Math 8

Summer Review Packet

Name: _____

Term	Definition	
base (of an exponent)	When numbers are written using exponents, the base is the larger bottom value, whereas the exponent is the smaller superscript to the top right of the base. The base number represents the factor being raised to a certain power (represented by the exponent value). For example, in the number 5^3 , the base is 5 and the exponent is 3.	
center of a dilation	The center of a dilation is a fixed point on a plane. It is the starting point from which we measure distances in a dilation. The center of dilation in this example is point <i>A</i> .	
clockwise	Clockwise means to turn in the same direction as the hands of a clock. It is a turn to the right.	
cone	A cone is a three-dimensional figure that tapers from a circular base to a point.	
congruent	One figure is congruent to another if it can be moved with translations, rotations, and reflections to fit exactly over the other. $30^{\circ} 52^{\circ} 52^{\circ} 10 10$	
constant term	 In an expression like 5x + 2, the number 2 is called the constant term because it doesn't change when x changes. In the expression 7x + 9, 9 is the constant term. In the expression 5x + (-8), -8 is the constant term. In the expression 12 - 4x, 12 is the constant term. 	
corresponding	When part of an original figure matches up with part of a copy, we call them corresponding parts. These could be points, segments, angles, or distances.	

Term	Definition	
counterclockwise	Counterclockwise means to turn opposite of the way the hands of a clock turn.	
cube root	The cube root of a number <i>n</i> is the number whose cube is <i>n</i> . It is also the edge length of a cube with a volume of <i>n</i> . We write the cube root of <i>n</i> as $\sqrt[3]{n}$. The cube root of 64 is 4 because 4^3 is 64. $\sqrt[3]{64}$ is also the edge length of a cube that has a volume of 64.	
cylinder	A cylinder is a three-dimensional figure like a prism, but with bases that are circles.	
dependent variable	A dependent variable is a variable representing the output of a function.	
dilation	A dilation is a transformation in which each point on a figure moves along a line and changes its distance from a fixed point. The fixed point is the center of the dilation. All of the original distances are multiplied by the same scale factor.	
exponent	The value that a number or expression is raised to. When this value is a positive integer, it tells us how many times the number or expression is multiplied by itself.	
function	A function is a rule that assigns exactly one output to each possible input.	
hypotenuse	The hypotenuse is the side of a right triangle that is opposite the right angle. It is the longest side of a right triangle.	
image	An image is the result of translations, rotations, and reflections on an object. Every part of the original object moves in the same way to match up with a part of the image.	

Term	Definition		
independent variable	An independent variable is a variable representing the input of a function.		
irrational number	Irrational numbers are numbers that are not rational; they cannot be written as a fraction of two integers. For example, 2 is a rational number because it can be written as $\frac{2}{1}$, whereas π or $\sqrt{3}$ are irrational because they cannot be written as a fraction of two integers.		
legs	The legs of a right triangle are the sides that make the right angle. They are the two sides that are not the hypotenuse.		
linear relationship	In a linear relationship, one quantity has a constant rate of change with respect to the other. The relationship is called "linear" because its graph is a line. If you travel 6 miles every hour, the relationship between time and distance traveled is linear.		
negative association	A negative association is a relationship between two quantities where one tends to decrease as the other increases.		
outlier	An outlier is a data value that is far from the other values in the data set. $\int_{20}^{11} \int_{20}^{11} \int_{20}^{11$		

Term	Definition	
positive association	A positive association is a relationship between two quantities where one tends to increase as the other increases.	
power of ten	A number written as a power of ten means that it is in the form 10^n , where <i>n</i> represents the exponent of 10 needed to remain equivalent. For example, 10 000 written as a power of ten is 10^4 , since $10\ 000 = 10^4$.	
Pythagorean theorem	The Pythagorean theorem describes the relationship between the side lengths of right triangles. The square of the hypotenuse is equal to the sum of the squares of the legs. This is written as $a^2 + b^2 = c^2$.	
radius	A radius is a line segment that goes from the center of a circle to any point on the circle. A radius can go in any direction. Every radius of a circle is the same length. We also use the word <i>radius</i> to mean the length of this segment.	
rate of change	The rate of change in a linear relationship is the amount y changes when x increases by 1. The rate of change in a linear relationship is also the slope of its graph. 12 9 6 1 1 1 1 1 1 1 1 1 1	
rational number	Rational numbers are numbers that can be written as a fraction of two integers. Some examples of rational numbers are: $\frac{1}{3}$, $\frac{-7}{4}$, 0, 0.2, - 5, and $\sqrt{9}$.	

Term	Definition		
reflection	A reflection across a line moves every point on a figure to a point directly on the opposite side of the line. The new point is the same distance from the line as it was in the original figure.		
relative frequency	The relative frequency of a category tells us the fraction or percent of the data set that is in this category.MeditatedCalm66%Agitated34%Total100%		
rigid transformation	A rigid transformation is a move that does not of figure. Translations, rotations, and reflections ar any sequence of these.	change any measurements of a re rigid transformations, as is	
rotation	A rotation moves every point on a figure around a center by a given angle in a specific direction.	A 45° B	
scale factor	In a dilation, a scale factor is the ratio between the lengths in a dilated figure and in the original figure. For example, the scale factor from polygon <i>ABCD</i> to <i>AB</i> ' <i>C</i> ' <i>D</i> ' is 2.	$ \begin{array}{cccccccccccccccccccccccccccccccccccc$	
scatter plot	A scatter plot is a set of disconnected data points plotted on a coordinate plane. It allows us to investigate connections between two variables.	s 75 0 50 0 60 0 77 77 76 80 82 84 86 88 90 Temperature (°F)	

Term	Definition	
scientific notation	Scientific notation is a way to write very large or very small numbers. We write these numbers by multiplying a number between 1 and 10 by a power of 10. For example, the number 425, 000, 000 in scientific notation is 4.25×10^{8} . The number 0.000000000783 in scientific notation is 7.83×10^{-11} .	
segmented bar graph	A segmented bar graph compares two categories within a data set. The whole bar represents all the data within one category. Then, each bar is separated into parts (segments) that show the percentage of each part in the second category.	
sequence of transformations	A sequence of transformations is a set of translations, rotations, reflections, and dilations on a figure, performed in a given order.	
similar	Two figures are similar if one can fit exactly over the other after rigid transformations and dilations.	
slope	The slope of a line is a number we can calculate using any two points on the line. To find the slope, divide the vertical distance between the points by the horizontal distance. The slope of this line is $\frac{k}{h} = \frac{6}{12} = \frac{1}{2}$.	
solution to an equation with two variables	A solution to an equation with two variables is a pair of values that make the equation true. One solution to the equation $4x + 3y = 24$ is (6, 0) because $4(6) + 3(0) = 24$.	
sphere	A sphere is a three-dimensional figure in which all cross-sections in every direction are circles.	

Term	Definition	
square root	The square root of a positive number n is the positive number whose square is n . It is also the the side length of a square whose area is n . We write the square root of n as \sqrt{n} . The square root of 16 is 4 because 4^2 is 16. $\sqrt{16}$ is also the side length of a square that has an area of 16.	
system of equations	A system of equations is a set of two or more equations. Each equation contains two or more variables. We want to find values for the variables that make all the equations true. These equations make up a system of equations: • $x + y = -2$ • $x - y = 12$ The solution to this system is $x = 5$ and $y = -7$ because when these values are substituted for x and y , each equation is true: $5 + (-7) = -2$ and 5 - (-7) = 12.	
term	A term is a part of an expression. It can be a single number, a variable, or a number and a variable that are multiplied together. For example, the expression $5x + 18$ has two terms. The first term is $5x$ and the second term is 18.	
transformation	A transformation is a translation, rotation, reflection, or dilation, or a combination of these.	
translation	A translation moves every point in a figure a given distance in a given direction.	
transversal	A transversal is a line that cuts across parallel lines.	

Term	Definition				
two-way table	A two-way table provides a way to compare two categorical variables. It shows one of the variables across the top and the other down one side. Each entry in the table is the frequency or relative frequency of the category shown by the column and row headings.	Calm Agitated Total	Meditated 45 23 68	Did Not Meditate 8 21 29	Total 53 44 97
vertical angles	Vertical angles are opposite angles that share the same vertex. They are formed by a pair of intersecting lines. Their angle measures are equal.		141°	399	39°
vertical intercept	The vertical intercept, sometimes called the <i>y</i> -intercept, is the point where the graph of a line crosses the vertical axis. x				
volume	The volume is the number of cubic units that fill a three-dimensional region without any gaps or overlaps.				

Unit 1 Summary

Prior Learning	Math 8, Unit 1	Future Learning
Grades 3–6 • Measuring angles • Parallel lines	 Rigid transformations (translations, rotations, reflections) 	Math 8, Unit 2 Dilations and similarity
 Graphing points Math 7 Sketching geometric shapes Angle relationships 	 Congruent figures Angle relationships on parallel lines 	High SchoolFunction transformationsTriangle congruence theorems

Transformations

There are three types of rigid transformations: translations, rotations, and reflections.



Rotation







To take the pre-image A BCD to the image A'B'C'D', **reflect** the polygon over the *y*-axis and then **translate** 2 units down, or translate first and then reflect.



Defining Congruence



Triangle EFD is congruent to triangle ABC because you can reflect ABC across a horizontal line and then translate to fit it on top of EFD.



Applying Congruence

We can use what we know about congruence and transformations to understand other relationships, particularly relationships between angles in triangles and on parallel lines.

Lines that cross parallel lines are called *transversals*.

We can translate and rotate the line DE to see that both the angles marked x are congruent.

We can use this strategy to see that the sum of all of the angles in a triangle is equal to a half-circle or 180° .



Try This at Home

Transformations



- 1.1 Use the language of transformations (translation, rotation, reflection) to describe how the figure changes from one panel to the next.
- 1.2 Draw a fifth panel that shows the last figure rotated 180° counterclockwise around the middle of the panel.

Defining Congruence

- 2.1 Reflect triangle *ABC* across line k to form a new triangle, *DEF*.
- 2.2 Is triangle *DEF* congruent to triangle *ABC*? Explain your thinking.
- 2.3 What is the measure of angle *D*?
- 2.4 Name at least one pair of sides that have the same length.



Applying Congruence

Here is a pair of parallel lines and a transversal.

3. Use what you know about angle relationships to determine the measurements of all of the other angles in the diagram.



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Unit 8.1, Family Resource

Solutions:

1.1 **Panel 1** \rightarrow **2:** Rotate 90° degrees clockwise around the center of the shape. **Panel 2** \rightarrow **3:** Translate to the right.

Panel 3 \rightarrow **4:** Reflect across a horizontal line in the middle of the panel.

1.2



- 2.1 See image on the right.
- 2.2 Yes.

Explanations vary. I created triangle *DEF* using a rigid transformation (a reflection), so it must be congruent.

- 2.3 45 degrees
- 2.4 Pairs of sides that are the same length:
 - AB and DE
 - BC and EF
 - AC and DF

- 3. Use what you know about angle relationships to determine the measurements of all of the other angles in the diagram.



Unit 2 Summary

Prior Learning	Math 8, Unit 2	Future Learning
Math 7 Proportional relationships 	 Non-rigid transformations (dilations) 	Math 8, Unit 3 Linear relationships
Math 8, Unit 1Rigid transformationsCongruent figures	Similar figuresIntroduction to slope	High SchoolFunction transformationsTrigonometryAverage rate of change

Dilations



The scale factor from figure ABCD to figure AB'C'D' is 2.

Each point in AB'C'D' is twice as far from the center of dilation (*A*) as it is in *ABCD*.

The scale factor from figure AB'C'D' to figure ABCD is $\frac{1}{2}$.



Similarity



These figures are not similar. Corresponding sides have the same scale factor, but corresponding angles are not congruent.



Triangles are similar if they have two pairs of congruent corresponding angles.



Slope

All triangles on the same line are similar. Triangles like these are often called **slope triangles**.

The slope of a line is a measure of its steepness.

We calculate the slope as the ratio of the vertical to the horizontal length of the triangle.

The slope of this line is $\frac{k}{h} = \frac{6}{12} = \frac{1}{2}$.



Try This at Home

Dilations

- 1. Rectangle *A* is 10 cm by 24 cm. Rectangle *B* is a dilation of rectangle *A*. If rectangle *B* is 25 cm by 60 cm, what is the scale factor?
- 2.1 Rectangle WXYZ is dilated with center *P* to W'X'Y'Z'. What was the scale factor used?

Explain your thinking.

2.2 Draw a dilation of *WXYZ* with a center of point *P* using a scale factor of $\frac{3}{2}$.



Similarity

3.1 Quadrilateral *ABCD* is similar to quadrilateral *GHEF*. What is the perimeter of *EFGH*?

Explain your thinking.

3.2 How could you use transformations to show that the two quadrilaterals are similar?





Slope

- 4.1 What is the slope of the line on the right?
- 4.2 Is the point (30, 18) on the line?

Explain your thinking.

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Solutions:

- 1. The scale factor is 2.5 because $10 \cdot 2.5 = 15$ and $24 \cdot 2.5 = 36$.
- 2.1 The scale factor is $\frac{1}{2}$. *Explanations vary.* Each point in W'X'Y'Z' is half as far from *P* as it is in *WXYZ*.





- 3.1 42 units. *Explanations vary.* Segment *GH* corresponds with segment *AB*, so we can use their relationship to find the scale factor. $6 \cdot 1.5 = 9$, so the scale factor is 1.5. We can use the scale factor to determine that the other segments in quadrilateral *EFGH* are 10.5, 7.5, and 15 units. 9 + 10.5 + 7.5 + 15 = 42.
- 3.2 *Responses vary.* You could translate figure *ABCD* until two corresponding points are aligned, such as point *C* and point *E*. Then, dilate figure *ABCD* with center *C* using a scale factor of 1.5. Then, rotate the shape until it is directly on top of *EFGH*.
- 4.1 $\frac{1}{3}$
- 4.2 No. *Explanations vary.* If you draw a slope triangle between (0, 4) and (30, 18), the vertical distance would be 14 units and the horizontal distance would be 30 units. This triangle would have a slope of $\frac{14}{30}$, which is not equal to $\frac{1}{3}$.

Prior Learning	Math 8, Unit 3	Future Learning
 Math 6 Calculating unit rates Math 7 Exploring proportional relationships Math 8, Unit 2 Calculating slope 	 Revisit proportional relationships Slope-intercept form (y = mx + b) Solutions and standard form (ax + by = c) 	 Math 8, Units 4 and 5 Systems of linear equations Piecewise-defined functions Math 8, Unit 6 Scatter plots and bivariate data High School Quadratic and exponential functions

Unit 3 Summary

Proportionality Revisited

Here is an example of a proportional relationship between the amount of carpet bought and its cost. We can identify the **constant of proportionality** or **slope** (1.5) in every representation.

Table

Equation

Graph

Carpets (sq. ft.)	Cost (dollars)
0	0
1	1.50
4	6





Slope-Intercept Form

A relationship between two quantities is called a **linear relationship** if its graph is a line.

For example, Irelle and Omar save some of the money they earned.

Let w represent the number of weeks passed. Let s represent the amount saved.

Equation for Irelle's savings: s = 20w + 10

Equation for Omar's savings: s = 10w + 20

Another example is measuring the amount of money on a public transit fare card over time.

The steepness of this line (called the slope) is

 $\frac{vertical change}{horizontal change} = \frac{40}{16} = -2.5.$

The y-intercept of this line is (0, 40), which means the card started out with \$40 on it.

One equation for this relationship is

y = -2.5x + 40, where x represents the number of rides you take and y represents the money left on the card.



In general, the slope-intercept form of a linear equation looks like:

y = mx + b m represent the two related quantities. m represents the slope of the graphed line. b represents the y-intercept of the line.



Solutions and Standard Form

A solution to an equation is a value (or values) that makes the equation true.

The graph of an equation is all of the ordered pairs that make the equation true.

The point (2, -1) is on the line y = 1.5x - 4.

We can show that the point is a solution to the equation by substituting 2 and -1 for *x* and *y*.

y = 1.5x - 4-1 = 1.5(2) - 4 -1 = 3 - 4 -1 = -1 \checkmark

Another form for a linear equation is **standard form**. An equation for this line in standard form is 3x - 2y = 8.

Try This At Home

Proportionality

This table shows some lengths measured in centimeters and the equivalent lengths in millimeters.

1.1 Complete the table.

Length (cm)	Length (mm)
1	10
2.5	
4	
	55

1.2 Sketch a graph of the relationship between centimeters and millimeters.



1.3 How would the graph look different if the *y*-axis were scaled by 10s instead of by 5s?

Slope-Intercept Form

This graph shows the height, in inches, of a bamboo plant each month after it was planted.

- 2.1 What is the slope of this line? What does that value mean in this context?
- 2.2 At what point does the line intersect the *y*-axis? What does that value mean in this context?
- 2.3 Write an equation showing the relationship between the two variables. Use x for the time in months and y for the height in inches.



Solutions and Standard Form

A length of ribbon is cut into two pieces. The graph shows the length of the second piece, y, for each possible length of the first piece, x.

3.1 How long is the original ribbon?

Explain how you know.

3.2 What is the slope of the line?

What does it represent?

- 3.3 List two possible pairs of lengths for the two pieces and explain what they mean.
- 3.4 Write an equation for the relationship between the length of the first piece (x) and the length of the second piece (y).



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Unit 8.3, Family Resource

Solutions:

1.1

Length (cm)	Length (mm)
1	10
2.5	25
4	40
5.5	55



- 1.3 The graph would look less steep because each point would be twice as close to the *x*-axis.
- 2.1 3. Every month that passes, the bamboo plant grows an additional 3 inches.
- 2.2 (0, 12). This bamboo plant was planted when it was 12 inches tall.

2.3 y = 3x + 12

- 3.1 15 feet. When the first piece is 0 feet long, the second is 15 feet long, so that is the length of the ribbon.
- 3.2 -1. For each length the first piece increases by, the second piece must decrease by the same length. For example, if we want the first piece to be 1 foot longer, then the second piece must be 1 foot shorter.
- 3.3 *Responses vary*. Two possible pairs: (14. 5, 0. 5), which means the first piece is 14. 5 feet long, so the second piece is only a half foot long. (7. 5, 7. 5), which means each piece is 7. 5 feet long, so the original ribbon was cut in half.
- 3.4 x + y = 15

1.2

Unit 4 Summary

Prior Learning	Math 8, Unit 4	Future Learning
 Math 8, Unit 3 Writing linear equations, such as y = 2x + 3 	• Solving equations in one variable, such as $3x + 20 = 7x$	Math 8, Unit 5 Equations of functions High School
 Graphing equations Slope and <i>y</i>-intercept 	 Solving systems of two linear equations using graphs and symbols 	 Solving systems of equations using elimination Solving systems with nonlinear equations
		nonlinear equations

Solving Linear Equations

Solving an equation means finding all values that make the equation true.

x = 2 is a solution to the equation 3x = 6 because 3(2) = 6.



A true equation is like a balanced hanger—if you perform the same operations to both sides, the hanger remains balanced.

The equations 3x + 5 = 5x and 5 = 2x are equivalent because we subtracted 3x (removed three triangles) from both sides.

When an equation requires several operations in order to determine a solution, we write each equation on its own line.

Here we use the **distributive property**: Add 6x, subtract 2, and divide by 11 to both sides of the equation to determine a solution.

$$2(-3x+4) = 5x+2$$
$$-6x+8 = 5x+2$$
$$8 = 11x+2$$
$$6 = 11x$$
$$\frac{6}{11} = x$$

Systems of Linear Equations

A system of equations is a set of two (or more) equations where the variables represent the same values.

Solving a system of equations means finding values for the variables that make both equations true.

Here is an example of a situation where systems of equations are useful:

Yona is running home from school at 4 meters per second. Her brother Haruto is walking to school from home at 2 meters per second. They leave at the same time and their school is 600 meters from their home. When will Yona and Haruto pass each other? How far will they be from home?

If you write an equation for each child's distance from home, the two equations form a system:

y = 2x + 5

y = 3x + 1

Yona:
$$d = 600 - 4t$$

Haruto:
$$d = 2t$$

The solution to the system is the point where the lines cross on the graph.

The question asks when the distances will be equal, so you can set these expressions equal to each other and solve for the time.

Once you know the time, use the equations to find the children's distances at that time.

Yona and Haruto pass each other when 100 seconds have passed. They are 200 meters from home.

$$d = 600 - 4(100)$$

$$d = 2(100)$$

$$d = 600 - 400 \qquad d = 200$$

d = 200

600 - 4t = 2t

600 = 6t100 = t



Try This at Home

Solving Linear Equations

- 1. Solve this equation: 3(3 3x) = 2(x + 3) 30
- 2. Elena and Noah work on the equation $\frac{1}{2}(x + 4) = -10 + 2x$ together.

Here is their work:

Do you agree with their solutions? Explain or show your reasoning.

Elena:	Noah:
$\frac{1}{2}(x+4) = -10+2x$ $(x+4) = -20+2x$ $x+24 = 2x$ $24 = x$ $x = 24$	$\frac{1}{2}(x+4) = -10+2x$ x+4 = -20+2x -3x+4 = -20 -3x = -24 x = -8

Systems of Linear Equations

Tiam and Maneli are biking in the same direction on the same path, but they start at different times. Tiam is riding at a constant speed of 18 miles per hour. Maneli started riding at a constant speed of 12 miles per hour a quarter of an hour (15 minutes) before Tiam started.

- 3.1 Write equations to represent the relationship between time and distance biked for each person.
- 3.2 Graph both equations on the same set of axes.
- 3.3 Use the equations and/or the graph to find the time and distance that Tiam and Maneli meet.



Solutions:

1. There are many possible ways to solve this equation, all with a correct solution of x = 3.

Here is one example:

$$3(3 - 3x) = 2(x + 3) - 30$$

$$9 - 9x = 2x + 6 - 30$$

$$9 - 9x = 2x - 24$$

$$-9x = 2x - 33$$

$$-11x = -33$$

$$x = 3$$

- 2. No, they both have errors in their solutions.
 - Elena multiplied both sides of the equation by 2 in her first step. She did not multiply the 2x by the 2. The second line should be (x + 4) = -20 + 4x.
 - We can check Elena's solution by replacing x with 24 in the original equation to see if the equation is true. Since 14 is not equal to 38, Elena's solution is not correct.

$$\frac{1}{2}(x+4) = -10+2x$$
$$\frac{1}{2}(24+4) = -10+2(24)$$
$$\frac{1}{2}(28) = -10+48$$
$$14 = 38$$

• Noah divided both sides in his last step. He wrote -8 as the quotient on the right hand side instead of 8. $\frac{-24}{-3} = 8$. His last

line should be x = 8.

- We can also check Noah's solution by replacing *x* with −8 in the original equation to see if the equation is true. Noah's solution is not correct.
- 3.1 **Tiam:** d = 18t

Maneli:
$$d = 12(t + \frac{1}{4})$$
 or $d = 12t + 3$

- 3.2 See the graph on the right.
- 3.3 Using the graph, Tiam and Maneli are at the same time and distance when their graphs cross, which is 0.5 hours since Tiam started riding (0.75 hours since Maneli started riding). They meet after having biked 9 miles.



Using equations, you can set the two expressions equal to each other and write the equation $18t = 12(t + \frac{1}{4})$.

One way to solve this equation is shown on the right.

First, use the distributive property to rewrite the right-hand side of the equation.

Then, subtract 12t from both sides.

Finally, divide both sides of the equation by 6.

After a half hour, Tiam and Maneli have each ridden 9 miles. Both strategies—using the graph and the equations—give the same result.

$$18t = 12 (t + \frac{1}{4})$$
$$6t = 3$$
$$t = \frac{1}{2}$$

Unit 5 Summary

Prior Learning	Math 8, Unit 5	Future Learning
Math 7	Introduction to functions	High School
 Area of circles 	 Representing and interpreting functions 	 Function notation Cross–sections and volumes in context
Math 8, Unit 3 Linear relationships 	 Volume of cylinders, cones, and spheres 	

Defining Functions

A function is a rule that assigns exactly one output to each possible input.

Examples	Non-Examples
Input: Name Output: First letter of that name (e.g., Sneha \rightarrow S)	Input: Letter Output: A name beginning with that letter (e.g., $S \rightarrow Sora$)
Input : Any number Output: Three more than the input (e.g., $7 \rightarrow 10$)	Input: Digit Output: A number whose last digit is the input (e.g., $7 \rightarrow 207$)

Here are some more **examples** of functions:

y = 4 - 3x

Input	Output
- 2	4π
- 1	1π
0	0
1	1π
2	4π



Representing and Interpreting Functions

A function can represent a story. Here is one example:



Volume of Cylinders, Cones, and Spheres

Volume is the number of cubic units that fill a 3-D region without any gaps or overlaps.



Try This at Home

Defining Functions

1.1 This table represents the total amount of data used compared to how many phone calls were made in a month.

# of Phone Calls	Total Data Used (GB)
10	4.3
19	6.2
35	7.5
10	8.3

- a. Name the independent variable (input) and dependent variable (output).
- b. Decide whether the situation represents a function or not. Explain your thinking.

1.2 This graph represents the height of a basketball over time.



- a. Name the independent variable (input) and dependent variable (output).
- b. Decide whether the situation represents a function or not. Explain your thinking.
- 1.3 Brown rice costs \$2 per pound. Beans cost \$1.60 per pound. Jamar has \$10 to spend to make a large meal of beans and rice for a potluck dinner. The amount of brown rice he can buy, r, is related to the amount of beans he can buy, b.
 - a. Name the independent variable (input) and dependent variable (output).
 - b. Decide whether the situation represents a function or not. Explain your thinking.

Representing and Interpreting Functions

Match each of the following situations with a graph (you can use a graph multiple times). Name the independent and dependent variables.



- 2.1 Daeja takes a handful of popcorn out of the bag every 5 minutes.
- 2.2 A plant grows the same amount every week.
- 2.3 The day started very warm, but then it slowly got colder.
- 2.4 A cylindrical glass sits on a counter.The more water you pour in, the higher the water level is.
- 3. Write an equation in the form y = mx + b that could represent the plant's growth. Explain what each number means in terms of the situation.

Volume of Cylinders, Cones, and Spheres

This cylinder has a height and radius of 5 cm. Express your answers in terms of π .

- 4.1 What is the diameter of the base?
- 4.2 What is the area of the base?
- 4.3 What is the volume of the cylinder?



- 4.4 What would the volume of a cone with the same height and radius be?
- 4.5 What would the height be if the volume of the cylinder remained the same, but the radius doubled?

Solutions:

- 1.1a The *independent variable* represents the input of a function. The *dependent variable* represents the output of a function. Here, the independent variable is the number of phone calls; the dependent variable is the total data used.
- 1.1b This relationship is **not** a function because the number of calls does not uniquely determine the amount of data. For example, 10 phone calls results in both 4.3 GB and 8.3 GB of data.
- 1.2a By convention, the *independent variable* is represented on the horizontal axis and the *dependent variable* on the vertical axis. The independent variable in this situation is the time since launch. The dependent variable is the height of the basketball.
- 1.2b This relationship is a function because there is exactly one height for each time.
- 1.3a It is possible for either variable to be the independent variable. In this case, we are wondering about how much rice can be bought, so the independent variable is the amount of brown rice purchased. The dependent variable is the amount of beans purchased.
- 1.3b This relationship is a function because for every amount of beans, there is only one possible amount of rice Lin can buy if he wants to spend exactly \$10.
- 2.1 Graph B, Independent variable = Time (minutes), Dependent variable = Amount of popcorn left in bag
- 2.2 Graph A, Independent variable = Time (weeks),Dependent variable = Height of the plant

- 2.3 Graph C, Independent variable = Time (hours), Dependent variable = Temperature outside
- 2.4 Graph A, Independent variable = Volume of water poured in the glass, Dependent variable = Height of water in the glass
- 3. The equations vary. An example equation is y = 2x + 5, where 5 represents the height of the plant when you start measuring and 2 represents the number of inches the plant grows every week.
- 4.1 10 cm. The diameter is twice the length of the radius, and 2(5) = 10.
- 4.2 25π cm². The area of a circle is π times the radius squared, or $(5)^2 \cdot \pi$.
- 4.3 125π cm³. The volume is the area of the base times the height. The area of the base here is 25π , so the volume is 125π cm³ since $25\pi \cdot 5 = 125\pi$.
- 4.4 $\frac{125\pi}{3}$ cm³. The volume of a cone is one-third the volume of the corresponding cylinder.
- 4.5 1. 25 cm. If the radius doubled, then it would be 10 cm. There are various methods for finding the height. One method is to organize each quantity using a table. A sample table is shown below. Radius (cm): 10

Base area (sq. cm): 100π

Height (cm): $\frac{125\pi}{100\pi} = 1.25$

Cylinder volume (cubic cm): 125π

Unit 6 Summary

Prior Learning	Math 8, Unit 6	Future Learning
 Grade 3 Drawing and interpreting bar graphs 	 Organizing numerical data Analyzing numerical data Categorical data 	 High School Fitting data with nonlinear functions Correlation coefficients
 Math 6 Plotting points in the coordinate plane Shape, center, spread, and outliers of data in one variable 		
Math 8, Unit 3 Linear relationships		

Organizing Numerical Data

Lists, sorted tables, scatter plots, and dot plots are all ways we can organize numerical data.

Scatter plots show us how two different variables are related.

This is data for an ice cream stand collected on temperature and number of customers over time.



With a scatter plot, we can investigate visual patterns and make predictions.

Some questions scatter plots can help answer:

- Is there an association between the outside temperature and the number of customers at an ice cream store?
- What is the predicted number of customers if the temperature is 78°F?



Analyzing Numerical Data

We say that there is a **positive association** between foot length and foot width because in general, longer feet are wider than shorter feet.

The line drawn on the graph shows the overall trend and can help us find the predicted foot width for a given foot length.

Points that are not close to the line and to most of the data are called **outliers**.

Categorical Data

When we collect data by counting things in various categories, such as tall or short, we call that categorical data. To help organize categorical data, we can use two-way tables and bar graphs.

This table shows states of mind of athletes during a meet and whether or not they meditated beforehand.

23 of the people who meditated were anxious. Only 21 of the people who did not meditate were anxious.

Does this mean that meditation has no impact or even a slight negative association with mood?

	Meditated	Did Not Meditate	Total
Calm	45	8	53
Anxious	23	21	44
Total	68	29	97

It can be more helpful to examine the percentages (called *relative frequencies*) in each category.

Of the people who meditated, 66% were calm. 28% of the people who did not meditate were calm.

The group that meditated has a lower percentage of athletes who are anxious.

	Meditated	Did Not Meditate
Calm	66%	28%
Anxious	34%	72%
Total	100%	100%

Try This at Home

Organizing Numerical Data

This scatter plot shows the heights and weights of 35 dogs.

- 1.1 Add a point to represent a dog that is 15 inches tall and weighs 35 pounds.
- 1.2 Add a point to represent a dog that weighs 100 pounds.
- 1.3 How many dogs in the set are about 25 inches tall? Explain how you know.
- 1.4 What is the heaviest weight for a dog in the set? Explain how you know.



Analyzing Numerical Data

Here is data on the weight of 21 cars and their fuel efficiency (miles driven for each gallon of gas).

- 2.1 How many cars have a fuel efficiency that is greater than 22 miles per gallon? Explain your thinking.
- 2.2 Do the variables in the scatter plot show a positive association or a negative association? Explain your thinking.
- 2.3 What is the predicted fuel efficiency of a car that weighs 1.5 tons?
- 2.4 Add an outlier to the scatter plot. Explain why this point is an outlier.



Categorical Data

This data is about people in various age groups and whether they use their cell phone as their alarm clock.

	Uses Cell Phone as Alarm	Does Not Use Cell Phone as Alarm	Total
18 to 29 Years Old	47	16	63
30 to 49 Years Old	66	23	87
50 + Years Old	31	39	70
Total	144	78	220

3.1 Fill in the blanks with the relative frequencies for each row in the table below. In other words, calculate the percent of people in each age group who use their phone as an alarm.

	Uses Cell Phone as Alarm	Does Not Use Cell Phone as Alarm	Total
18 to 29 Years Old	75%		100%
30 to 49 Years Old			
50 + Years Old			

- 3.2 Is there an association between cell phone alarm use and age for 18- to 29-year-olds and 30to 49-year-olds? Explain your thinking.
- 3.3 Is there an association between cell phone alarm use and age for the youngest age bracket and the 50 + age bracket? Explain your thinking.

Solutions:



1.2 A point that represents a dog with a weight of 100 pounds will have a *y* -value of 100, such as the point in the graph below.



- 1.3 Each point on the scatter plot represents one dog. Since the *x*-axis represents the height of a dog in inches, we can find the number of points that have an *x*-value of 25 to help us determine how many dogs are about 25 inches tall. There are 5 points that have an *x* value of roughly 25, so there are 5 dogs that are about 25 inches tall.
- 1.4 Since the *y*-axis represents the weight of a dog in pounds, we can find the heaviest dog by finding the *y*-value of the highest point on the scatter plot, which is about 110. This means that the heaviest dog in the set is about 110 pounds.
- 2.1 Since the *y*-axis represents fuel efficiency in miles per gallons, we can find the points on the graph that describe cars with a fuel efficiency greater than 22 miles per gallon by finding the number of points that have a *y*-value greater than 22. There are nine points with a *y*-value greater than 22, so there are nine cars with a fuel efficiency that is greater than 22 miles per gallon.
- 2.2 There is a negative association between weight and fuel efficiency because as the weight of the car increases, the fuel efficiency decreases.
- 2.3 Using the line of fit, we can predict that a 1.5-ton car will have a fuel efficiency of about 25.5 miles per gallon.

2.4 (2, 30) is one example of an outlier. It is far from the rest of the data and has far greater fuel efficiency than the line of fit predicts for a car that weighs 2 tons.



3.1 $\frac{47}{63}$ is approximately 0.75, so the percentage of 18- to 29-years-olds who use a cell phone as an alarm clock is about 75%.

	Uses Cell Phone as Alarm	Does Not Use Cell Phone as Alarm	Total
18 to 29 Years Old	75%	25%	100%
30 to 49 Years Old	76%	24%	100%
50 + Years Old	44%	56%	100%

- 3.2 There is not an association between cell phone alarm use and age for 18- to 29-year-olds and 30- to 49-year-olds because the relative frequencies are very similar (75% vs. 76% and 25% vs. 24%).
- 3.3 Using a cell phone as an alarm is associated with being in the younger age brackets. About 75% of 18- to 29-year olds use their cell phone as an alarm, but only 44% of people 50 years or older do.

Unit 7 Summary

Prior Learning	Math 8, Unit 7	Future Learning
 Math 6 Operations with whole number exponents (e.g., using V = s³) 	Exponent propertiesScientific notation	 Math 8, Unit 8 Square and cube roots High School Exponential and polynomial functions Rational exponents

Exponent Properties

$8^{\circ} = 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8 \cdot 8$	Exponents are a way of keeping track of how many
Base = 8	times a number has been repeatedly multiplied.
Exponent = 6	Using our understanding of repeated multiplication,
Power = 8^6	we can figure out several properties of exponents.

$$10^{3} \cdot 10^{4} = (10 \cdot 10 \cdot 10) \cdot (10 \cdot 10 \cdot 10)$$

 $10^{3} \cdot 10^{4} = 10^{7}$

¹⁰ Another way to multiply powers with the same base is to add their exponents together.

$$(7^2)^3 = (7 \cdot 7) \cdot (7 \cdot 7) \cdot (7 \cdot 7)$$

 $(7^2)^3 = 7^6$

Another way to express powers of powers can be found by multiplying the exponents together.

$$10^{-3} = \frac{1}{10} \cdot \frac{1}{10} \cdot \frac{1}{10} = \frac{1}{10 \cdot 10 \cdot 10} = \frac{1}{10^{3}}$$
$$\frac{1}{10^{3}}$$
$$8^{0} = 1$$

Negative exponents and zero exponents extend from the properties of positive exponents.

Scientific Notation

The United States mint has made over 500, 000, 000, 000 pennies.

A single carbon atom weighs 0.00000000000000000000002 grams.

The distance from Earth to the moon is 240,000 miles.

 $500,000,000,000 = 5 \cdot 10^{11}$

Another way to write very large and very small numbers is as multiples of powers of 10.

Writing numbers in this way helps avoid errors since it would be easy to accidentally add or take away a zero when writing out the decimal.

Scientific notation is one specific way to write numbers.

Numbers in scientific notation are written as a number between 1 and 10 multiplied by a power of 10.

It is more efficient to compare numbers when they are both written in this form.

 $240\ 000 = 2 \cdot 10^5 + 4 \cdot 10^4 \text{ or } 24 \cdot 10^4$ Scientific notation: $2.4 \cdot 10^5$

 $2.4 \cdot 10^5 < 1.5 \cdot 10^6$

Try This at Home

Exponent Properties

- 1.1 Carlos and Amara were trying to understand the expression $3^4 \cdot 3^5$. Amara said, "Since we are multiplying, we will get 3^{20} ." Carlos said, "But I don't think you can get 3 twenty times by multiplying everything together." Do you agree with either of them?
- 1.2 Next, Carlos and Amara were thinking about the expression $(3^4)^5$. Amara said, "Okay, this one will be 3^{20} because you will have five groups of four 3s." Carlos said, "I agree it will be 3^{20} , but it's because there will be four groups of five 3s." Do you agree with either of them?

Scientific Notation

Vehicle	Speed (kilometers per hour)
Sports car	$4.15 \cdot 10^2$
Apollo command and service module (Mother ship of the Apollo spacecraft)	$3.99 \cdot 10^4$
Jet boat	$5.1 \cdot 10^2$
Autonomous drone	$2.1 \cdot 10^4$

This table shows the top speeds of different vehicles.

- 2.1 Order the vehicles from fastest to slowest.
- 2.2 The top speed of a rocket sled is 10, 326 kilometers per hour. Is this faster or slower than the autonomous drone?
- 2.3 Estimate how many times as fast the Apollo command and service module is as the sports car.

desmos

Unit 8.7, Family Resource

Solutions:

1.1 Carlos is correct. Rewriting $3^4 \cdot 3^5$ to show all the factors looks like:

3 · 3 · 3 · 3 · 3 · 3 · 3 · 3 · 3

We can see that there are a total of 3s multiplied together nine times. This helps us understand what's going on when we use the rule to write $3^4 \cdot 3^5 = 3^{4+5} = 3^9$.

1.2 This time, Amara is correct. When we look at $(3^4)^5$, the outside exponent of 5 tells us that there are five factors of 3^4 being multiplied together. So $(3^4)^5 = 3^4 \cdot 3^4 \cdot 3^4 \cdot 3^4 \cdot 3^4 \cdot 3^4$. We could write this out the long way as:

 $(3^{4})^{5} = (3 \cdot 3 \cdot 3 \cdot 3) \cdot (3 \cdot 3 \cdot 3 \cdot 3).$

This helps us understand what's going on when we use the rule to write $(3^4)^5 = 3^{4 \cdot 5} = 3^{20}$.

- 2.1 The order from fastest to slowest is:
 - Apollo CSM
 - Autonomous drone
 - Jet boat
 - Sports car

Since all of these values are in scientific notation, we can look at the power of 10 to compare. Both the speeds of the Apollo CSM and the autonomous drone have the highest power of ten , 10^4 , so they are the fastest. The Apollo CSM is faster than the drone because 3.99 is greater than 2.1. Similarly, the jet boat is faster than the sports car because 5.1 is greater

than 4.15, even if their speeds both have the same power of ten, 10^2 .

- 2.2 The drone is faster than the rocket sled. One approach is to convert the rocket sled's speed into scientific notation. 10, 326 is equivalent to 1032.6 \cdot 10, which is equivalent to 103.26 \cdot 10 \cdot 10. By continuing that process, we can determine that the rocket sled's speed is 1.0326 \cdot 10⁴. The drone's speed is 2.1 \cdot 10⁴ kilometers per hour and 2.1 is greater than 1.0326, so the drone must be faster.
- 2.3 To compare the speeds of the Apollo CSM and the sports car, we can try to find the missing number in this equation: $? \cdot 4.15 \cdot 10^2 = 3.99 \cdot 10^4$. To find the missing value (?), we need to compute $\frac{3.99 \cdot 10^4}{4.15 \cdot 10^2}$. Since we are estimating, we can simplify the calculation to $\frac{4 \cdot 10^4}{4 \cdot 10^2}$.

Using properties of exponents and our understanding of fractions, we can conclude that

 $\frac{4 \cdot 10^4}{4 \cdot 10^2} = 1 \cdot 10^2$, so the Apollo CSM is about 100 times as fast as the sports car!

Unit	8	Summary
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Prior Learning	Math 8, Unit 8	Future Learning
 Math 6 Areas of parallelograms and triangles Math 7 Operations with rational numbers Converting fractions to decimals using long division 	 Square roots and cube roots The Pythagorean theorem Rational and irrational numbers 	 High School Trigonometry Rational exponents Square root and cube root functions Complex numbers

Square Roots and Cube Roots

We call the length of the side of a square whose area is a square units \sqrt{a}

(pronounced "the square root of a").

$$\sqrt{9} = 3$$
 because $3^2 = 9$.

$$\sqrt{16} = 4$$
 because $4^2 = 16$.

 $\sqrt{10}$ is between 3 and 4 because 10 is between 9 and 16.

						\mathbf{N}				
				1	0			1	6	
	9			1	U			Т	0	
	3				$\sqrt{1}$.0		4	Ŀ	

We call the length of the edge of a cube whose volume is a cubic units $\sqrt[3]{a}$

(pronounced "the cube root of a").

$$\sqrt[3]{64} = 4$$
 because $4^3 = 64$.
 $\sqrt[3]{70} > 4$ because $\sqrt[3]{70} > \sqrt[3]{64} = 4$.
 $\sqrt[3]{70} \approx 4.12$ because $(4.12)^3 \approx 69.93 \approx 70$.

Pythagorean Theorem

In triangle D, the square of the hypotenuse is equal to the sum of the squares of the legs.

This relationship is true for all **right triangles**.

We can describe this relationship as $a^2 + b^2 = c^2$, where *a* and *b* are the lengths of the legs, and *c* is the length of the hypotenuse of a right triangle.



What can the Pythagorean theorem be used for?

- Deciding if a triangle is a right triangle.
- Calculating one side length of a right triangle if we know the other two side lengths.

Rational and Irrational Numbers

Rational numbers are numbers that can be written as a fraction of two integers. We call numbers that cannot be written this way irrational numbers.





Try This at Home

Square Roots and Cube Roots

1.1 If each grid square represents 1 square unit, what is the area of this titled square?

1.2 What is the side length of this tilted square?



2. Draw a square so that segment *AB* is along one side of the square.

Exact length of AB:_____

Approximate length of AB:_____



3. Plot the following numbers on the number line below: $\sqrt{27}$, $\sqrt[3]{27}$, $\sqrt[3]{5}$, $\sqrt{5}$



Pythagorean Theorem

- 4.1 Label the hypotenuse of this triangle with the letter *c*.
- 4.2 Calculate the length of k.

Then determine its length.



Rational and Irrational Numbers

- 5. Write each rational number as a decimal. $\frac{3}{5}$, $\frac{6}{11}$, $\frac{17}{6}$.
- 6.1 Write some examples of rational numbers. Try to include examples of numbers that are rational but that someone might think are irrational.
- 6.2 Write some examples of irrational numbers.

Solutions:

1.1 The area of the square is 26 square units.

One way to find the area of a tilted square is to enclose the square in a larger square whose area you do know. The side length of this square is 6. Its area is $6 \cdot 6 = 36$ square units.

To find the area of the tilted square, subtract out the areas of the four triangles between the larger

square and the original $(4 \cdot \frac{1}{2} \cdot 1 \cdot 5 = 10 \text{ square})$ units).

1.2 The side length of the square is $\sqrt{26}$ units because the square root of the area is the side length of a square.



2. Exact length of *AB* (as a square root): $\sqrt{50}$ units

Area of the large square: $10^2 = 100$ square units Area of the triangles: $4 \cdot \frac{1}{2} \cdot 5 \cdot 5 = 50$ square units Area of the tilted square: 100 - 50 = 50 square units Side length of the tilted square: $\sqrt{50}$ units

Approximate length of *AB*: $\sqrt{50}$ is between 7 and 8 because 50 is between 49 or 7² and 64 or 8².

3. Plot the following numbers on the number line below: $\sqrt{27}$, $\sqrt[3]{27}$, $\sqrt[3]{5}$, $\sqrt{5}$



4.1	The length of the hypotenuse is $\sqrt{50}$ units. $a^{2} + b^{2} = c^{2}$ $(5)^{2} + (5)^{2} = c^{2}$ $25 + 25 = c^{2}$ $50 = c^{2}$ $c = \sqrt{50}$	4.2	The length of k is 7 units. $a^{2} + b^{2} = c^{2}$ $(k)^{2} + (24)^{2} = 25^{2}$ $k^{2} + 576 = 625$ $k^{2} = 49$ k = 7
4.3	Line segment p is 5 units long. $a^{2} + b^{2} = c^{2}$ $(3)^{2} + (4)^{2} = p^{2}$ $9 + 16 = p^{2}$ $25 = p^{2}$ p = 5	4.4	This is not _a right triangle because the Pythagorean theorem is not true. $9^2 + 12^2 \neq 14^2$ $81 + 144 \neq 196$ $225 \neq 196$ If the hypotenuse were 15, the triangle would be a right triangle.

5.



6.1 *Responses vary.* Some examples: $\frac{3}{5}$, 0. 16, $\frac{\sqrt{16}}{\sqrt{100}}$, $\sqrt[3]{8}$, 7, . $1\overline{66}$ 6.2 *Responses vary.* Some examples: $\frac{\sqrt{3}}{5}$, $\sqrt{8}$, $\sqrt[3]{16}$, 7π , $16 \cdot \sqrt{7}$