A Story of Functions: A Curriculum Overview for Grades 9-12

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Introduction

The Common Core Learning Standards (CCLS) define progressions of learning that develop the major content of school mathematics over grades Pre-K through 12. When those standards are further connected to each other within a grade and throughout a sequence of lessons, a coherent story emerges of mathematics as an elegant subject in which the collective body of knowledge results from reasoning from a cohesive set of principles. The word *story* in the title *A Story of Functions* is meant to capture this notion of coherence as students study functions and model with them.

This document provides an overview of the academic year for Grades 9 through 12, beginning with a curriculum map and followed by detailed grade level descriptions. Courses for Algebra I, Geometry, and Algebra II were designed in accordance with PARCC Model Content Frameworks for High School Mathematics.[1] The courses outlined in this document were informed by, but are not identical to, Appendix A of the Common Core State Standards.[2] A Precalculus course is provided as a fourth course.

Each course description begins with a list of the modules that comprise the instruction of the course. The list is followed by five sections of information:

- Summary of Year, which describes the focus of the course[3]
- Recommended Fluencies for the course, as stated in the PARCC Model Content Frameworks for High School Mathematics (Note that this information is not available for Precalculus.)
- CCLS Major Emphasis Clusters for the course, as stated in the PARCC Model Content Frameworks for High School Mathematics (Note that this information is not available for Precalculus.)
- Rationale for the Module Sequence of the course
- Alignment Chart of the course standards

[1] http://www.parcconline.org/sites/parcc/files/PARCCMCFMathematicsNovember2012V3_FINAL.pdf
[2] http://www.corestandards.org/assets/CCSSI_Mathematics_Appendix_A.pdf
[3] Text in the summary paragraphs and Rationale for Module Sequencing for Algebra I, Geometry, and Algebra II were informed by, but are not identical to, Appendix A of the Common Cores State Standards.

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Key for reading this document:

(★) According to the CCLS, "Modeling is best interpreted not as a collection of isolated topics but rather in relation to other standards. Making mathematical models is a Standard for Mathematical Practice, and specific modeling standards appear throughout the high school standards indicated by a star symbol (★)." Opportunities for modeling are woven throughout all four courses and are also indicated by (★) in this document.

(+) The CCLS notes, "Standards beginning with the (+) symbol form a starting point for fourth year courses in Precalculus and in Probability and Statistics." A few key (+) standards are included in the Geometry and Algebra II courses to provide coherence to the curriculum. They can be used to effectively extend a topic (e.g., G-GMD.2 as an extension of G-GMD.1) or to introduce a theme/concept that will be fully covered in the Precalculus course. ***Note: None of the (+) standard in the Geometry or Algebra II course will be assessed on the Regents Exam in those courses.*** All (+) standards are in the Precalculus course where they are assessed.

Timeline

The curriculum map on the next page shows the approximate number of instructional days designated for each module of each grade. The number of instructional days and dates will vary due to different school calendars, school holidays, snow days, and especially student needs.

To accommodate the January and June Regents Exam periods, the modules are based 150 instructional days instead of 180. The remaining 30 days takes into consideration 15 days of test administration and at least 10 days for review. ***Note: For the first administration of the Regents Exams, there will be less than 150 instructional days because the Regents Exams are given early (impacting Algebra I and Geometry in the 2013–2014 school year and all courses in the 2014–2015 school year).***

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Curriculum Map

	Grade 9 -- Algebra I	Grade 10 -- Geometry	Grade 11 -- Algebra II	Grade 12 -- Precalculus	
20 days	M1: Relationships Between Quantities and Reasoning with Equations and Their Graphs (40 days)	M1: Congruence, Proof, and Constructions (45 days)	M1: Polynomial, Rational, and Radical Relationships (45 days)	M1: Complex Numbers and Transformations (40 days)	20 days
20 days					20 days
20 days	M2: Descriptive Statistics (25 days)	M2: Similarity, Proof, and Trigonometry (45 days)	M2: Trigonometric Functions (20 days)	M2: Vectors and Matrices (40 days)	20 days
20 days	M3: Linear and Exponential Functions		M3: Functions (45 days)		20 days
20 days	State Examinations (35 days)	State Examinations	State Examinations	State Examinations	20 days
20 days	M4: Polynomial and Quadratic Expressions, Equations and Functions (30 days)	M3: Extending to Three Dimensions (10 days) M4: Connecting Algebra and Geometry through Coordinates (25 days)	M4: Inferences and Conclusions from Data (40 days)	M3: Rational and Exponential Functions (25 days) M4: Trigonometry (20 days)	20 days
20 days					20 days
20 days	M5: A Synthesis of Modeling with Equations and Functions (20 days)	M5: Circles with and Without Coordinates (25 days)		M5: Probability and Statistics (25 days)	20 days
20 days	Review and Examinations	Review and Examinations	Review and Examinations	Review and Examinations	20 days

Key:	Number and Quantity and Modeling	Geometry and Modeling	Algebra and Modeling	Statistics and Probability and Modeling	Functions and Modeling

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Standards for Mathematical Practice

The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

MP.1 **Make sense of problems and persevere in solving them.** Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships, and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables, and graphs or draw diagrams of important features and relationships, graph data, and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method, and they continually ask themselves, "Does this make sense?" They can understand the approaches of others so solving complex problems and identify correspondences between different approaches.

MP.2 **Reason abstractly and quantitatively.** Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to *decontextualize*—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents, and the ability to *contextualize*—to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand; considering the 6 units involved; attending to the meaning of quantities, not just how to compute them; and knowing and flexibly using different properties of operations and objects (exemplified in Topic D).

MP.3 **Construct viable arguments and critique the reasoning of others.** Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams, and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the argument (exemplified in Topics A and E).

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MP.4 **Model with mathematics.** Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society, and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students, who can apply what they know, are comfortable making assumptions and approximations to simplify a complicated situation and realize that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using tools, such as diagrams, two-way tables, graphs, flowcharts, and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MP.5 **Use appropriate tools strategically.** Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package, or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences, and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website, and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MP.6 **Attend to precision.** Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently and express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school, they have learned to examine claims and make explicit use of definitions (exemplified in Topics C and F).

MP.7 **Look for and make use of structure.** Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more, or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y (exemplified in Topic B).

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MP.8 **Look for and express regularity in repeated reasoning.** Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$, and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results (exemplified in Topic G).

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Sequence of Algebra I Modules Aligned with the Standards

Module 1: Relationships Between Quantities and Reasoning with Equations and Their Graphs
Module 2: Descriptive Statistics
Module 3: Linear and Exponential Functions
Module 4: Polynomial and Quadratic Expressions, Equations and Functions
Module 5: A Synthesis of Modeling with Equations and Functions

Summary of Year

The fundamental purpose of this course is to formalize and extend the mathematics that students learned in the middle grades. Because it is built on the middle grades standards, this is a more ambitious version of Algebra I than has generally been offered. The modules deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Recommended Fluencies for Algebra I

- Solving characteristic problems involving the analytic geometry of lines, including, writing the equation of a line given a point and a slope.
- Adding, subtracting and multiplying polynomials.
- Transforming expressions and chunking (seeing the parts of an expression as a single object) as used in factoring, completing the square, and other algebraic calculations.

CCLS Major Emphasis Clusters
Seeing Structure in Expressions • Interpret the structure of expressions Arithmetic with Polynomials and Rational Expressions • Perform arithmetic operations on polynomials Creating Equations • Create equations that describe numbers or relationships Reasoning with Equations and Inequalities • Understand solving equations as a process of reasoning and explain the reasoning • Solve equations and inequalities in one variable • Represent and solve equations and inequalities graphically Interpreting Functions • Understand the concept of a function and use function notation • Interpret functions that arise in applications in terms of the context Interpreting Categorical and Quantitative Data • Interpret linear models

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Rationale for Module Sequence in Algebra I

Module 1: By the end of eighth grade, students have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Now, students analyze and explain precisely the process of solving an equation. Students, through reasoning, develop fluency writing, interpreting, and translating between various forms of linear equations and inequalities and make conjectures about the form that a linear equation might take in a solution to a problem. They reason abstractly and quantitatively by choosing and interpreting units in the context of creating equations in two variables to represent relationships between quantities. They master the solution of linear equations and apply related solution techniques and the properties of exponents to the creation and solution of simple exponential equations. They learn the terminology specific to polynomials and understand that polynomials form a system analogous to the integers.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Module 2: This module builds upon students' prior experiences with data, providing students with more formal means of assessing how a model fits data. Students display and interpret graphical representations of data, and if appropriate, choose regression techniques when building a model that approximates a linear relationship between quantities. They analyze their knowledge of the context of a situation to justify their choice of a linear model. With linear models, they plot and analyze residuals to informally assess the goodness of fit.

Module 3: In earlier grades, students defined, evaluated, and compared functions in modeling relationships between quantities. In this module, students learn function notation and develop the concepts of domain and range. They explore many examples of functions, including sequences; they interpret functions given graphically, numerically, symbolically, and verbally, translate between representations, and understand the limitations of various representations. Students build on their understanding of integer exponents to consider exponential functions with integer domains. They compare and contrast linear and exponential functions, looking for structure in each and distinguishing between additive and multiplicative change. Students explore systems of equations and inequalities, and they find and interpret their solutions. They interpret arithmetic sequences as linear functions and geometric sequences as exponential functions. In building models of relationships between two quantities, students analyze the key features of a graph or table of a function.

Module 4: In this module, students build on their knowledge from Module 3. Students strengthen their ability to discern structure in polynomial expressions. They create and solve equations involving quadratic and cubic expressions. In this module's modeling applications, students reason abstractly and quantitatively in interpreting parts of an expression that represent a quantity in terms of its context; they also learn to make sense of problems and persevere in solving them by choosing or producing equivalent forms of an expression (e.g., completing the square in a quadratic expression to reveal a maximum value). Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They learn through repeated reasoning to anticipate the graph of a quadratic function by interpreting the structure

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of various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function.

Module 5: In this module, students expand their experience with functions to include more specialized functions—linear, exponential, quadratic, square, and cube root, and those that are piecewise-defined, including absolute value and step. Students select from among these functions to model phenomena using the modeling cycle (see page 61 of the CCLS).

Alignment Chart

Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra I Modules
Module 1: **Relationships Between Quantities and Reasoning with Equations and Their Graphs** (40 days)	**Reason quantitatively and use units to solve problems.** **N-Q.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **N-Q.2**[4] Define appropriate quantities for the purpose of descriptive modeling. **N-Q.3**[5] Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **Interpret the structure of expressions** **A-SSE.1** Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients.[6] b. Interpret complicated expressions by viewing one or more of their parts as a single entity.

[4] This standard will be assessed in Algebra I by ensuring that some modeling tasks (involving Algebra I content or securely held content from Grades 6-8) require the student to create a quantity of interest in the situation being described.

[5] The greatest precision for a result is only at the level of the least precise data point (e.g., if units are tenths and hundredths, then the appropriate level of precision is tenths). Calculation of relative error is not included in this standard (in preparation for Regents Exams).

[6] The "such as" listed are not the only parts of an expression students are expected to know; others include, but are not limited to, degree of a polynomial, leading coefficient, constant term, and the standard form of a polynomial (descending exponents) (in preparation for Regents Exams).

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra I Modules
	For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P. **A-SSE.2**[7] Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* **Perform arithmetic operations on polynomials** **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **Create equations that describe numbers or relationships** **A-CED.1**[8] Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★ **A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ **A-CED.3** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*★ **A-CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*★

[7] In Algebra I, tasks are limited to numerical expressions and polynomial expressions in one variable. Examples: Recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form $(53 - 47)(53 + 47)$. See an opportunity to rewrite $a^2 + 9a + 14$ as $(a + 7)(a + 2)$. This does not include factoring by grouping and factoring the sum and difference of cubes (in preparation for Regents Exams).

[8] In Algebra I, tasks are limited to linear, quadratic, or exponential equations with integer exponents.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra I Modules
	Understand solving equations as a process of reasoning and explain the reasoning **A-REI.1** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **Solve equations and inequalities in one variable** **A-REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. **Solve systems of equations** **A-REI.5** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. **A-REI.6**[9] Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **Represent and solve equations and inequalities graphically** **A-REI.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). **A-REI.12** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
Module 2: **Descriptive Statistics** (25 days)	**Summarize, represent, and interpret data on a single count or measurement variable** **S-ID.1** Represent data with plots on the real number line (dot plots, histograms, and box plots).★ **S-ID.2** Use statistics appropriate to the shape of the data distribution to compare center (median,

[9] Tasks have a real-world context. In Algebra I, tasks have hallmarks of modeling as a mathematical practice (less defined tasks, more of the modeling cycle, etc.).

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra I Modules
	mean) and spread (interquartile range, standard deviation) of two or more different data sets.★ **S-ID.3** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).★ **Summarize, represent, and interpret data on two categorical and quantitative variables** **S-ID.5** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.★ **S-ID.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★ a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*[10] b. Informally assess the fit of a function by plotting and analyzing residuals.[11] c. Fit a linear function for a scatter plot that suggests a linear association.[12] **Interpret linear models** **S-ID.7** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.★ **S-ID.8** Compute (using technology) and interpret the correlation coefficient of a linear fit.★ **S-ID.9** Distinguish between correlation and causation.★

[10] Tasks have a real-world context. In Algebra I, exponential functions are limited to those with domains in the integers. Includes the use of the regression capabilities of the calculator (in preparation for Regents Exams).
[11] Includes creating residual plots using the capabilities of the calculator (not manually) (in preparation for Regents Exams).
[12] Both correlation coefficient and residuals will be addressed in this standard (in preparation for Regents Exams).

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra I Modules
Module 3: **Linear and Exponential Functions** (35 days)	**Write expressions in equivalent forms to solve problems** **A-SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*[13] **Create equations that describe numbers or relationships** **A-CED.1**[14] Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.★ **Represent and solve equations and inequalities graphically** **A-REI.11**[15] Explain why the *x*-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ **Understand the concept of a function and use function notation** **F-IF.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then $f(x)$ denotes the output of *f* corresponding to the input *x*. The

[13] Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation such that choosing and producing an equivalent form of the expression reveals something about the situation. In Algebra I, tasks are limited to exponential expressions with integer exponents.
[14] In Algebra I, tasks are limited to linear, quadratic, or exponential equations with integer exponents.
[15] In Algebra I, tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. Finding the solutions approximately is limited to cases where $f(x)$ and $g(x)$ are polynomial functions.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra I Modules
	graph of f is the graph of the equation $y = f(x)$.
	F-IF.2 Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
	F-IF.3[16] Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n–1)$ for $n \geq 1$.*
	Interpret functions that arise in applications in terms of the context
	F-IF.4[17] For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★
	F-IF.5 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★
	F-IF.6[18] Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★
	Analyze functions using different representations
	F-IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple

[16] This standard is part of the Major Content in Algebra I and will be assessed accordingly.

[17] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. The focus in this module is on linear and exponential functions.

[18] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. The focus in this module is on linear and exponential functions.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra I Modules
	cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. **F-IF.9**[19] Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **Build a function that models a relationship between two quantities** **F-BF.1**[20] Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. **Build new functions from existing functions** **F-BF.3**[21] Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* **Construct and compare linear, quadratic, and exponential models and solve problems** **F-LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.★ a. Prove that linear functions grow by equal differences over equal intervals, and that

[19] In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. The focus in this module is on linear and exponential functions.

[20] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.

[21] In Algebra I, identifying the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x+k)* for specific values of *k* (both positive and negative) is limited to linear and quadratic functions. Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. Tasks do not involve recognizing even and odd functions. The focus in this module is on linear and exponential functions.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra I Modules
	exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **F-LE.2**[22] Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★ **F-LE.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.★ **Interpret expressions for functions in terms of the situation they model** **F-LE.5**[23] Interpret the parameters in a linear or exponential function in terms of a context.★
Module 4: **Polynomial and Quadratic Expressions, Equations and Functions** (30 days)	**Use properties of rational and irrational numbers.** **N-RN.3** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. **Interpret the structure of expressions** **A-SSE.1** Interpret expressions that represent a quantity in terms of its context.★ a. Interpret parts of an expression, such as terms, factors, and coefficients.[24]

[22] In Algebra I, tasks are limited to constructing linear and exponential functions in simple context (not multi-step).
[23] Tasks have a real-world context. In Algebra I, exponential functions are limited to those with domains in the integers.
[24] The "such as" listed are not the only parts of an expression students are expected to know; others include, but are not limited to, degree of a polynomial, leading coefficient, constant term, and the standard form of a polynomial (descending exponents) (in preparation for Regents Exams).

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra I Modules
	b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret $P(1+r)^n$ as the product of P and a factor not depending on P.* **A-SSE.2**[25] Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.* **Write expressions in equivalent forms to solve problems** **A-SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ a. Factor a quadratic expression to reveal the zeros of the function it defines.[26] b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. **Perform arithmetic operations on polynomials** **A-APR.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. **Understand the relationship between zeros and factors of polynomials** **A-APR.3**[27] Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

[25] In Algebra I, tasks are limited to numerical expressions and polynomial expressions in one variable. Examples: Recognize $53^2 - 47^2$ as a difference of squares and see an opportunity to rewrite it in the easier-to-evaluate form $(53 - 47)(53 + 47)$. See an opportunity to rewrite $a^2 + 9a + 14$ as $(a + 7)(a + 2)$. This does not include factoring by grouping and factoring the sum and difference of cubes (in preparation for Regents Exams).

[26] Includes trinomials with leading coefficients other than 1 (in preparation for Regents Exams).

[27] In Algebra I, tasks are limited to quadratic and cubic polynomials in which linear and quadratic factors are available. For example, find the zeros of $(x - 2)(x^2 - 9)$.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra I Modules
	Create equations that describe numbers or relationships **A-CED.1**[28] Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*★ **A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ **Solve equations and inequalities in one variable** **A-REI.4**[29] Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.[30] **Represent and solve equations and inequalities graphically** **A-REI.11**[31] Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute

[28] In Algebra I, tasks are limited to linear, quadratic, or exponential equations with integer exponents.
[29] Solutions may include simplifying radicals (in preparation for Regents Exams).
[30] Tasks do not require students to write solutions for quadratic equations that have roots with nonzero imaginary parts. However, tasks can require the student to recognize cases in which a quadratic equation has no real solutions.
[31] In Algebra I, tasks that assess conceptual understanding of the indicated concept may involve any of the function types mentioned in the standard except exponential and logarithmic functions. Finding the solutions approximately is limited to cases where $f(x)$ and $g(x)$ are polynomial functions.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra I Modules
	value, exponential, and logarithmic functions.★ **Interpret functions that arise in applications in terms of the context** **F-IF.4**[32] For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*★ **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*★ **F-IF.6**[33] Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ **Analyze functions using different representations** **F-IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

[32] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. The focus in this module is on linear and exponential functions.

[33] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. The focus in this module is on linear and exponential functions.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra I Modules
	F-IF.8 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. **F-IF.9**[34] Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **Build new functions from existing functions** **F-BF.3**[35] Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.*

[34] In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

[35] In Algebra I, identifying the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x+k)* for specific values of *k* (both positive and negative) is limited to linear and quadratic functions. Experimenting with cases and illustrating an explanation of the effects on the graph using technology is limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers. Tasks do not involve recognizing even and odd functions.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra I Modules
Module 5: **A Synthesis of Modeling with Equations and Functions** (20 days)	**Reason quantitatively and use units to solve problems.** **N-Q.3**[36] Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **Create equations that describe numbers or relationships** **A-CED.1**[37] Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.★ **A-CED.2** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.★ **Interpret functions that arise in applications in terms of the context** **F-IF.4**[38] For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★ **F-IF.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. *For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate*

[36] The greatest precision for a result is only at the level of the least precise data point (e.g., if units are tenths and hundredths, then the appropriate level of precision is tenths). Calculation of relative error is not included in this standard (in preparation for Regents Exams).
[37] In Algebra I, tasks are limited to linear, quadratic, or exponential equations with integer exponents.
[38] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra I Modules
	domain for the function.★ **F-IF.6**[39] Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ **Build a function that models a relationship between two quantities** **F-BF.1**[40] Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context. **Construct and compare linear, quadratic, and exponential models and solve problems** **F-LE.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.★ b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **F-LE.2**[41] Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★

[39] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, square root functions, cube root functions, piecewise-defined functions (including step functions and absolute value functions), and exponential functions with domains in the integers.
[40] Tasks have a real-world context. In Algebra I, tasks are limited to linear functions, quadratic functions, and exponential functions with domains in the integers.
[41] In Algebra I, tasks are limited to constructing linear and exponential functions in simple context (not multi-step).

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Sequence of Geometry Modules Aligned with the Standards

Module 1: Congruence, Proof, and Constructions
Module 2: Similarity, Proof, and Trigonometry
Module 3: Extending to Three Dimensions
Module 4: Connecting Algebra and Geometry through Coordinates
Module 5: Circles with and Without Coordinates

Summary of Year

The fundamental purpose of the course in Geometry is to formalize and extend students' geometric experiences from the middle grades. Students explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Close attention should be paid to the introductory content for the Geometry conceptual category found in the high school CCSS. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Recommended Fluencies for Geometry

- Triangle congruence and similarity criteria.
- Using coordinates to establish geometric results.
- Calculating length and angle measures.
- Using geometric representations as a modeling tool.
- Using construction tools, physical and computational to draft models of geometric phenomenon.

CCLS Major Emphasis Clusters
Congruence • Understand congruence in terms of rigid motions • Prove geometric theorems Similarity, Right Triangles, and Trigonometry • Understand similarity in terms of similarity transformations • Prove theorems using similarity • Define trigonometric ratios and solve problems involving right triangles Expressing Geometric Properties with Equations • Use coordinates to prove simple geometric theorems algebraically Modeling with Geometry • Apply geometric concepts in modeling situations

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Rationale for Module Sequence in Geometry

Module 1: In previous grades, students were asked to draw triangles based on given measurements. They also have prior experience with rigid motions—translations, reflections, and rotations—and have strategically applied a rigid motion to informally show that two triangles are congruent. In this module, students establish triangle congruence criteria, based on analyses of rigid motions and formal constructions. They build upon this familiar foundation of triangle congruence to develop formal proof techniques. Students make conjectures and construct viable arguments to prove theorems—using a variety of formats—and solve problems about triangles, quadrilaterals, and other polygons. They construct figures by manipulating appropriate geometric tools (compass, ruler, protractor, etc.) and justify why their written instructions produce the desired figure.

Module 2: Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, make sense of and persevere in solving similarity problems, and apply similarity to right triangles to prove the Pythagorean Theorem. Students attend to precision in showing that trigonometric ratios are well defined, and apply trigonometric ratios to find missing measures of general (not necessarily right) triangles. Students model and make sense out of indirect measurement problems and geometry problems that involve ratios or rates.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Module 3: Students' experience with two-dimensional and three-dimensional objects is extended to include informal explanations of circumference, area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line. They reason abstractly and quantitatively to model problems using volume formulas.

Module 4: Building on their work with the Pythagorean Theorem in 8th grade to find distances, students analyze geometric relationships in the context of a rectangular coordinate system, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines, relating back to work done in the first module. Students attend to precision as they connect the geometric and algebraic definitions of parabola. They solve design problems by representing figures in the coordinate plane, and in doing so, they leverage their knowledge from synthetic geometry by combining it with the solving power of algebra inherent in analytic geometry.

Module 5: In this module, students prove and apply basic theorems about circles, such as: a tangent line is perpendicular to a radius theorem, the inscribed angle theorem, and theorems about chords, secants, and tangents dealing with segment lengths and angle measures. They study relationships among segments on chords, secants, and tangents as an application of similarity. In the Cartesian coordinate system, students explain the correspondence between the definition of a circle and the equation of a circle written in terms of the distance formula, its radius, and coordinates of its center. Given an equation of a circle, they draw the graph in the coordinate plane and apply techniques for solving quadratic equations. Students visualize, with the aid of appropriate software tools, changes to a three-dimensional model by exploring the consequences of varying parameters in the model.

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Alignment Chart

Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Geometry Modules
Module 1: **Congruence, Proof, and Constructions** (45 days)	**Experiment with transformations in the plane** **G-CO.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. **G-CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). **G-CO.3**[42] Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. **G-CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **G-CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. **Understand congruence in terms of rigid motions** **G-CO.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. **G-CO.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.

[42] Trapezoid is defined as "A quadrilateral with at least one pair of parallel sides" (in preparation for Regents Exams).

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Geometry Modules
	G-CO.8 Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. **Prove geometric theorems** **G-CO.9** Prove theorems about lines and angles. *Theorems include*[43]*: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.* **G-CO.10** Prove theorems about triangles. *Theorems include*[44]*: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.* **G-CO.11** Prove theorems about parallelograms. *Theorems include*[45]*: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals*. **Make geometric constructions** **G-CO.12**[46] Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). *Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.* **G-CO.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle.

[43] Theorems include but are not limited to the listed theorems. Example: Theorems that involve complementary or supplementary angles (in preparation for Regents Exams).
[44] Theorems include but are not limited to the listed theorems. Example: An exterior angle of a triangle is equal to the sum of the two interior opposite angles of the triangle (in preparation for Regents Exams).
[45] Theorems include but are not limited to the listed theorems. Example: A rhombus is a parallelogram with perpendicular diagonals (in preparation for Regents Exams).
[46] The constructions include, but are not limited to, the listed constructions (in preparation for Regents Exams).

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Geometry Modules
Module 2: **Similarity, Proof, and Trigonometry** (45 days)	**Understand similarity in terms of similarity transformations** **G-SRT.1** Verify experimentally the properties of dilations given by a center and a scale factor: a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **G-SRT.2** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **G-SRT.3** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **Prove theorems involving similarity** **G-SRT.4** Prove theorems about triangles. *Theorems include*[47]*: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity*. **G-SRT.5** Use congruence and similarity criteria[48] for triangles to solve problems and to prove relationships in geometric figures. **Define trigonometric ratios and solve problems involving right triangles** **G-SRT.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

[47] Theorems include, but are not limited to, the listed theorems. Example: the length of the altitude drawn from the vertex of the right angle of a right triangle to its hypotenuse is the geometric mean between the lengths of the two segments of the hypotenuse (in preparation for Regents Exams).
[48] ASA, SAS, SSS, AAS, and Hypotenuse-Leg theorems are valid criteria for triangle congruence. AA, SAS, and SSS are valid criteria for triangle similarity (in preparation for Regents Exams).

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Geometry Modules
	G-SRT.7 Explain and use the relationship between the sine and cosine of complementary angles. **G-SRT.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★ **Apply geometric concepts in modeling situations** **G-MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★ **G-MG.2** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).★ **G-MG.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).★
Module 3: **Extending to Three Dimensions** (10 days)	**Explain volume formulas and use them to solve problems**[49] **G-GMD.1** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. *Use dissection arguments, Cavalieri's principle, and informal limit arguments*. **G-GMD.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.★ **Visualize relationships between two-dimensional and three-dimensional objects** **G-GMD.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. **Apply geometric concepts in modeling situations** **G-MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a

[49] The (+) standard on the volume of the sphere is an extension of G-GMD.1. It is explained by the teacher in this grade and used by students in G-GMD.3. Note: Students are not assessed on proving the volume of a sphere formula until Precalculus.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Geometry Modules
	tree trunk or a human torso as a cylinder).★
Module 4: **Connecting Algebra and Geometry through Coordinates** (25 days)	**Use coordinates to prove simple geometric theorems algebraically** **G-GPE.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).* **G-GPE.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point). **G-GPE.6** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **G-GPE.7** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.★
Module 5: **Circles with and Without Coordinates** (25 days)	**Understand and apply theorems about circles** **G-C.1** Prove that all circles are similar. **G-C.2** Identify and describe relationships among inscribed angles, radii, and chords. *Include*[50] *the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.* **G-C.3** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **Find arc lengths and areas of sectors of circles**

[50] Relationships include but are not limited to the listed relationships. Example: angles involving tangents and secants (in preparation for Regents Exams).

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Geometry Modules
	G-C.5 Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector **Translate between the geometric description and the equation for a conic section** **G-GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **Use coordinates to prove simple geometric theorems algebraically** **G-GPE.4** Use coordinates to prove simple geometric theorems algebraically. *For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2)*. **Apply geometric concepts in modeling situations** **G-MG.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).★

Extensions to the Geometry Course

The (+) standards below are included in the Geometry course to provide coherence to the curriculum. They can be used to effectively extend a topic or to introduce a theme/concept that will be fully covered in the Precalculus course. *Note: None of the (+) standard below will be assessed on the Regents Exam or PARRC Assessments until Precalculus.*

Module 2. These standards can be taught as applications of similar triangles and the definitions of the trigonometric ratios.	**Apply trigonometry to general triangles** **G-SRT.9** (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **G-SRT.10** (+) Prove the Laws of Sines and Cosines and use them to solve problems.

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	G-SRT.11 (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).
Module 3. This standard on the volume of the sphere is an extension of G-GMD.1. In this course, it is explained by the teacher and used by students in G-GMD.3.	**Explain volume formulas and use them to solve problems** **G-GMD.2** (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
Module 5. This standard is an immediate extension of G-C.2 and can be given as a homework assignment (with an appropriate hint).	**Understand and apply theorems about circles** **G-C.4** (+) Construct a tangent line from a point outside a given circle to the circle.

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Sequence of Algebra II Modules Aligned with the Standards

Module 1: Polynomial, Rational, and Radical Relationships
Module 2: Trigonometric Functions
Module 3: Functions
Module 4: Inferences and Conclusions from Data

Summary of Year

Building on their work with linear, quadratic, and exponential functions, students extend their repertoire of functions to include polynomial, rational, and radical functions. Students work closely with the expressions that define the functions and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

Recommended Fluencies for Algebra II

- Divide polynomials with remainder by inspection in simple cases.
- See structure in expressions and use this structure to rewrite expressions (e.g., factoring, grouping).
- Translate between recursive definitions and closed forms for problems involving sequences and series.

CCLS Major Emphasis Clusters
The Real Number System • Extend the properties of exponents to rational exponents Seeing Structure in Expressions • Interpret the structure of expressions • Write expressions in equivalent forms to solve problems Arithmetic with Polynomials and Rational Expressions • Understand the relationship between zeros and factors of polynomials Reasoning with Equations and Inequalities • Understand solving equations as a process of reasoning and explain the reasoning • Represent and solve equations and inequalities graphically Interpreting Functions • Interpret functions that arise in applications in terms of the context Building Functions • Build a function that models a relationship between two quantities Making Inferences and Justifying Conclusions • Make inferences and justify conclusions from sample surveys, experiments and observational studies

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Rationale for Module Sequence in Algebra II

Module 1: In this module, students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect the structure inherent in multi-digit whole number multiplication with multiplication of polynomials and similarly connect division of polynomials with long division of integers. Students identify zeros of polynomials, including complex zeros of quadratic polynomials. Through regularity in repeated reasoning, they make connections between zeros of polynomials and solutions of polynomial equations. Students analyze the key features of a graph or table of a polynomial function and relate those features back to the two quantities in the problem that the function is modeling. A theme of this module is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Module 2: Building on their previous work with functions, and on their work with trigonometric ratios and circles in Geometry, students extend trigonometric functions to all (or most) real numbers. To reinforce their understanding of these functions, students begin building fluency with the values of sine, cosine, and tangent at $\pi/6$, $\pi/4$, $\pi/3$, $\pi/2$, etc. Students make sense of periodic phenomena as they model with trigonometric functions.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Module 3: In this module, students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore (with appropriate tools) the effects of transformations on graphs of diverse functions, including functions arising in an application. They notice, by looking for general methods in repeated calculations, that the transformations on a graph always have the same effect regardless of the type of the underlying function. These observations lead to students to conjecture and construct general principles about how the graph of a function changes after applying a function transformation to that function. Students identify appropriate types of functions to model a situation, they adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as, "*the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions,*" is at the heart of this module. In particular, through repeated opportunities in working through the modeling cycle (see page 61 of the CCLS), students acquire the insight that the same mathematical or statistical structure can sometimes model seemingly different situations.

Module 4: In this module, students see how the visual displays and summary statistics they learned in earlier grades relate to different types of data and to probability distributions. They identify different ways of collecting data, including sample surveys, experiments, and simulations, and the role that randomness and careful design play in the conclusions that can be drawn. Students create theoretical and experimental probability models following the modeling cycle (see page 61 of CCLS). They compute and interpret probabilities from those models for compound events, attending to mutually exclusive events, independent events, and conditional probability.

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Alignment Chart

Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra II Modules
Module 1: **Polynomial, Rational, and Radical Relationships** (45 days)	**Reason quantitatively and use units to solve problems.** **N-Q.2**[51] Define appropriate quantities for the purpose of descriptive modeling. **Perform arithmetic operations with complex numbers.** **N-CN.1** Know there is a complex number i such that $i^2 = –1$, and every complex number has the form $a + bi$ with a and b real. **N-CN.2** Use the relation $i^2 = –1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **Use complex numbers in polynomial identities and equations.** **N-CN.7** Solve quadratic equations with real coefficients that have complex solutions. **Interpret the structure of expressions** **A-SSE.2**[52] Use the structure of an expression to identify ways to rewrite it. *For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.* **Understand the relationship between zeros and factors of polynomials** **A-APR.2**[53] Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.

[51] This standard will be assessed in Algebra II by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude.

[52] In Algebra II, tasks are limited to polynomial, rational, or exponential expressions. Examples: see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. In the equation $x^2 + 2x + 1 + y^2 = 9$, see an opportunity to rewrite the first three terms as $(x+1)^2$, thus recognizing the equation of a circle with radius 3 and center (–1, 0). See $(x^2 + 4)/(x^2 + 3)$ as $((x^2+3) + 1)/(x^2+3)$, thus recognizing an opportunity to write it as $1 + 1/(x^2 + 3)$. Includes the sum or difference of cubes (in one variable), and factoring by grouping.

[53] Include problems that involve interpreting the Remainder Theorem from graphs and in problems that require long division.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra II Modules
	A-APR.3[54] Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **Use polynomial identities to solve problems** **A-APR.4** Prove[55] polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. **Rewrite rational expressions** **A-APR.6**[56] Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x),* where *a(x), b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system. **Understand solving equations as a process of reasoning and explain the reasoning** **A-REI.1**[57] Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. **A-REI.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. **Solve equations and inequalities in one variable** **A-REI.4**[58] Solve quadratic equations in one variable.

[54] In Algebra II, tasks include quadratic, cubic, and quartic polynomials and polynomials for which factors are not provided. For example, find the zeros of $(x^2 – 1)(x^2 + 1)$.
[55] Prove *and apply* (in preparation for Regents Exams).
[56] Include rewriting rational expressions that are in the form of a complex fraction.
[57] In Algebra II, tasks are limited to simple rational or radical equations.
[58] In Algebra II, in the case of equations having roots with nonzero imaginary parts, students write the solutions as $a \pm bi$, where a and b are real numbers.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra II Modules
	b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. **Solve systems of equations** **A-REI.6**[59] Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. **A-REI.7** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. *For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.* **Analyze functions using different representations** **F-IF-7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★ c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. **Translate between the geometric description and the equation for a conic section** **G-GPE.2** Derive the equation of a parabola given a focus and directrix.
Module 2: **Trigonometric Functions** (20 days)	**Extend the domain of trigonometric functions using the unit circle** **F-TF.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.

[59] In Algebra II, tasks are limited to 3 × 3 systems.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra II Modules
	F-TF.2[60] Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **Model periodic phenomena with trigonometric functions** **F-TF.5**[61] Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.★ **Prove and apply trigonometric identities** **F-TF.8** Prove the Pythagorean identity $\sin^2(\Theta) + \cos^2(\Theta) = 1$ and use it to find $\sin(\Theta)$, $\cos(\Theta)$, or $\tan(\Theta)$ given $\sin(\Theta)$, $\cos(\Theta)$, or $\tan(\Theta)$ and the quadrant of the angle. **Summarize, represent, and interpret data on two categorical and quantitative variables** **S-ID.6**[62] Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.★ a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.*
Module 3: **Functions** (45 days)	**Extend the properties of exponents to rational exponents.** **N-RN.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. *For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.*

[60] Also extend trigonometric functions to their reciprocal functions.
[61] Including specified phase shift.
[62] Tasks have a real-world context. In Algebra II, tasks include exponential functions with domains not in the integers and trigonometric functions.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra II Modules
	N-RN.2[63] Rewrite expressions involving radicals and rational exponents using the properties of exponents. **Reason quantitatively and use units to solve problems.** **N-Q.2**[64] Define appropriate quantities for the purpose of descriptive modeling. **Write expressions in equivalent forms to solve problems** **A-SSE.3**[65] Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★ c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15^t *can be rewritten as* $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%*. **A-SSE.4**[66] Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. *For example, calculate mortgage payments*.★ **Create equations that describe numbers or relationships** **A-CED.1**[67] Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions*.★

[63] Including expressions where either base or exponent may contain variables.
[64] This standard will be assessed in Algebra II by ensuring that some modeling tasks (involving Algebra II content or securely held content from previous grades and courses) require the student to create a quantity of interest in the situation being described (i.e., this is not provided in the task). For example, in a situation involving periodic phenomena, the student might autonomously decide that amplitude is a key variable in a situation, and then choose to work with peak amplitude.
[65] Tasks have a real-world context. As described in the standard, there is an interplay between the mathematical structure of the expression and the structure of the situation, such that choosing and producing an equivalent form of the expression reveals something about the situation. In Algebra II, tasks include exponential expressions with rational or real exponents.
[66] This standard includes using the summation notation symbol.
[67] Tasks have a real-world context. In Algebra II, tasks include exponential equations with rational or real exponents, rational functions, and absolute value functions.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra II Modules
	Represent and solve equations and inequalities graphically **A-REI.11**[68] Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★ **Understand the concept of a function and use function notation** **F-IF.3**[69] Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. *For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$.* **Interpret functions that arise in applications in terms of the context** **F-IF.4**[70] For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. *Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity*.★ **F-IF.6**[71] Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★ **Analyze functions using different representations** **F-IF.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

[68] In Algebra II, tasks may involve any of the function types mentioned in the standard.
[69] This standard is Supporting Content in Algebra II. This standard should support the Major Content in F-BF.2 for coherence.
[70] Tasks have a real-world context. In Algebra II, tasks may involve polynomial, exponential, logarithmic, and trigonometric functions.
[71] Tasks have a real-world context. In Algebra II, tasks may involve polynomial, exponential, logarithmic, and trigonometric functions.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra II Modules
	e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **F-IF.8**[72] Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. b. Use the properties of exponents to interpret expressions for exponential functions. *For example, identify percent rate of change in functions such as* $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, *and classify them as representing exponential growth or decay.* **F-IF.9**[73] Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **Build a function that models a relationship between two quantities** **F-BF.1** Write a function that describes a relationship between two quantities.★ a. Determine an explicit expression, a recursive process, or steps for calculation from a context.[74] b. Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*[75] **F-BF.2** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

[72] Tasks include knowing and applying $A = Pe^{rt}$ and $A = P\left(1 + \frac{r}{n}\right)^{nt}$.

[73] In Algebra II, tasks may involve polynomial, exponential, logarithmic, and trigonometric functions.

[74] Tasks have a real-world context. In Algebra II, tasks may involve linear functions, quadratic functions, and exponential functions.

[75] Combining functions also includes composition of functions.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra II Modules
	Build new functions from existing functions **F-BF.3**[76] Identify the effect on the graph of replacing *f(x)* by *f(x) + k*, *k f(x)*, *f(kx)*, and *f(x + k)* for specific values of *k* (both positive and negative); find the value of *k* given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. *Include recognizing even and odd functions from their graphs and algebraic expressions for them.* **F-BF.4** Find inverse functions. a. Solve an equation of the form *f(x) = c* for a simple function *f* that has an inverse and write an expression for the inverse. *For example, f(x) =2 x^3 or f(x) = (x+1)/(x–1) for x ≠ 1.* **Construct and compare linear, quadratic, and exponential models and solve problems** **F-LE.2**[77] Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).★ **F-LE.4**[78] For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where *a*, *c*, and *d* are numbers and the base *b* is 2, 10, or *e*; evaluate the logarithm using technology.★ **Interpret expressions for functions in terms of the situation they model** **F-LE.5**[79] Interpret the parameters in a linear or exponential function in terms of a context.★
Module 4: **Inferences and Conclusions from Data**	**Summarize, represent, and interpret data on a single count or measurement variable** **S-ID.4** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure

[76] In Algebra II, tasks may involve polynomial, exponential, logarithmic, and trigonometric functions. Tasks may involve recognizing even and odd functions.
[77] In Algebra II, tasks will include solving multi-step problems by constructing linear and exponential functions.
[78] Students learn terminology that logarithm without a base specified is base 10 and that natural logarithm always refers to base *e*.
[79] Tasks have a real-world context. In Algebra II, tasks include exponential functions with domains not in the integers.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra II Modules
(40 days)	is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.★ **Understand and evaluate random processes underlying statistical experiments** **S-IC.1** Understand statistics as a process for making inferences about population parameters based on a random sample from that population.★ **S-IC.2** Decide if a specified model is consistent with results from a given date-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*.★ **Make inferences and justify conclusions from sample surveys, experiments, and observational studies** **S-IC.3** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.★ **S-IC.4** Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.★ **S-IC.5** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.★ **S-IC.6** Evaluate reports based on data.★ **Understand independence and conditional probability and use them to interpret data** **S-CP.1** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions intersections, or complements of other events (“or,” “and,” “not”).★ **S-CP.2** Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.★ **S-CP.3** Understand the conditional probability of *A* given *B* as *P*(*A* and *B*)/*P*(*B*), and interpret

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Algebra II Modules
	independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.★
	S-CP.4 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*★
	S-CP.5 Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*★
	Use the rules of probability to compute probabilities of compound events in a uniform probability model
	S-CP.6 Find the conditional probability of *A* given *B* as the fraction of *B*'s outcomes that also belong to *A*, and interpret the answer in terms of the model.★
	S-CP.7 Apply the Addition Rule, P(*A* or *B*) = P(*A*) + P(*B*) – P(*A* and *B*), and interpret the answer in terms of the model.★

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Extensions to the Algebra II Course

The (+) standards below are included in the Algebra II course to provide coherence to the curriculum. They can be used to effectively extend a topic or to introduce a theme/concept that will be fully covered in the Precalculus course. *Note: None of the (+) standard below will be assessed on the Regents Exam or PARRC Assessments until Precalculus.*

Module 1. Students will be working with zeros of polynomials in this module, which offers teachers an opportunity to introduce Standard N-CN.9. A major theme of the module is A-APR.7. Teachers should continually remind students of the connections between rational expressions and rational numbers as students add, subtract, multiply and divide rational expressions.	**Use complex numbers in polynomial identities and equations.** **N-CN.8** (+) Extend polynomial identities to the complex numbers. *For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.* **N-CN.9** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. **Rewrite rational expressions** **A-APR.7** (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
Module 2. In F-TF.3, students begin fluency exercises with trigonometric ratios of the special angles. Teachers present proofs of formulas in F-TF.9. Students use the formulas in Algebra II; they prove the formulas in Precalculus.	**Extend the domain of trigonometric functions using the unit circle** **F-TF.3** (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π–*x*, π+*x*, and 2π–*x* in terms of their values for *x*, where *x* is any real number. **Prove and apply trigonometric identities** **F-TF.9** (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

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Sequence of Precalculus Modules Aligned with the Standards

Module 1: Complex Numbers and Transformations
Module 2: Vectors and Matrices
Module 3: Rational and Exponential Functions
Module 4: Trigonometry
Module 5: Probability and Statistics

Summary of Year

Extending their understanding of complex numbers to points in the complex plane, students come to understand that multiplying a given set of points by a complex number amounts to rotating and dilating those points in the complex plane about zero. Matrices are studied as tools for performing rotations and reflections of the coordinate plane, as well as for solving systems of linear equations. Inverse functions are explored as students study the relationship between exponential and logarithmic functions and restrict the domain of the trigonometric functions to allow for their inverses. The year concludes with a capstone module on modeling with probability and statistics. The Mathematical Practice Standards apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, useful, and logical subject that makes use of their ability to make sense of problem situations.

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Rationale for Module Sequence in Precalculus

In Algebra II, students extended their understanding of number to include complex numbers as they studied polynomials with complex zeros. In Module 1, students graph complex numbers in the complex plane and translate between rectangular and polar forms of complex numbers. In particular, through repeated reasoning, they come to realize that multiplying a given set of points by a complex number amounts to rotating and dilating those points in the complex plane around the point zero. Thinking of a complex number, $a + bi$, once again as a point (a,b) in the coordinate plane, students investigate how multiplying by a complex number can be thought of as a map from the coordinate plane to itself. That study, in turn, leads to matrix notation and a natural definition for multiplying a vector by a matrix:

$$(a+bi)(c+di)=(ac-bd)+(ad+bc)i \text{ is equivalent to } \begin{pmatrix} a & -b \\ b & a \end{pmatrix}\begin{pmatrix} c \\ d \end{pmatrix}=\begin{pmatrix} ac-bd \\ ad+bc \end{pmatrix}.$$

Thus, students discern structure in the operations with matrices and vectors by comparing them to arithmetic with complex numbers.

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Students began the study of transformations in Grade 8, and precisely defined rigid motions in the plane in terms of angles, circles, perpendicular lines, parallel lines, and segments in Geometry. In this module, students precisely define rotations, reflections and dilations in the coordinate plane using 2 × 2 matrices (and translations by vector addition). These well-defined definitions of transformations of the coordinate plane shed light on how geometry software and video games efficiently perform rigid motion calculations.

In the first module, students viewed matrices as tools for performing rotations and reflections of the coordinate plane. In Module 2, they move beyond this viewpoint to study matrices and vectors as objects in their own right. Students interpret the properties and operations of matrices to learn multiple ways to solve problems with them, including solving systems of linear equations. They construct viable arguments using matrices to once again derive equations for conic sections, this time by translating and rotating the locus of points into a "standard" position using matrix operations. (For example, applying rigid motions to move the directrix of a parabola to one of the coordinate axes.)

Students study rational and exponential functions in Module 3. They graph rational functions by extending what they learned about graphing polynomials functions. Students, through repeatedly exploiting the relationship between exponential and logarithmic functions, learn the meaning of inverse functions. Additionally, students learn to explicitly build composite functions to model relationships between two quantities. In particular, they analyze the composite of two functions in describing the relationship of three or more quantities in modeling activities in this module.

In Module 4, students visualize graphs of trigonometric functions with the aid of appropriate software and interpret how a family of graphs defined by varying a parameter in a given function changes based upon that parameter. They analyze symmetry and periodicity of trigonometric functions. They extend their knowledge of inverse functions to trigonometric functions by restricting domains to create the inverses, and apply inverse functions to

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solve trigonometric equations that arise in modeling contexts. Students also construct viable arguments to prove the Law of Sines, Law of Cosines, and the addition and subtraction formulas for the trigonometric functions.

This course concludes with Module 5, a capstone module on modeling with probability and statistics in which students consolidate their study of statistics as they analyze decisions and strategies using newly refined skills in calculating expected values.

Alignment Chart

Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Precalculus Modules
Module 1: **Complex Numbers and Transformations** (40 days)	**Perform arithmetic operations with complex numbers.** **N-CN.3** (+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers. **Represent complex numbers and their operations on the complex plane.** **N-CN.4** (+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number. **N-CN.5** (+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. *For example,* $(-1 + \sqrt{3}\,i)^3 = 8$ *because* $(-1 + \sqrt{3}\,i)$ *has modulus 2 and argument 120°.* **N-CN.6** (+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints. **Perform operations on matrices and use matrices in applications.** **N-VM.10**[80] (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square

[80] N.VM and G.CO standards are included in the context of defining transformations of the plane rigorously using complex numbers and 2 × 2 matrices and linking rotations and reflections to multiplication by complex number and/or by 2 × 2 matrices to show how geometry software and video games work.

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Module and Approximate Number of Instructional Days	Common Core Learning Standards Addressed in Precalculus Modules
	matrix is nonzero if and only if the matrix has a multiplicative inverse. **N-VM.11** (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. **N-VM.12** (+) Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. **Experiment with transformations in the plane** **G-CO.2** Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). **G-CO.4** Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. **G-CO.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.
Module 2: **Vectors and Matrices** (40 days)	**Represent and model with vector quantities.** **N-VM.1** (+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\mathbf{v}$, $\lvert\mathbf{v}\rvert$, $\lvert\lvert\mathbf{v}\rvert\rvert$, v). **N-VM.2** (+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point. **N-VM.3** (+) Solve problems involving velocity and other quantities that can be represented by vectors.

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	Perform operations on vectors. **N-VM.4** (+) Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction **v** – **w** as **v** + (–**w**), where –**w** is the additive inverse of **w**, with the same magnitude as **w** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise. **N-VM.5** (+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(v_x, v_y) = (cv_x, cv_y)$. b. Compute the magnitude of a scalar multiple c**v** using $\|\|c\mathbf{v}\|\| = \|c\|v$. Compute the direction of c**v** knowing that when $\|c\|v \neq 0$, the direction of c**v** is either along **v** for ($c > 0$) or against **v** (for $c < 0$). **Perform operations on matrices and use matrices in applications.** **N-VM.6** (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network. **N-VM.7** (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled. **N-VM.8** (+) Add, subtract, and multiply matrices of appropriate dimensions. **N-VM.9** (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices

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	is not a commutative operation, but still satisfies the associative and distributive properties. **N-VM.10** (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse. **N-VM.11** (+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors. **N-VM.12** (+) Work with 2 x 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area. **Solve systems of equations** **A-REI.8** (+) Represent a system of linear equations as a single matrix equation in a vector variable. **A-REI.9** (+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 x 3 or greater). **Translate between the geometric description and the equation for a conic section** **G-GPE.1** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **G-GPE.2** Derive the equation of a parabola given a focus and directrix. **G-GPE.3** (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
Module 3: **Rational and Exponential Functions** (25 days)	**Use complex numbers in polynomial identities and equations.** **N-CN.8** (+) Extend polynomial identities to the complex numbers. *For example, rewrite* $x^2 + 4$ *as* $(x + 2i)(x – 2i)$. **N-CN.9** (+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

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	Use polynomial identities to solve problems **A-APR.5** (+) Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[81] **Rewrite rational expressions** **A-APR.7** (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. **Analyze functions using different representations** **F-IF.7** Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases.★ d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. **F-IF.9**[82] Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.* **Build a function that models a relationship between two quantities** **F-BF.1** Write a function that describes a relationship between two quantities.★ c. (+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time*.

[81] The Binomial Theorem can be proved by mathematical induction or by a combinatorial argument.
[82] This standard is to be applied to rational functions.

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	Build new functions from existing functions **F-BF.4** Find inverse functions. b. (+) Verify by composition that one function is the inverse of another. c. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse. d. (+) Produce an invertible function from a non-invertible function by restricting the domain. **F-BF.5** (+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents. **Explain volume formulas and use them to solve problems** **G-GMD.2** (+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
Module 4: **Trigonometry** (20 days)	**Extend the domain of trigonometric functions using the unit circle** **F-TF.3** (+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi–x$, $\pi+x$, and $2\pi–x$ in terms of their values for x, where x is any real number. **F-TF.4** (+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. **Model periodic phenomena with trigonometric functions** **F-TF.6** (+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed. **F-TF.7** (+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.★

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	Prove and apply trigonometric identities **F-TF.9**[83] (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems. **Apply trigonometry to general triangles** **G-SRT.9** (+) Derive the formula $A = 1/2\ ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. **G-SRT.10** (+) Prove the Laws of Sines and Cosines and use them to solve problems. **G-SRT.11** (+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces). **Understand and apply theorems about circles** **G-C.4** (+) Construct a tangent line from a point outside a given circle to the circle.
Module 5: **Probability and Statistics** (25 days)	**Use the rules of probability to compute probabilities of compound events in a uniform probability model** **S-CP.8** (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.★ **S-CP.9** (+) Use permutations and combinations to compute probabilities of compound events and solve problems.★ **Calculate expected values and use them to solve problems** **S-MD.1** (+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.★

[83] Students are now responsible for proofs of angle addition and subtraction formulas.

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	S-MD.2 (+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.★ **S-MD.3** (+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. *For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes*.★ **S-MD.4** (+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. *For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*★ **Use probability to evaluate outcomes of decisions** **S-MD.5** (+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.★ a. Find the expected payoff for a game of chance. *For example, find the expected winnings from a state lottery ticket or a game at a fast-food restaurant*. b. Evaluate and compare strategies on the basis of expected values. *For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident*. **S-MD.6** (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).★ **S-MD.7** (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).★

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