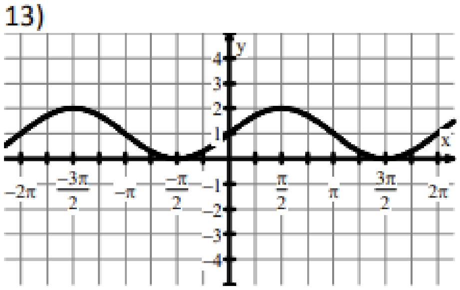
12)



y = ____________________________

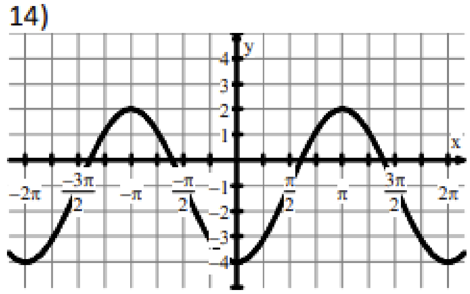
For 13-15, write the equation of the following ***cosine*** curves. Use a positive leading coefficient a and the closest phase possible (left or right). For some problems, it may be equal to move left or right.

13)



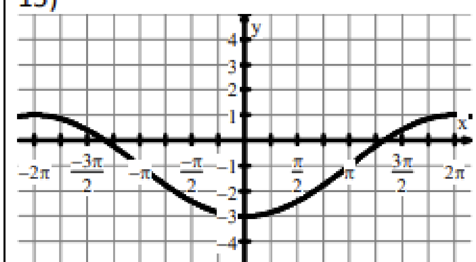
y = ____________________________

14)



y = ____________________________

15)

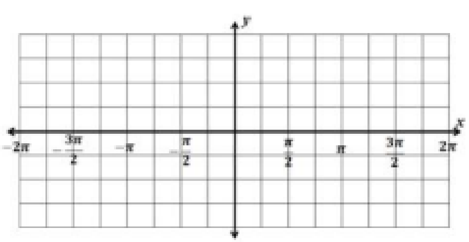


y = ____________________________

For 1-6, graph the trig function.

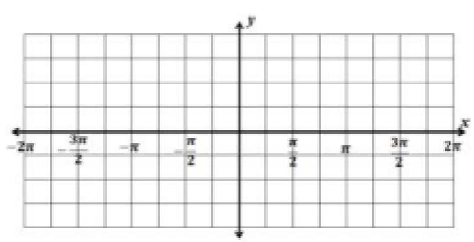
1. $y = 3\csc\theta - 1$

Amp Period P.S. V.S.



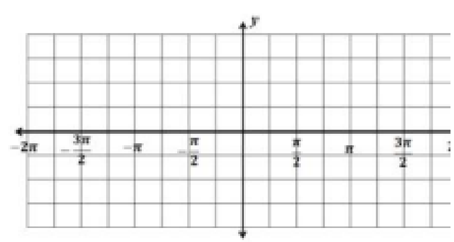
2. $y = -1 + 2\cot\theta$

Amp Period P.S. V.S.



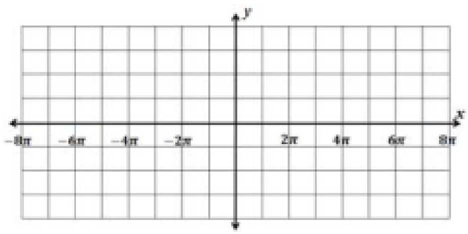
3. $y = 3\sec 2\theta$

Amp Period P.S. V.S.



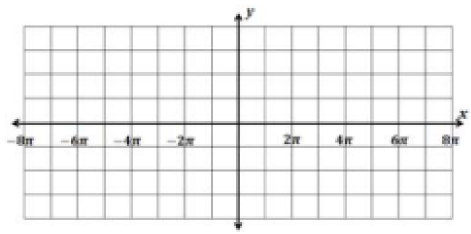
4. $y = \cot\left(\frac{\theta}{6} + \frac{\pi}{6}\right) + 2$

<u>Amp</u> <u>Period</u> <u>P.S.</u> <u>V.S.</u>



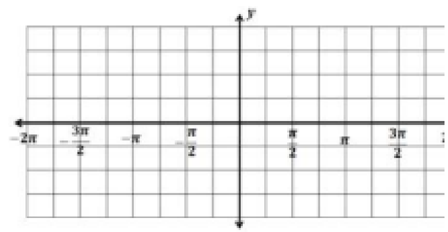
5. $y = \sec\left(\frac{\theta}{4} - \frac{\pi}{2}\right) + 2$

<u>Amp</u> <u>Period</u> <u>P.S.</u> <u>V.S.</u>



6. $y = -2 + 2\csc\left(2\theta + \frac{\pi}{2}\right)$

<u>Amp</u> <u>Period</u> <u>P.S.</u> <u>V.S.</u>



Find the exact value. Only radian please.

7) $\sin\left(\operatorname{arccsc}\left(\frac{2\sqrt{3}}{3}\right)\right)$

8) $\cos\left(\arcsin\left(\frac{\sqrt{2}}{2}\right)\right)$

9) $\cos\left(\arccos\left(\frac{1}{2}\right)\right)$

10) $\tan\left(\arcsin\left(\frac{\sqrt{2}}{2}\right)\right)$

11) $\sec\left(\arctan\left(\frac{\sqrt{3}}{3}\right)\right)$

12) $\sin\left(\arcsin\left(\frac{\sqrt{3}}{2}\right)\right)$

13) $\cos(\operatorname{arcsec}(2))$

14) $\csc(\arctan(\sqrt{3}))$

For 23-26 , use a reference triangle to find the exact value of the expression.

23) $\tan\left(\sin^{-1}\left(\frac{4}{11}\right)\right)$

24) $\sec\left(\operatorname{arccsc}\left(\frac{7}{2}\right)\right)$

25) $\csc\left(\cot^{-1}\left(\frac{5}{6}\right)\right)$

26) $\sec\left(\sin^{-1}\left(\frac{3}{7}\right)\right)$

Topic: Trig Identities (with Double Angle Formula)

Tangent and Cotangent Identities

$\tan(\theta) = \dfrac{\sin(\theta)}{\cos(\theta)}$ $\cot(\theta) = \dfrac{\cos(\theta)}{\sin(\theta)}$

Reciprocal Identities

$\csc(\theta) = \dfrac{1}{\sin(\theta)}$ $\sin(\theta) = \dfrac{1}{\csc(\theta)}$

$\sec(\theta) = \dfrac{1}{\cos(\theta)}$ $\cos(\theta) = \dfrac{1}{\sec(\theta)}$

$\cot(\theta) = \dfrac{1}{\tan(\theta)}$ $\tan(\theta) = \dfrac{1}{\cot(\theta)}$

Pythagorean Identities

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\tan^2(\theta) + 1 = \sec^2(\theta)$

$1 + \cot^2(\theta) = \csc^2(\theta)$

Even/Odd Formulas

$\sin(-\theta) = -\sin(\theta)$ $\csc(-\theta) = -\csc(\theta)$

$\cos(-\theta) = \cos(\theta)$ $\sec(-\theta) = \sec(\theta)$

$\tan(-\theta) = -\tan(\theta)$ $\cot(-\theta) = -\cot(\theta)$

Sum and Difference Formulas

$\sin(\alpha \pm \beta) = \sin(\alpha)\cos(\beta) \pm \cos(\alpha)\sin(\beta)$

$\cos(\alpha \pm \beta) = \cos(\alpha)\cos(\beta) \mp \sin(\alpha)\sin(\beta)$

$\tan(\alpha \pm \beta) = \dfrac{\tan(\alpha) \pm \tan(\beta)}{1 \mp \tan(\alpha)\tan(\beta)}$

Double Angle Formulas

$\sin(2\theta) = 2\sin(\theta)\cos(\theta)$

$\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$

$= 2\cos^2(\theta) - 1$

$= 1 - 2\sin^2(\theta)$

$\tan(2\theta) = \dfrac{2\tan(\theta)}{1 - \tan^2(\theta)}$

Directions: Simplify to a single trig expression.

1) $\cos\beta \csc\beta$	2) $\cot\theta \sec\theta$	3) $\dfrac{\tan\alpha \cot\alpha}{\sin\alpha}$

Directions: Verify the identity.

4) $\cos\theta \tan\theta \csc\theta = 1$	5) $\tan x \cos x = \sin x$	6) $\cot x(\sec x - \sin x) = \csc x - \cos x$

7) $\frac{1+\csc\theta}{\cot\theta} = \tan\theta + \sec\theta$	8) $\frac{\tan\theta}{\sin\theta} = \sec\theta$	9) $\frac{sinx+\cot x}{\cos x} = \tan x + cscx$

Directions: Multiply or Factor.

10) $\tan x(\cos x + \csc x)$	11) $sin^2x - 10\sin x - 24$	12) $10\, csc^3x + 15\, csc^2x$

10. Find $\sin 2x$, $\cos 2x$, and $\tan 2x$ from the given information.

(a) $\sin x = \frac{5}{13}$, $\sec x > 0$

(b) $\cot x = \frac{2}{3}$, $\sin x > 0$

12. Prove the following identities.

(a) $\sin 4x = 2\sin 2x \cos 2x$

(b) $\cos 6x = 2\cos^2 3x - 1$

(c) $\sin 3x = \sin x\left(4\cos^2 x - 1\right)$

Proof the given identities.

1. $(\sin A + \cos A)^2 = 1 + 2\cos A \sin A$

2. $\sec x(\sec x + 1) = \dfrac{\tan^2 x}{1 - \cos x}$

3. $\dfrac{\sin x}{\csc x} + \dfrac{\cos x}{\sec x} = 1$

4. $\dfrac{\sec^2 x - 7\tan x + 11}{\sec^2 x - 17} = \dfrac{\tan x - 3}{\tan x + 4}$

5. $\dfrac{1 - \cos B}{\sin B} = \dfrac{\sin B}{1 + \cos B}$

6. $\dfrac{\sin A}{1 + \sec A} = \dfrac{\sin A \cos A}{\cos A + 1}$

7. $\sec^4 \delta - 2\sec^2 \delta \tan^2 \delta + \tan^4 \delta = 1$

8. $\sqrt[3]{\tan^2 x - \sec^2 x} = -1$

9. $\dfrac{1}{\csc y + \cot y} = \dfrac{1 - \cos y}{\sin y}$

10. $(\sec x + \tan x)^3 (\sec x - \tan x)^4 = \dfrac{1 - \sin x}{\cos x}$

17. $\tan^2 x \sin^2 x = \tan^2 x - \sin^2 x$

18. $\dfrac{\cos x + 1}{\sin^3 x} = \dfrac{\csc x}{1 - \cos x}$

Topic: Solving Trig Equations

Solve each equation for $0 \le x \le 2\pi$. Find the exact value(s) using the unit circle.

1. $2\sin x + 3 = 4$

2. $4 - 3\cos x = 7$

3. $\tan^2 x = 1$

4. $-1 = \cos^2 x + 2\cos x$

Solve each equation for $0 \le \theta \le 2\pi$. Find the approximate value(s) using a calculator.

5. $6 = 3\cos\theta + 7$

6. $5\sin^2\theta + 3 = 6$

7. $6\cos^2\theta + 4\cos\theta = 0$

8. $\cos\theta = 3\cos^2\theta$

Solve each equation. Find ALL exact value(s) using the unit circle.

9. $2\sin^2\theta = \sin\theta$

10. $4\tan^2(2x) = 12$

Solve each equation. Find ALL approximate value(s) using a calculator.

11. $7\sin^2 x = 5$

12. $5\sin\theta + 3 = 4$

Solve each equation for $0 \le \theta \le 2\pi$. Find the exact value(s) using the unit circle

1. $6\sin\theta = 3\sqrt{3}$

2. $4\cos^2\theta - 2\cos\theta = 0$

Solve each equation for $0 \le x \le 2\pi$. Find the approximate value(s) using a calculator.

3. $5 - 4\sin x = 8$

4. $\tan^2 x + 2\tan x = 15$

Solve each equation. Find ALL exact value(s) using the unit circle

5. $-2 = 2\cos(2x) - 1$

6. $4\sin^2\theta - 1 = 1$

Solve each equation. Find ALL approximate value(s) using a calculator.

7. $5 + 4\cos\theta = 6$

8. $\tan^2 x = 4\tan x$

16. What are all values of θ, for $0 \le \theta \le 2\pi$, where $4\sin^2\theta = 2\sin\theta$?

(A) $0, \frac{\pi}{3}, \pi, \frac{2\pi}{3}, 2\pi$

(B) $0, \frac{\pi}{6}, \pi, \frac{5\pi}{6}, 2\pi$

(C) $\frac{\pi}{3}, \frac{\pi}{2}, \frac{2\pi}{3}, \frac{3\pi}{2}$

(D) $\frac{\pi}{6}, \frac{\pi}{2}, \frac{5\pi}{6}, \frac{3\pi}{2}$

17. The function f is given by $f(x) = 2\cos(3x) - 1$. For how many values of x where $0 \le x \le 2\pi$ does $f(x) = 0$?

(A) None

(B) Two

(C) Four

(D) Six

20. What are all values of θ, for $0 \le \theta \le 2\pi$, where $\sin^2\theta - \cos\theta\sin\theta = 0$?

(A) $\frac{\pi}{2}, \frac{3\pi}{2}, \frac{\pi}{4}, \frac{3\pi}{4}, \frac{5\pi}{4}, \frac{7\pi}{4}$

(B) $0, \frac{\pi}{4}, \pi, \frac{5\pi}{4}, 2\pi$

(C) $0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$

(D) $\frac{\pi}{4}$ and $\frac{5\pi}{4}$

Topic: Misc. Concepts needed for Calculus
3 forms of linear equation:
Slope intercept:

Point-slope:

Standard:

1. Write an equation of the (a) vertical line and (b) horizontal line passing through the point $(-7,\pi)$.

3. Write an equation of the line through the point $\left(4,\frac{1}{2}\right)$ that is (a) parallel to and

(b) perpendicular to the line $4x-3y=6$. Then (c) both the *x*- and *y*- intercepts of the line found in part (b).

4. Find the value of *x* for which line through $(-8,-3)$ and $(x,4)$ has a slope of 3.

I. **Multiple Choice**

_____ 1. If $p(x)=(x+2)(x+k)$ and if the remainder is 12 when $p(x)$ is divided by $x-1$, then $k=$

(A) 2 (B) 3 (C) 6 (D) 11 (E) 13

_____ 2. If $f(x)=\frac{4}{x-1}$ and $g(x)=2x$, then the solution set of $f(g(x))=g(f(x))$ is

(A) $\left\{\frac{1}{3}\right\}$ (B) $\{2\}$ (C) $\{3\}$ (D) $\{-1,2\}$ (E) $\left\{\frac{1}{3},2\right\}$

_____ 5. If $f(x)=2x^3+Ax^2+Bx-5$ and if $f(2)=3$ and $f(-2)=-37$, what is the value of $A+B$?

(A) -6 (B) -3 (C) -1 (D) 2 (E) cannot be determined from given info

_____ 6. Dividing the polynomial $f(x)=x^3+3x^2-12$ by the polynomial $p(x)=x+1$ gives a remainder of what?

(A) 0 (B) -10 (C) 10 (D) -8 (E) none of these

_____ 7. Find the inverse of the function $f(x)=\frac{3x+2}{x}$, where $x \neq 0$. $f^{-1}(x)=$

(A) $\frac{1}{3x}$ (B) $\frac{x}{2x-3}$ (C) $\frac{x}{2x+3}$ (D) $\frac{2}{x-3}$ (E) none of these

10. For $f(x)=2x^2-kx^2+3x-k$, find the value of k so that when $f(x)$ is divided by $x+1$ the remainder is $\frac{2}{3}$.

_____ 3. Rationalize the numerator of $\frac{\sqrt{x+4}-\sqrt{x-2}}{x}$

(A) $\frac{2}{x\left(\sqrt{x+4}+\sqrt{x-2}\right)}$ (B) $\frac{6}{x\left(\sqrt{x+4}-\sqrt{x-2}\right)}$ (C) $\frac{6}{x\left(\sqrt{x+4}+\sqrt{x-2}\right)}$

(D) $\frac{2x}{\sqrt{x+4}+\sqrt{x-2}}$ (E) $\frac{6x}{\sqrt{x+4}-\sqrt{x-2}}$

_____ 4. Which, if any, of the following statements are true when a, b are real numbers?

I. For all positive a and b, $\sqrt{a+b}=\sqrt{a}+\sqrt{b}$.

II. For all a and b, $\sqrt{(a+b)^2}=|a+b|$.

III. For all positive a and b, $\frac{a-b}{\sqrt{a}+\sqrt{b}}=\sqrt{a}+\sqrt{b}$.

(A) III only (B) all of them (C) I and II only (D) II only (E) II and III only (F) none of them (G) I and III only (H) I only

_____ 5. Simplify the expression $\frac{1+\frac{2}{x-3}}{5+40\left(\frac{x}{x^2-9}\right)}$

(A) $\frac{1}{5}\left(\frac{x+3}{2x+9}\right)$ (B) $\frac{x+3}{x-9}$ (C) $\frac{1}{5}\left(\frac{x+3}{x+9}\right)$ (D) $\frac{x+3}{2x-9}$ (E) $\frac{1}{5}\left(\frac{x-3}{x+9}\right)$ (F) $\frac{x-3}{x-9}$

_____ 9. Simplify the difference quotient $\frac{f(x+h)-f(x)}{h}$, $(h \neq 0)$, when $f(x)=2x^2-4x-4$.

(A) $4x+4+2h$ (B) $4x-4+2h$ (C) $2x+4+2h$ (D) $2x-4+2h$ (E) $4x-4$

_____ 11. If $f(x)=2x-1$ and $g(x)=x+3$, which of the following gives $(f \circ g)(2)$?

(A) 2 (B) 6 (C) 7 (D) 9 (E) 10

_____ 12. Which of the following is a solution of the equation $2-3^x=-1$?

(A) $x=-2$ (B) $x=-1$ (C) $x=0$ (D) $x=1$ (E) No solution

_____ 13. The length L of a rectangle is twice as long as its width W. Which of the following gives the area A of the rectangle as a function of its width?

(A) $A(W)=3W$ (B) $A(W)=\frac{1}{2}W^2$ (C) $A(W)=2W^2$

(D) $A(W)=W^2+2W$ (E) $A(W)=W^2-2W$

_____ 15. The set of all points (e^t, t), where t is a real number, is the graph of $y=$

(A) $\frac{1}{e^x}$ (B) $e^{1/x}$ (C) $xe^{1/x}$ (D) $\frac{1}{\ln x}$ (E) $\ln x$

_____ 20. If $f(g(x))=x^3+3x^2+4x+5$ and $g(x)=5$, then $g(f(x))=$

(A) $5x^2+15x+25$ (B) $5x^3+15x^2+20x+25$ (C) 1125 (D) 225 (E) 5

_____ 22. Suppose that *f* is a function that is defined for all real numbers. Which of the following conditions assures that *f* has an inverse function?

(A) The function *f* is periodic (B) The function *f* is symmetric with respect to the *y*-axis
(C) The function *f* is concave up (D) The function *f* is a strictly increasing function
The function *f* is continuous

_____ 23. If $\log_a(2^a)=\frac{a}{4}$, then $a=$

(A) 2 (B) 4 (C) 8 (D) 16 (E) 32

_____ 24. If $f(g(x))=\ln(x^2+4)$, $f(x)=\ln(x^2)$, and $g(x)>0$ for all real x, then $g(x)=$

(A) $\frac{1}{\sqrt{x^2+4}}$ (B) $\frac{1}{x^2+4}$ (C) $\sqrt{x^2+4}$ (D) x^2+4 (E) $x+2$

_____ 29. If $e^{g(x)}=\frac{x^x}{x^2-1}$, then $g(x)=$

(A) $x\ln x-2x$ (B) $\frac{\ln x}{2}$ (C) $(x-2)\ln x$ (D) $\frac{x\ln x}{\ln(x^2-1)}$ (E) $x\ln x-\ln(x^2-1)$

Topic: Domain and range of combination of functions

1. Find the x- and y-intercepts and domain, then sketch the graph and find the range.

   (a) $f(x)=\sqrt{2-x}$ (b) $g(x)=\sqrt{4-x^2}$

   (c) $f(t)=\dfrac{|t-3|}{t-3}$ (d) $h(m)=\begin{cases}(m-1)^2, & m\geq 1\\ 3m-3, & m<1\end{cases}$

2. Sketch the following piecewise functions, then find the domain and range of each.

   (a) $f(x)=\begin{cases}3-x, & x\leq 1\\ 2x, & 1<x\end{cases}$ (b) $g(x)=\begin{cases}2, & x<0\\ \sqrt{x}, & x\geq 0\end{cases}$ (c) $h(t)=\begin{cases}t^2, & t<0\\ t^3, & 0\leq t\leq 1\\ 2t-1, & t>1\end{cases}$

_____ 5. The domain of the function defined by $g(x)=\ln\left(x^2-4\right)$ is the set of all real numbers x such that

(A) $|x|<2$ (B) $|x|\leq 2$ (C) $|x|>2$ (D) $|x|\geq 2$ (E) x is a real number

_____ 6. The graph of $y^2=x^2+9$ is symmetric to which of the following?

I. The x-axis
II. The y-axis
III. The origin

(A) I only (B) II only (C) III only (D) I and II only (E) I, II, and III

______ 7. What is the domain of the function f given by $f(x)=\dfrac{\sqrt{x^2-4}}{x-3}$?

(A) $\{x:x\neq 3\}$ (B) $\{x:|x|\leq 2\}$ (C) $\{x:|x|\geq 2\}$

(D) $\{x:|x|\geq 2 \text{ and } x\neq 3\}$ (E) $\{x:x\geq 2 \text{ and } x\neq 3\}$

_____ 11. Find the domain of the function $f(x)=\dfrac{\sqrt{x+1}}{x-5}$.

(A) $D_f:\{x|x\geq 1\}$ (B) $D_f:\{x|x<1,\ x\neq -5\}$ (C) $D_f:\{x|x\leq -1,\ x\neq -5\}$

(D) $D_f:\{x|x>-1,\ x\neq -5\}$ (E) $D_f:\{x|x\leq 1\}$ (F) $D_f:\{x|x\geq -1,\ x\neq -5\}$

_____ 12. Which of the following functions has the following graph of $x \in [-6,6]$, $x \neq 1$

(A) $f(x) = -\frac{x^2-1}{|x+1|}$

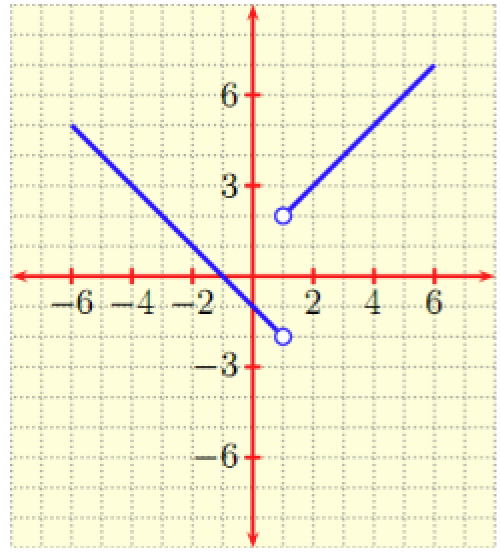
(B) $f(x) = \frac{|x^2-1|}{x-1}$

(C) $f(x) = -\frac{|x^2-1|}{x-1}$

(D) $f(x) = \frac{x^2-1}{|x+1|}$

(E) $f(x) = \frac{x^2-1}{|x-1|}$

(F) $f(x) = -\frac{x^2-1}{|x-1|}$



_____ 13. Which of the following gives the domain of $f(x) = \frac{x}{\sqrt{9-x^2}}$?

(A) $x \neq \pm 3$ (B) $(-3,3)$ (C) $[-3,3]$ (D) $(-\infty,-3)\cup(3,\infty)$ (E) $(3,\infty)$

_____ 15. Which of the following gives the range of $y = 4 - 2^{-x}$?

(A) $(-\infty,2)$ (B) $(-\infty,4)$ (C) $[-4,\infty)$ (D) $(-\infty,4]$ (E) all reals

_____ 17. The domain of the function $f(x) = \ln(x^2 - x - 6)$ is the set of all real numbers x such that

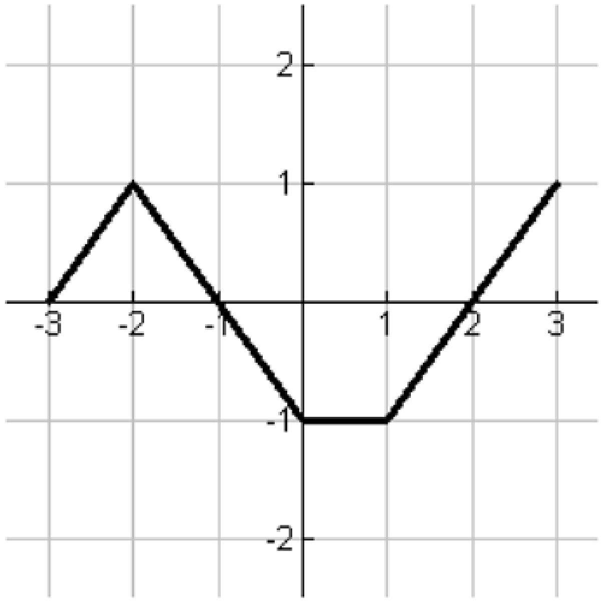
(A) $x > 0$ (B) $-2 \leq x \leq 3$ (C) $x \geq -2$ or $x \geq 3$ (D) $x < -2$ or $x > 3$ (E) $-2 < x < 3$

_____ 18. The domain of $y = \sqrt{(x-1)(x-2)}$ is

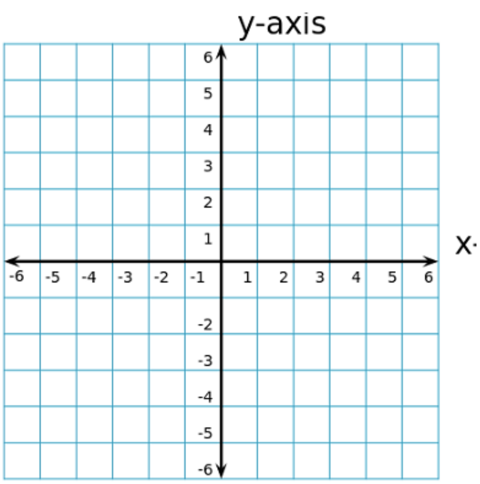
(A) $|x| < 2$ (B) $(1,2)$ (C) $|x| > 2$ (D) $(-\infty,1]\cup[2,\infty)$ (E) $[1,2]$

Transformation with parent function:

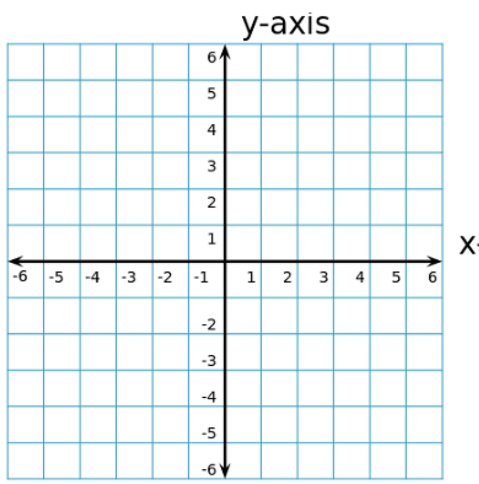
Given the graph of parent function of $f(x)$, graph $g(x)$ using transformation.



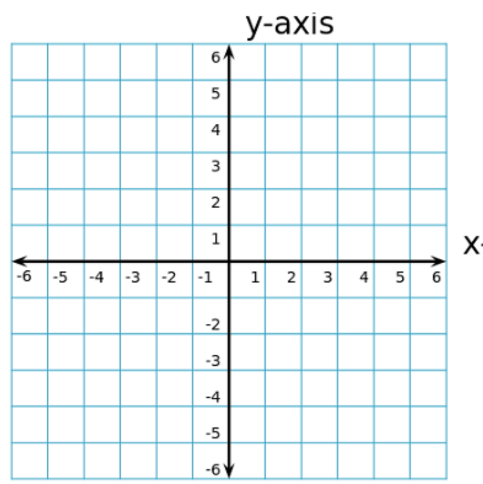
10. $g(x) = -3f(x) + 1$



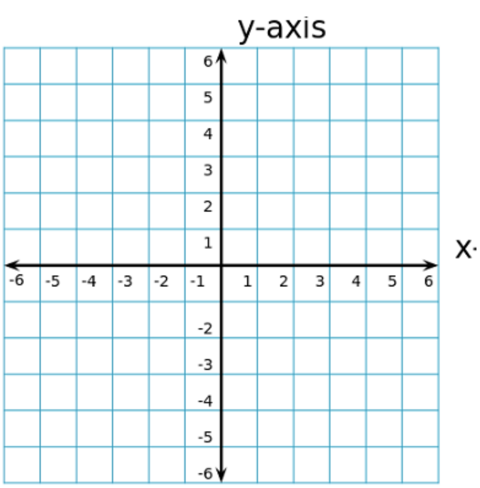
11. $g(x) = f(-3x - 3)$



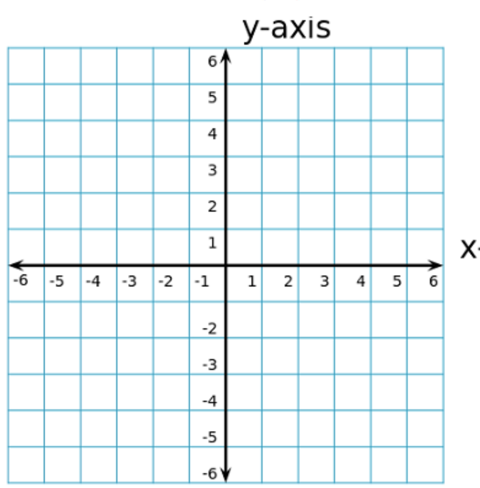
12. $g(x) = 1 - 2f\left(\frac{x}{2}\right)$



13. $g(x) = |f(x)|$



14. $g(x) = f(|x|)$



15. $g(x) = |f(|x|)|$