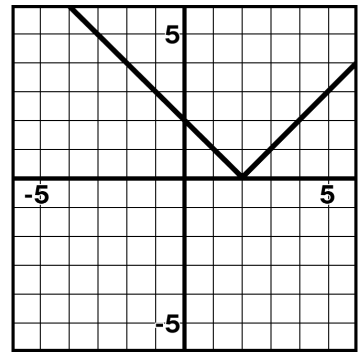
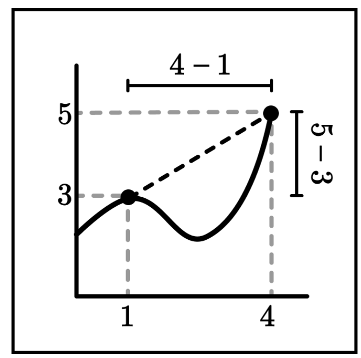
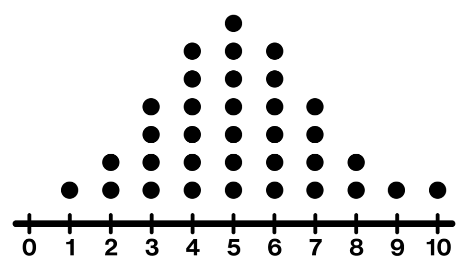
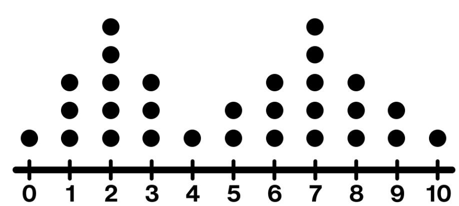
Algebra 1
Review
Summer Packet

Algebra 1 Glossary

Term	Definition
absolute value function	A function whose output is the distance of its input from a given value. For example, $f(x) = \lvert x - 2 \rvert$ outputs the distance from 2. 
average rate of change	The slope of the line between two points on a function. The average rate of change of $f(x)$ between 1 and 4 is $\frac{2}{3}$ because $\frac{5-3}{4-1} = \frac{2}{3}$. 
bell-shaped	A data distribution whose dot plot or histogram takes the form of a bell with most of the data at the center and fewer points farther from the center. 
bimodal	A data distribution with two very common data values, seen in a dot plot or histogram as two peaks. In the dot plot on the right, the two peaks are at 2 and 7. 

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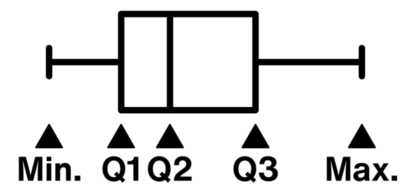
box plot	One way to visualize numerical data. The data is divided into four sections using five values: the minimum, Q1, Q2 (the median), Q3, and the maximum. A box is drawn between Q1 and Q3, and the line inside the box represents the median. 
categorical data	Categorical data has values that are categories, such as colors, words, or ZIP Codes, instead of numbers that are used to add and subtract. *What kind of pet do you have?* is a question that produces categorical data.
causation	Causation is one type of correlation. In a causal relationship, a change in one variable causes a change in the other variable.
completing the square	Completing the square is the process of rewriting a quadratic expression or equation to include a perfect square. Here are some examples of completing the square. $x^2 + 10x = 39$ $+25 \quad +25$ $(x+5)^2 = 64$ $x^2 + 10x - 39$ $(x^2 + 10x + 25) - 25 - 39$ $(x+5)^2 - 64$
compound inequality	A compound inequality is two or more inequalities joined together. It can be written using symbols or words. The numbers greater than 3 and less than or equal to 10 can be written as: • $x > 3$ and $x \leq 10$ • $3 < x \leq 10$

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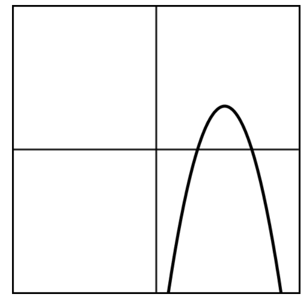
<table>
<tr><td>compound interest</td><td>Compound interest is calculated based on the initial amount and the interest from previous periods. It is calculated at regular intervals (daily, monthly, annually, etc.) The balance in an account with compound interest is modeled by an exponential function.</td></tr>
<tr><td>concave down</td><td>A parabola that opens downward.

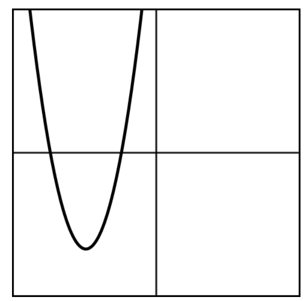
A negative value of a will produce a concave down parabola.

The parabola to the right has an equation of $f(x) = -1(x - 4)^2 + 2$.

</td></tr>
<tr><td>concave up</td><td>A parabola that opens upward.

A positive value of a will produce a concave up parabola.

The parabola to the right has an equation of $f(x) = 2(x + 6)^2 - 5$.

</td></tr>
<tr><td>constant difference</td><td>When the difference between any two consecutive values in a pattern is the same, there is a constant difference.

The table on the right has a constant difference of 2.

<table><tr><th>x</th><th>y</th><th></th></tr><tr><td>0</td><td>5</td><td></td></tr><tr><td>1</td><td>7</td><td>+2</td></tr><tr><td>2</td><td>9</td><td>+2</td></tr><tr><td>3</td><td>11</td><td>+2</td></tr></table></td></tr>
<tr><td>constant ratio</td><td>When the ratio between any two consecutive values in a pattern is the same, there is a constant ratio.

The table on the right has a constant ratio of 3.

<table><tr><th>x</th><th>y</th><th></th></tr><tr><td>0</td><td>1</td><td></td></tr><tr><td>1</td><td>3</td><td>·3</td></tr><tr><td>2</td><td>9</td><td>·3</td></tr><tr><td>3</td><td>27</td><td>·3</td></tr></table></td></tr>
</table>

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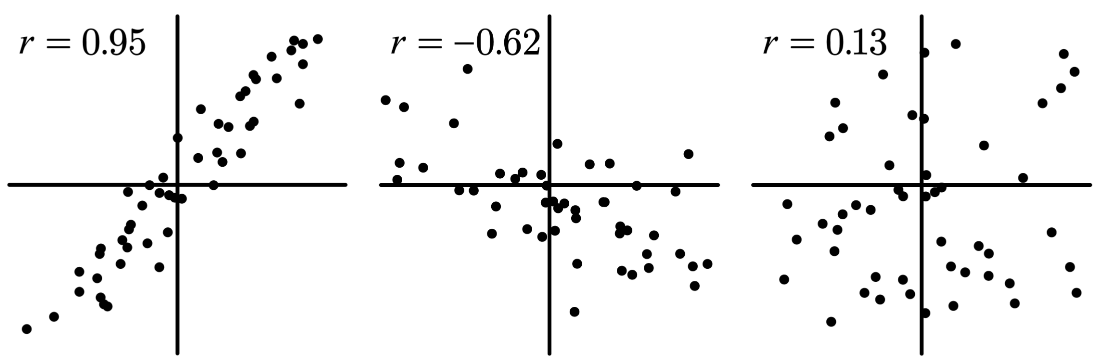
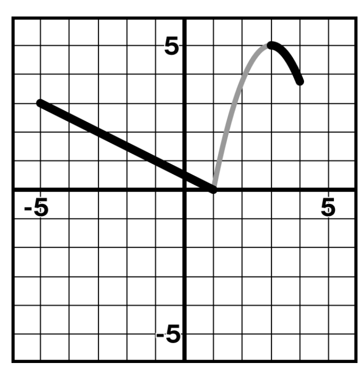
constraint	A constraint is a limitation on the possible values of variables in a model. We often use equations or inequalities to represent constraints. The constraint that "you must be 36 inches or taller to ride the Ferris wheel" can be represented by the inequality $h \geq 36$.
correlation	A relationship between two or more variables. Also called an *association*.
correlation coefficient (r-value)	A correlation coefficient is a number between -1 and 1 that describes the strength and direction of a linear association between two numerical variables. The closer the correlation coefficient is to 0, the weaker the linear association. The closer the correlation coefficient is to -1 or 1, the stronger the linear association. These graphs show examples of different correlation coefficients. 
decreasing (interval or function)	A function is decreasing when its outputs decrease as its inputs increase. A function can be decreasing for its entire domain or over a particular interval. For example, the function $f(x)$ is decreasing when $-5 < x < 1$ and when $3 < x < 4$. 

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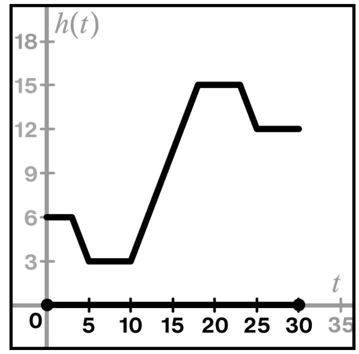
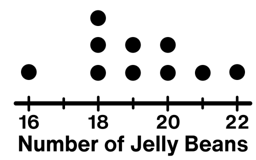
domain	The domain of a function is the set of all possible input values. The domain can be described in words and in symbols. The domain of this graph can be described as: • All numbers from 0 to 30 • $0 \leq t \leq 30$	
dot plot	A dot plot is one way to visualize data. Each data point is shown as a dot above its value, stacking on top of other dots with the same value. For example, this dot plot shows that 3 students guessed that there were 18 jelly beans in a jar.	
elimination	A method of solving systems of equations where you add or subtract the equations to produce a new equation with fewer variables. In the example on the right, subtraction is used to eliminate y and create an equation that can be solved for x.	$2x + y = 30$ $-\ (x + y = 23)$ $x + 0 = 7$ $x = 7$ $(7) + y = 23$ $y = 16$
equivalent equations	Equivalent equations have the same solutions. These equations are equivalent because if you multiply the top equation by 3, you create the bottom one. The solution to each equation is $x = 2$.	$3x + 4 = 10$ $9x + 12 = 30$

Algebra 1 Glossary

exponential decay

An exponential relationship shows exponential decay when it decreases. An exponential decay relationship has a growth factor between 0 and 1.

For example, $f(x) = 320 \bullet (\frac{4}{5})^x$ is an example of exponential decay because the growth factor, $\frac{4}{5}$, is between 0 and 1.

exponential relationship

Relationships (patterns, tables, scenarios) that increase or decrease by a constant ratio.

For example, $y = 2 \cdot 4^x$ represents an exponential relationship that starts at 2 and grows by a constant ratio of 4.

The table on the right shows $y = 2 \cdot 4^x$.

x	y	
0	2	
1	8	$\cdot 4$
2	32	$\cdot 4$
3	128	$\cdot 4$

factored form (of quadratic expressions)

Factored form is one of three common forms of a quadratic expression.

Factored form looks like $a(x - m)(x - n)$.

Here are some examples of factored-form quadratic expressions.

$$g(x) = x(x + 10)$$

$$2(x - 1)(x + 3) = y$$

$$(5x + 2)(3x - 3)$$

function

A rule that assigns exactly one output to each possible input.

function notation

Function notation is a way of writing the inputs and outputs of a function.

$f(4) = 9$ says that when the input of the function f is 4, the output is 9.

$$f(x) = 2x + 1$$

$$f(4) = 9$$

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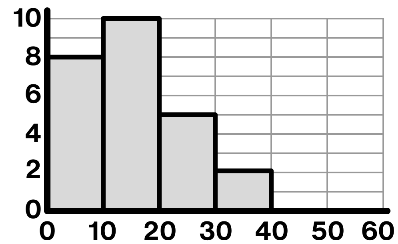
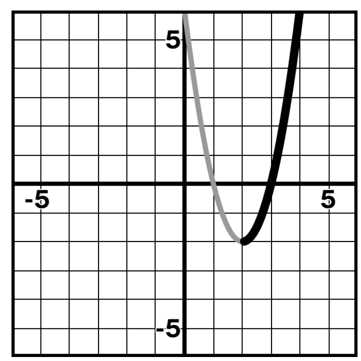
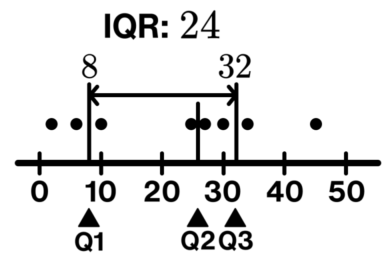
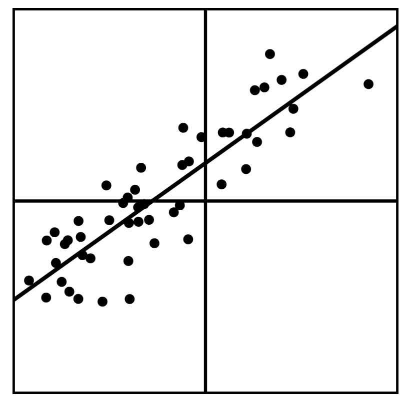
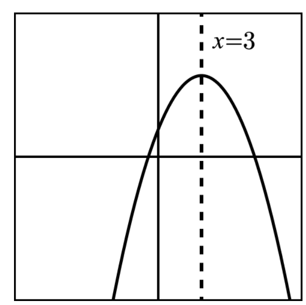
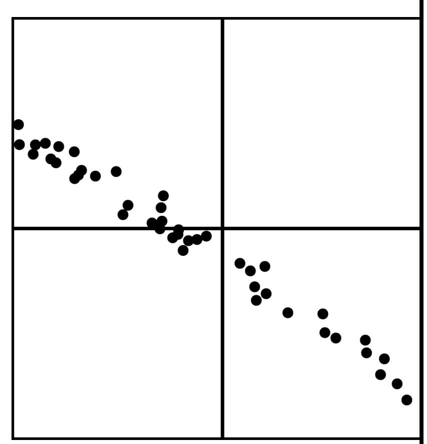
growth factor	The constant ratio (or common factor) that each term is multiplied by to generate an exponential pattern.
histogram	A histogram is one way to visualize numerical data. The data in a histogram is grouped into bins, each shown by a rectangle. The height of each rectangle shows how many values are in that bin. For example, this histogram shows that there are 8 values between 0 and 10. 
increasing (interval or function)	A function is increasing when its outputs increase as its inputs increase. A function can be increasing for its entire domain or over a particular interval. For example, the function $f(x)$, whose graph is shown, is increasing when $x > 2$. 
irrational number	Irrational numbers are numbers that are not rational; they cannot be written as a fraction with an integer numerator and denominator. For example, 2 is a rational number because it can be written as $\frac{2}{1}$, whereas π and $\sqrt{3}$ are irrational because they cannot be written as a fraction of two integers.

Algebra 1 Glossary

interquartile range (IQR)	A measure of spread. It is the distance from Q1 to Q3, and the width of the box in a box plot. For example, the IQR of this data set is $32 - 8 = 24$.	
line of best fit	The line on a scatter plot that best represents the trend created by the points in a data set.	
line of symmetry	If you fold a parabola along the *line of symmetry*, the two halves are identical. This is sometimes called an *axis of symmetry.* The equation of this *line of symmetry* is $x = 3$.	
linear association	When the points on a scatter plot follow a line, we say there is a linear association between x and y. Linear associations are categorized as strong or weak, depending on how closely the points follow a line of best fit. Linear associations are also categorized as positive and negative. The graph on the right shows a strong, negative linear association.	

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Algebra 1 Glossary

<table>
<tr>
<td>linear function</td>
<td>A function that grows by equal differences over equal intervals.

For example, $y = 5 + 6x$ represents a linear function that starts at 5 and grows by a constant difference of 6.

The table on the right shows $y = 5 + 6x$.</td>
<td><table><tr><th>x</th><th>y</th><th></th></tr><tr><td>0</td><td>5</td><td></td></tr><tr><td>1</td><td>11</td><td>+6</td></tr><tr><td>2</td><td>17</td><td>+6</td></tr><tr><td>3</td><td>23</td><td>+6</td></tr></table></td>
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<td>linear relationship</td>
<td>Relationships (patterns, tables, scenarios) that increase or decrease by a constant difference.

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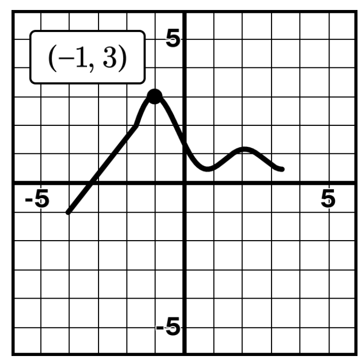
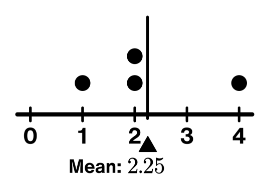
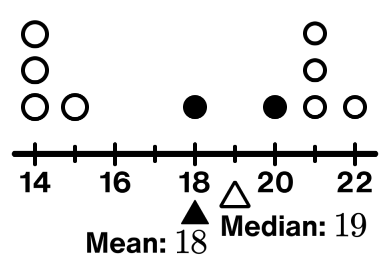
</tr>
<tr>
<td>maximum</td>
<td>The maximum of the graph of a function is the highest point on the graph.

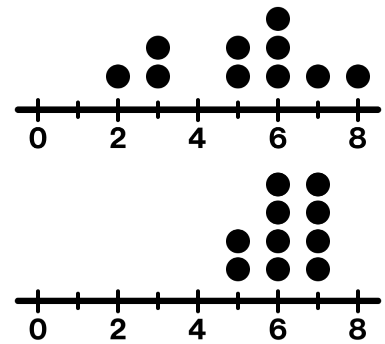
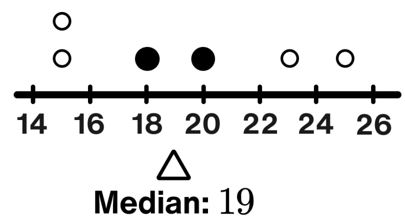
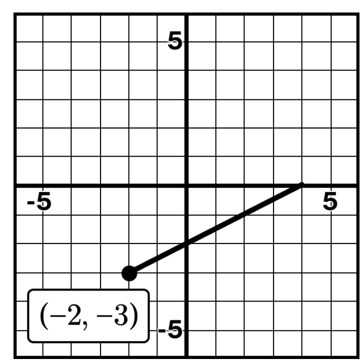
The maximum of this function is at $(-1, 3)$.</td>
<td>
</td>
</tr>
<tr>
<td>mean</td>
<td>The mean, or average, is a measure of center.

To calculate the mean, you can add up all the data values, and divide by the number of data points.

In this data set, the mean is 2.25.</td>
<td>

$1 + 2 + 2 + 4 = 9$
$9 \div 4 = 2.25$</td>
</tr>
<tr>
<td>measure of center</td>
<td>A single number that summarizes all of the values in a data set. It is usually a typical value for a data set.

Mean and median are measures of center.</td>
<td>
</td>
</tr>
</table>

measure of spread	This tells us how bunched up or spread out the values in a data set are. Interquartile range and standard deviation are measures of spread. For example, the dot plot on the top has a larger spread than the dot plot on the bottom. 
median	A measure of center. It is the middle value of a data set when the values are in numerical order. If there are two values in the middle of the data set, then the median is the mean of those two values. 
minimum	The minimum of the graph of a function is the lowest point on the graph. The minimum of this function is at $(-2, -3)$. 
model	A mathematical representation (graph, equation, relationship) of real-world data that you can use to make predictions and decisions, or to solve problems.

Algebra 1 Glossary

<table>
<tr>
<td>negative (interval or function)</td>
<td>

A function is negative when its outputs are below 0. A function can be negative for its entire domain or over a particular interval.

For example, the function $f(x)$ is negative when $1 < x < 3$.

</td>
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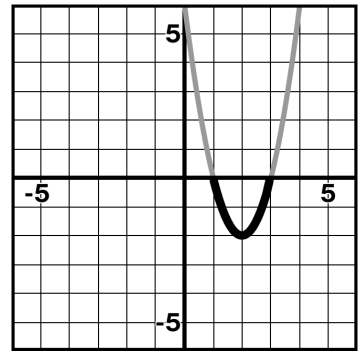

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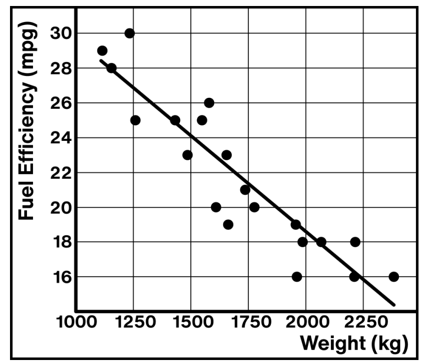
<tr>
<td>negative association</td>
<td>A relationship between two quantities where one tends to decrease as the other increases.</td>
<td>



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<tr>
<td>outlier</td>
<td>

A data value that is far from the other values in the data set.

The circled point is an outlier.

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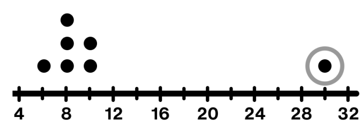
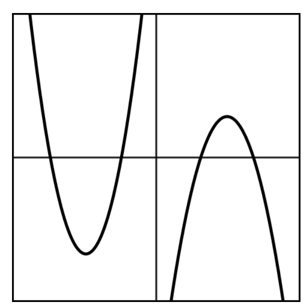


</td>
</tr>
<tr>
<td>parabola</td>
<td>The graph of a quadratic function.</td>
<td>



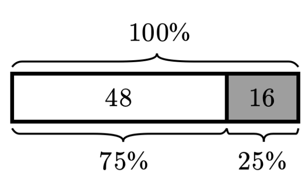
</td>
</tr>
<tr>
<td>percent decrease</td>
<td>

Describes how much a quantity goes down, expressed as a percent of the starting amount.

For example, a store had 64 hats in stock on Friday. They had 48 hats left on Saturday. The amount of hats went down by 16.

This was a 25% decrease because 16 is 25% of 64.

</td>
<td>



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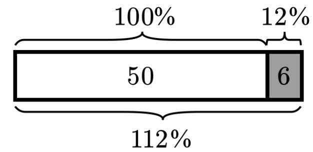
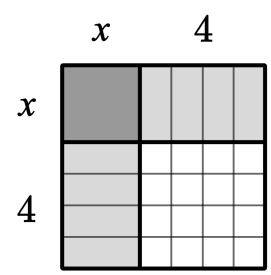
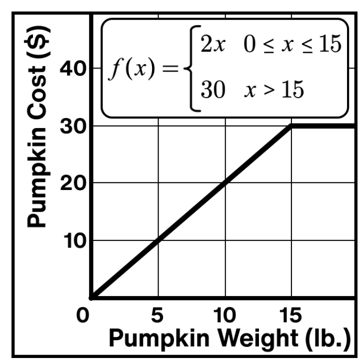
Algebra 1 Glossary

percent increase	Describes how much a quantity goes up, expressed as a percent of the starting amount. For example, Elena had \$50 on Monday. She helped a neighbor, so she had \$56 on Tuesday. The amount went up by \$6. This was a 12% increase because 6 is 12% of 50. $\frac{6}{50} = 0.12 = 12\%$. 
perfect square	A perfect square is an expression that can be represented as something multiplied by itself. Examples: 3^2, 9, $(x+4)^2$, $x^2 + 8x + 16$ 
piecewise-defined function	A function in which different rules apply to different intervals in its domain. For example, when $0 \le x \le 15$, $f(x) = 2x$. When $x > 15$, $f(x) = 30$. 
plus/minus symbol $\pm$	The plus/minus symbol ($\pm$) is used to represent both the positive and negative of a number. It also can be used to represent two expressions. For example: • $\pm\sqrt{9}$ represents $-\sqrt{9}$ and $+\sqrt{9}$. • 2 ± 3 represents $2 - 3$ and $2 + 3$.

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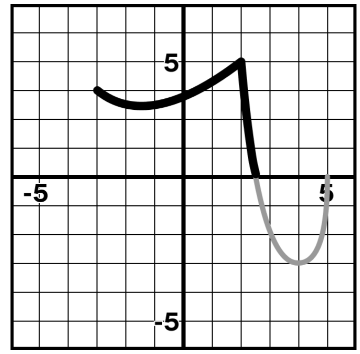
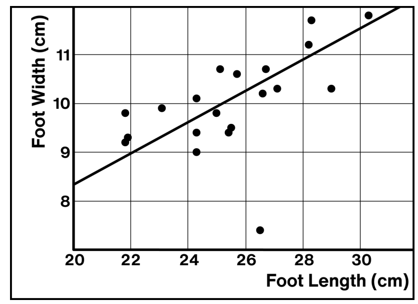
Algebra 1 Glossary

positive (interval or function)	A function is positive when its outputs are above 0. A function can be positive for its entire domain or over a particular interval. For example, the function $f(x)$ is positive when $-3 \leq x < 3$. 
positive association	A relationship between two quantities where one tends to increase as the other increases. 
quadratic equation	An equation used to represent a quadratic function. Quadratic equations contain a squared term. There are three common forms of quadratic equations: • Factored form: $f(x) = a(x - m)(x - n)$ • Standard form: $f(x) = ax^2 + bx + c$ • Vertex form: $f(x) = a(x - h)^2 + k$
quadratic formula	The quadratic formula can be used to determine the solutions of a quadratic equation $ax^2 + bx + c = 0$, where $a \neq 0$. The quadratic formula is: $$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

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Algebra 1 Glossary

<table>
<tr><td>quadratic function</td><td>A quadratic function has output values that change by a constant second difference.

Equations of quadratic functions have a squared term as the highest degree.

The graph of a quadratic function is a parabola.

<table><tr><th>x</th><th>f(x)</th><th></th><th></th></tr><tr><td>1</td><td>8</td><td></td><td></td></tr><tr><td></td><td></td><td>+3</td><td></td></tr><tr><td>2</td><td>11</td><td></td><td>+2</td></tr><tr><td></td><td></td><td>+5</td><td></td></tr><tr><td>3</td><td>16</td><td></td><td>+2</td></tr><tr><td></td><td></td><td>+7</td><td></td></tr><tr><td>4</td><td>23</td><td></td><td></td></tr></table></td></tr>
<tr><td>quantitative data</td><td>Quantitative data has values that are numbers, measurements, or quantities instead of words.

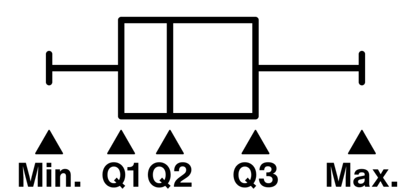
How many pets do you have? is a question that produces quantitative data.</td></tr>
<tr><td>quartile</td><td>Quartiles divide a data set into four sections.

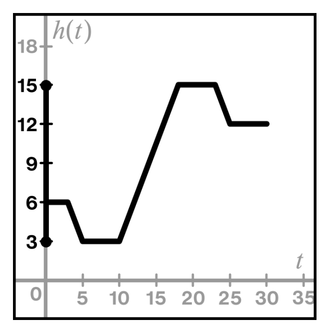
Quartile 1 is the median of the lower half of the data.

Q2 is also the median.

Q3 is the median of the upper half of the data.

Q4 is also the maximum.

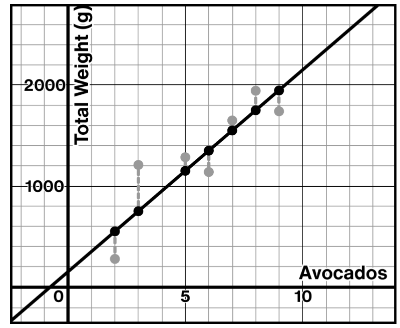

</td></tr>
<tr><td>range</td><td>The range of a function is the set of all possible output values. The range can be described in words and in symbols.

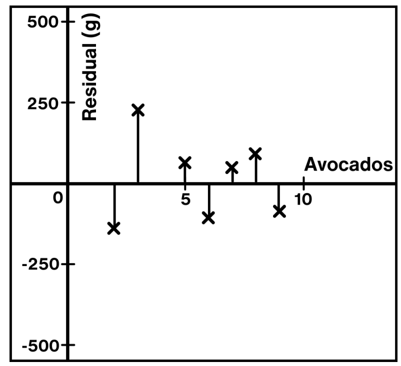
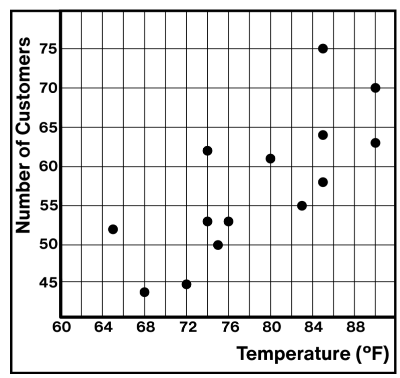
The range of this graph can be described as:
All numbers from 3 to 15.
$3 \le h(t) \le 15$

</td></tr>
<tr><td>rational number</td><td>Rational numbers are numbers that can be written as a fraction with an integer numerator and denominator.

Examples: $\frac{1}{3}$, $\frac{-7}{4}$, 0, 0.2, -5, and $\sqrt{9}$.</td></tr>
</table>

Algebra 1 Glossary

<table>
<tr>
<td>residual</td>
<td>The difference between the y-value for a point in a scatter plot and the value predicted by the line of best fit.

The dashed lines on the graph show the residuals for each data point.</td>
<td>
</td>
</tr>
<tr>
<td>residual plot</td>
<td>A scatter plot of residual values for a data set.

The x-axis represents the value predicted by the line of best fit, and the y-value of each point represents the value of the residual.</td>
<td>
</td>
</tr>
<tr>
<td>scatter plot</td>
<td>A graph of plotted points that show the relationship between two variables.</td>
<td>
</td>
</tr>
<tr>
<td>second difference</td>
<td>The differences between consecutive output values in the table of a function are called first differences.

The differences between those values are called second differences.

In this example, the first differences are 3, 6, 9, and 12. The second differences are all 3.</td>
<td>
<table>
<tr><th>x</th><th>f(x)</th><th></th><th></th></tr>
<tr><td>1</td><td>5</td><td></td><td></td></tr>
<tr><td></td><td></td><td>+3</td><td></td></tr>
<tr><td>2</td><td>8</td><td></td><td>+3</td></tr>
<tr><td></td><td></td><td>+6</td><td></td></tr>
<tr><td>3</td><td>14</td><td></td><td>+3</td></tr>
<tr><td></td><td></td><td>+9</td><td></td></tr>
<tr><td>4</td><td>23</td><td></td><td>+3</td></tr>
<tr><td></td><td></td><td>+12</td><td></td></tr>
<tr><td>5</td><td>35</td><td></td><td></td></tr>
</table>
</td>
</tr>
</table>

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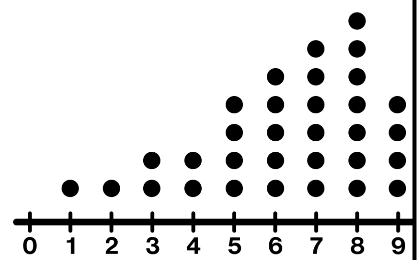
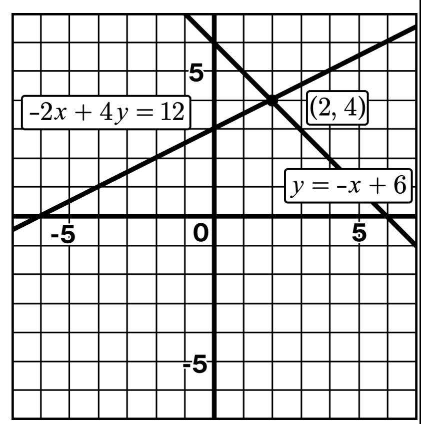
Algebra 1 Glossary

simple interest	Simple interest is calculated based on the initial amount. It is calculated once at the beginning of an investment. The balance in an account with simple interest is modeled by a linear function.	
skewed	A data distribution where there are more values concentrated on one end of the data, and few values on the other end.	
solution	A solution is a value or set of values that makes an equation or inequality true. For example, $x = 2$ is a solution to the equation $3x + 4 = 10$. $x > 2$ is the solution to the inequality $3x + 4 > 10$. The ordered pair $(1, 2)$ is a solution to the equation $3x + 4y = 11$.	
solution to a system of equations	A solution to a system of equations is a set of values that makes all equations in that system true. When the equations are graphed, the solution to the system is the intersection point. For example, $(2, 4)$ is the solution to this system of equations on the right, and the intersection point on the graph.	$y = -x + 6$ $-2x + 4y = 12$ 

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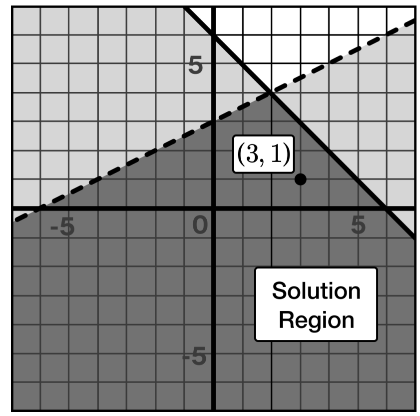
Algebra 1 Glossary

solutions to a system of inequalities (solution region)	Solutions to a system of inequalities are all the sets of values that make the inequalities in that system true. Every point in the region where their graphs overlap is a solution to the system: this is the *solution region*. For example, $(3, 1)$ is a solution to the system on the right because it falls in the region where the inequalities overlap. $y \geq -x + 6$ $-2x + 4y < 12$ 
square root $\sqrt{\ }$	The square root of a number n (written as: $\sqrt{n}$) is the positive number which can be squared to get n. It is also the side length of a square with an area of n. Example: $\sqrt{16} = 4$ because 4^2 is 16. $\sqrt{16}$ is also the side length of a square with an area of 16.
standard deviation	A measure of the variability, or spread, of a distribution of data. Standard deviation is calculated by a method similar to calculating the MAD (mean absolute deviation). The exact method is studied in more advanced courses.
standard form (of quadratic expressions)	Standard form is one of three common forms of a quadratic expression. Standard form looks like $ax^2 + bx + c$, where $a \neq 0$. Here are some examples of standard-form expressions. $2x^2 + 5x + 3$ $h(x) = x^2 + 3x$ $y = 4x^2 - 7$
standard form (of a linear equation)	Standard form of a linear equation looks like $ax + by = c$, where a, b, and c are constants and x and y are variables. The equations on the right are in standard form. The equation $y = 2x + 4$ is not in standard form. $3x + 4y = 10$ $9x - 12y = 4$

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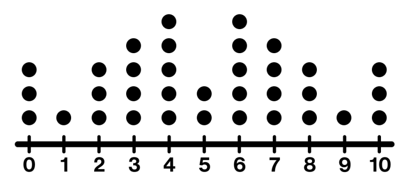
Algebra 1 Glossary

statistic	A single number that measures something about a data set. Examples of statistics: mean, median, IQR, and standard deviation.
substitution	A method of solving systems of equations where a variable is replaced with an equivalent expression in order to produce a new equation with fewer variables. $y = -4x + 6 \qquad y = 3x - 15$ $-4x + 6 = 3x - 15$ $-7x = -21$ $\boxed{x = 3}$ $y = 3(3) - 15$ $\boxed{y = -6}$
symmetric	A data distribution that has a vertical line of symmetry. 
systems of equations	Two or more equations that represent the constraints on a shared set of variables form a system of equations. $3b + c = -2$ $b - 5c = 12$
system of inequalities	Two or more inequalities that represent the constraints on a shared set of variables form a system of inequalities. $10m + 5n > -2$ $m - 5n \leq 12$
translation	A translation moves every point in a figure a given distance in a given direction. It changes the location of a function, but does not change its shape.

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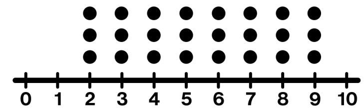
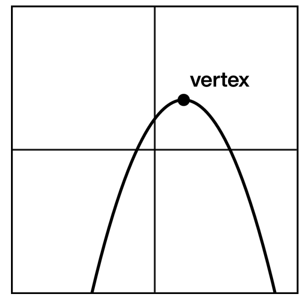
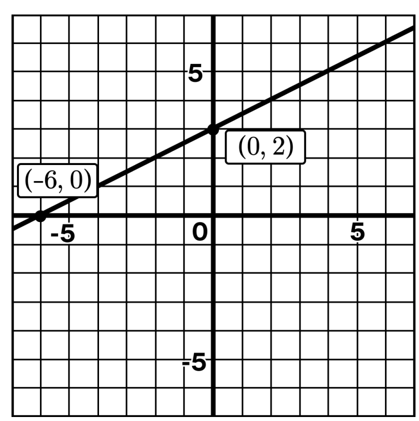
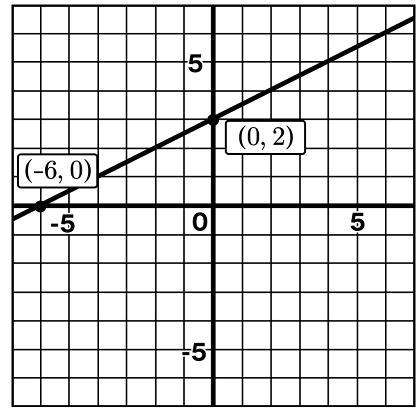
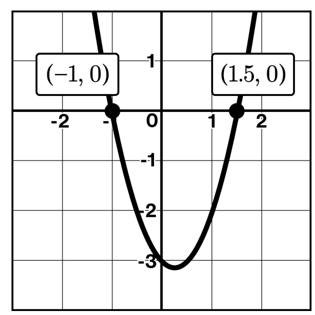
Algebra 1 Glossary

uniform	A data distribution that has the data values evenly distributed. 
vertex	The vertex is the point where a parabola changes from increasing to decreasing, or vice versa. It is the highest or lowest point on the graph. 
vertex form	One of three common forms of a quadratic equation. It looks like $f(x) = a(x - h)^2 + k$. The equations to the right are all in vertex form. $(x - 3)^2 + 10 = g(x)$ $y = 2(x + 8)^2 - 1$ $f(x) = -(x - 6)^2 + 15$
vertical stretch	The result of multiplying the output values of a function by a factor. When a function is vertically stretched, the y-values of its graph move away from or toward the x-axis.
x-intercept	The x-intercept is the point where its graph crosses the x-axis or when $y = 0$. The x-intercept of the graph of $-2x + 4y = 12$ is $(-6, 0)$, or just -6. 

y-intercept	The y-intercept is the point where its graph crosses the y-axis or when $x = 0$. The y-intercept of the graph $-2x + 4y = 12$ is $(0, 2)$, or just 2. 
zero-product property	The zero-product property says that if the product of two or more factors is 0, then at least one of the factors is 0. For example, if $(2x - 3)(x + 1) = 0$, then either $2x - 3 = 0$ or $x + 1 = 0$. This can be used to help solve the equation.
zeros (of a function)	The zeros of a function are the x-values that make $f(x) = 0$. For example, if the function $f(x) = (2x - 3)(x + 1)$, then its zeros are $\frac{3}{2}$ and -1. 

Unit 1
Representing Relationships

Unit 1 Summary

Prior Learning	Algebra 1, Unit 1	Future Learning
Grades 6–8 • Modeling relationships • Plotting points • Linear relationships • Evaluating exponents • Exponent properties	• Multiple representations of relationships • Linear and exponential relationships	Algebra 1, Unit 5 • Systems of linear equations Algebra 1, Unit 6 • Exponential functions • Percentage growth and decay

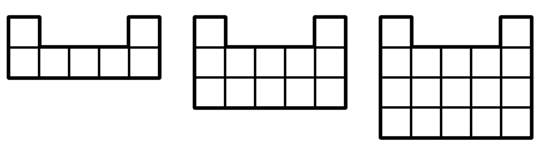
Multiple Representations of Relationships (Lessons 1–4)

Mathematical relationships can be represented in many different ways. Each representation has advantages and disadvantages, and can be used to solve problems and make predictions.

Here are four different representations of the same relationship:

Visual Pattern

Figure 1 Figure 2 Figure 3



Equation

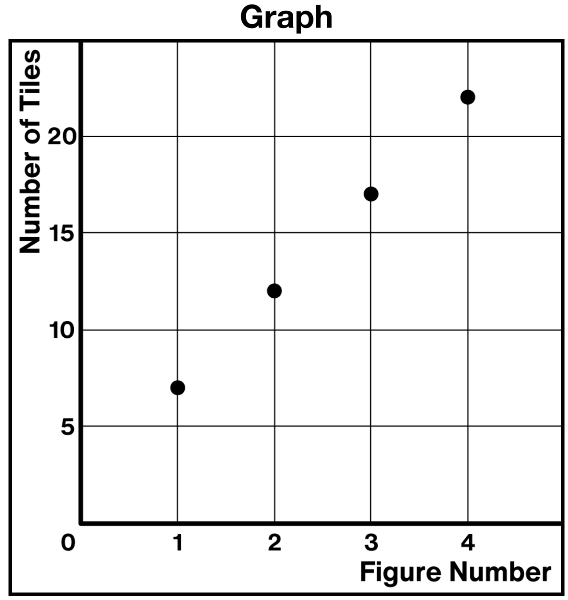
$$t = 2 + 5n$$

n is the figure number
t is the number of tiles

Table

Figure, n	Tiles, t
1	7
2	12
3	17

Graph



Linear and Exponential Relationships (Lessons 5–11)

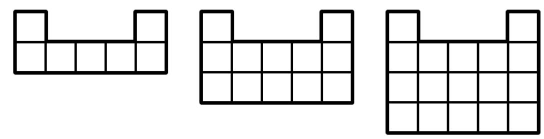
Lessons 5–7

There are two different types of relationships in this unit.

Linear relationships have a constant difference.

Example: The constant difference is 5 because each figure has 5 more tiles than the one before it.

Figure 1 **Figure 2** **Figure 3**



Here is a linear equation to represent this relationship.

$$t = 2 + 5n$$

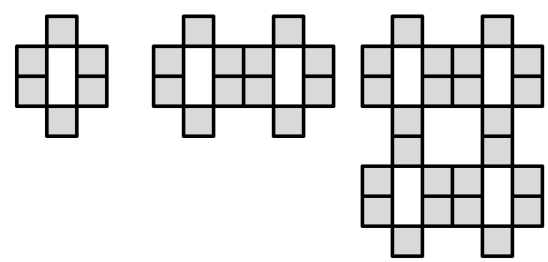
n is the figure number
t is the number of tiles

Figure, n	**Tiles, t**
1	$2 + 5$
2	$2 + 5 + 5$
3	$2 + 5 + 5 + 5$

Exponential relationships have a constant ratio.

Example: The constant ratio here is 2 because each figure has 2 times as many tiles as the one before it.

Figure 1 **Figure 2** **Figure 3**



Here is an exponential equation to represent this relationship.

$$t = 3 \cdot 2^n$$

n is the figure number
t is the number of tiles

Figure, n	**Tiles, t**
1	$3 \cdot 2$
2	$3 \cdot 2 \cdot 2$
3	$3 \cdot 2 \cdot 2 \cdot 2$

Lessons 8–10

Equations and graphs can be used to describe relationships and make predictions.

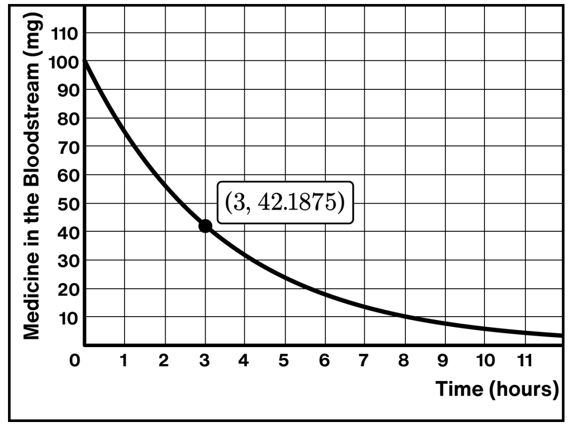
Example: Imagine you take 100 mg of medicine.

Each hour, $\frac{3}{4}$ of the medicine is left in your body.

The equation $y = 100 \cdot \left(\frac{3}{4}\right)^x$ models this relationship.

x is the number of hours after taking the medicine.

y is the number of milligrams of medicine left in your body.



Use the equation to answer questions like: *How much of the medicine is in your body after 3 hours?*

Based on the work on the right, you will have 42.2 mg of medicine in your body (a little less than half) after 3 hours.

$$y = 100 \cdot \left(\frac{3}{4}\right)^3$$

$$y = 100 \cdot \frac{27}{64}$$

$$y = 42.1875$$

Lesson 11

The equation and graph above are one example of a mathematical model.

The British statistician George Box once said: *All models are wrong, but some are useful.*

The model above about medicine in the body is wrong because it is overly simplistic. Bodies do not process medicine in a perfectly exponential way. However, the model is also useful because we can use it to make predictions and show general trends, even if those are not exact.

Try This at Home

Multiple Representations of Relationships (Lessons 1–4)

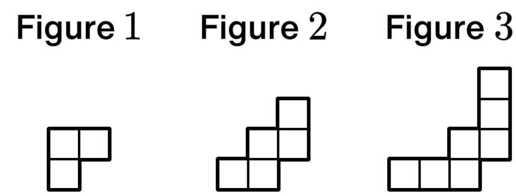
Try After Lesson 3

1.1 Sketch what you think figure 4 will look like.

1.2 How many tiles will there be in figure 4?

1.3 How many tiles will there be in figure 10?

1.4 How many tiles will there be in figure n?

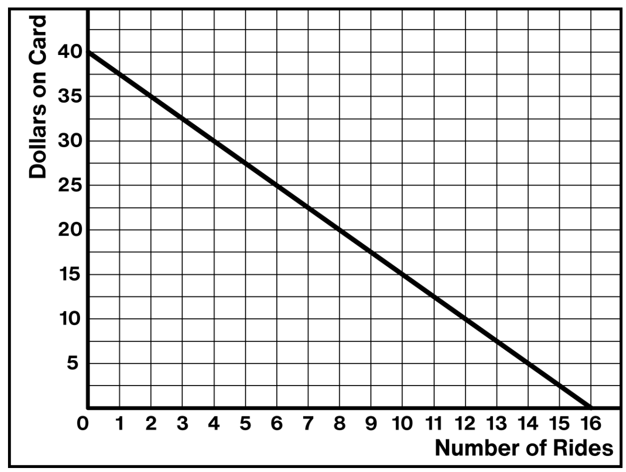


Try After Lesson 4

This graph shows the amount of money on a public transit-fare card over time. Use the graph to answer:

2.1 How much money does the card start with?

2.2 When there is $\$15$ left on the card, how many rides have been taken? Explain your thinking.



Linear and Exponential Relationships (Lessons 5–11)

Try After Lesson 6

Determine whether each table shows a linear relationship, an exponential relationship, or neither.

3.1

x	y
0	5
1	15
2	20
3	30

3.2

x	y
0	5
1	20
2	80
3	320

3.3

x	y
0	5
1	25
2	45
3	65

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Unit A1.1, Family Resource

Try After Lesson 8

This table shows the number of bacteria cells on a plate over time.

Hours	Number of Bacteria Cells
0	10
1	80
2	640
3	5 120

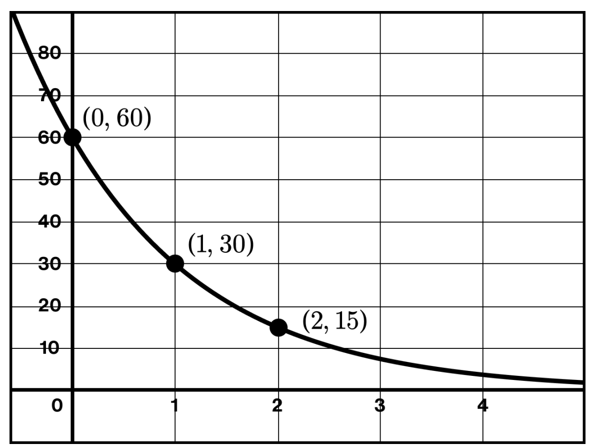
4.1 Write an equation to represent the relationship between the number of hours and number of bacteria.

4.2 How many bacteria cells will there be after 6 hours? Show or explain your thinking.

4.3 Does this number of bacteria surprise you? Why or why not?

Try After Lesson 10

Here is a graph of an exponential relationship.



5.1 Explain how you know this relationship is not linear.

5.2 Write an equation that represents this graph.

Unit 2
Linear Equations & Inequalities

Unit 2 Summary

Prior Learning	Algebra 1, Unit 2	Future Learning
Grades 6–8 • Solving equations • Adding and subtracting expressions • Inequalities • Graphing equations Algebra 1, Unit 1 • Modeling linear relationships	• Solving one-variable equations • Solving multi-variable equations • Representing situations with linear equations and inequalities • Solving and graphing inequalities	Algebra 1, Unit 5 • Solving systems of linear equations and inequalities Algebra 1, Unit 8 • Solving quadratic equations

One-Variable Equations (Lessons 1–5)

Lesson 1

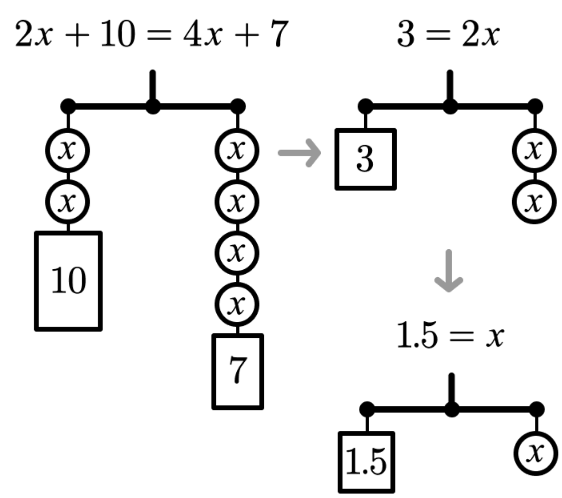
Solving an equation means determining all the values that make an equation true.

Hanger diagrams can be used to represent equations.

The first balanced hanger represents $2x + 10 = 4x + 7$.

The hanger $3 = 2x$ must also be balanced because 2 x's and 7 have been removed from both sides. These actions are called "balanced moves."

$x = 1.5$ is a *solution* to the equation $2x + 10 = 4x + 7$ because $2(1.5) + 10 = 4(1.5) + 7$.



Lesson 2

Working backwards using *inverse operations* can also help solve equations.

The table and equation show two strategies for solving $-4(x + 2) = 20$.

The *solution* to the equation $-4(x + 2) = 20$ is $x = -7$ because $-4(-7 + 2) = 20$.

-2 $\div -4$

x	$x+2$	$-4(x+2)$
-7	-5	20

$-4(x+2) = 20$
$-4 \quad -4$

$x + 2 = -5$
$-2 \quad -2$

$x = -7$

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Unit A1.2, Family Resource

Lessons 3–4

Equivalent equations have the same solutions.

Here is Lola's work to solve the equation $3 - 7x = \frac{-12x-10}{2}$.

Her work shows what she did at each step to create equivalent equations.

$$3 - 7x = \frac{-12x - 10}{2}$$

Step 1:	$6-14x=-12x-10$	multiply by 2
Step 2:	$16-14x=-12x$	add 10
Step 3:	$16=2x$	add 14x
Step 4:	$x=8$	divide by 2

Lola can check her *solution* by substituting 8 into the equation for x and figuring out whether or not the equation is true.

$$3-7(8) \stackrel{?}{=} \frac{-12(8)-10}{2}$$

$$3-56 \stackrel{?}{=} \frac{-96-10}{2}$$

$$3-56 \stackrel{?}{=} \frac{-106}{2}$$

$$-53=-53 \checkmark$$

Lesson 5

Do all equations have exactly one solution? No!

Linear equations can have *one solution*, *no solution*, or *infinitely many solutions*.

- When an equation has *no solution*, no value of x makes the equation true.
- When an equation has *infinitely many solutions*, every value of x makes the equation true.

Here are examples of equations with one solution, no solution, and infinitely many solutions.

One Solution	No Solution	Infinitely Many Solutions
$3x + 4 = 2x + 10$	$2x + 4 = 2x + 10$	$2(x + 5) = 2x + 10$
$3x = 2x + 6$	$4 = 10$	$2x + 10 = 2x + 10$
$x = 6$	**This is never true!**	$10 = 10$
		This is always true!

Multi-Variable Equations (Lessons 6–9)

Lessons 6–7
Sometimes equations have more than one variable in them.

To solve for a variable means to write an equivalent equation that isolates the variable.

Here is an example where solving for a variable might be helpful.

The seating capacity on a crowded bus can be modeled by the equation $6t + 3d = 300$, where t is the number of seats and d is the number of people who can stand.

The equation can be solved for t so that the city can calculate the standing capacity for any number of seats.

The 150 means that 150 can stand if there are no seats.

The -3 means for every seat added, 3 less people can stand.

$$\begin{aligned} 6t + 2d &= 300 \\ -\ 6t \qquad &\quad -\ 6t \\ 2d &= 300 - 6t \\ d &= 150 - 3t \end{aligned}$$

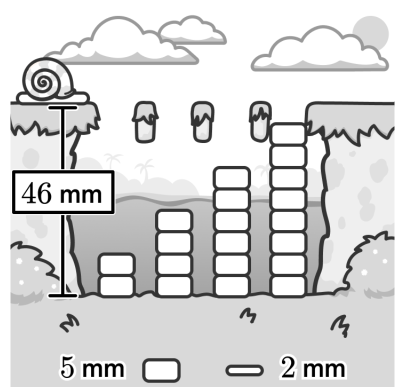
Lessons 8–9
Equations, graphs, and tables can be used to model the same relationship.

A snail needs to cross a gap that is 46 mm tall using 5 mm and 2 mm blocks.

This situation can be represented by this equation in standard form: $5x + 2y = 46$.

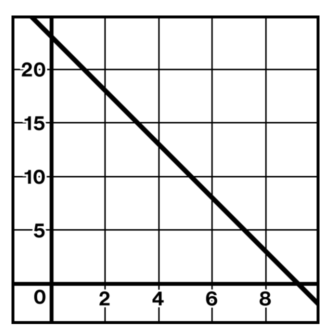
Solving the equation for y can help determine the number of 2 mm blocks given any number of 5 mm blocks.

$$y = 23 - \frac{5}{2}x.$$



The equation solved for y is connected to the graph.

The y-intercept of the graph is $(0,\ 23)$ and the slope is $-\frac{5}{2}$.



One-Variable and Two-Variable Inequalities

Lesson 10

Inequalities can model contexts with *constraints*.

Wey Wey drives a new electric bus for the city of Metropolis.

The cost of electricity for the bus is $\$0.05$ per mile.

If Metropolis wants to spend at most $\$400$ per year on electricity for each bus, how many miles can he drive?

The solutions to this inequality are $x \leq 8\,000$, which means that Wey Wey can drive at most $8\,000$ miles per year.

$$0.05x \leq 400$$
$$\div 0.05 \qquad \div 0.05$$
$$x \leq 8000$$

$8\,000$ **miles or less!**

Lessons 11–12

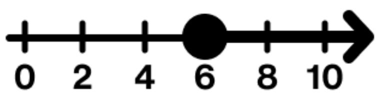
Solutions to one-variable inequalities can be represented on a number line.

Here are some one-variable inequalities and their solutions on a graph.

$2x - 4 \geq 8$

$2x \geq 12$

$x \geq 6$

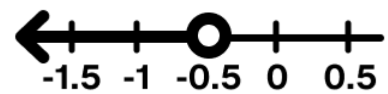


This graph has a closed circle at $x = 6$ because that point **is** included in the solution.

$2x - 2 > 6x$

$-2 > 4x$

$-0.5 > x$



This graph has an open circle at $x = -0.5$ because that point **is not** included.

One strategy for solving inequalities is to solve their corresponding equation, then test some values.

Here is an inequality: $-3(x - 4) \leq 9$.

The solution to its corresponding equation $-3(x - 4) = 9$ is $x = 1$.

To determine the solutions to $-3(x - 4) \leq 9$, test values less than and greater than 1.

The value less than 1 created a false inequality, so the solutions are $x \geq 1$.

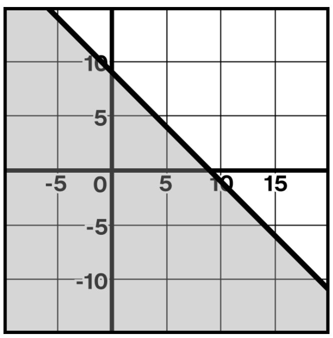
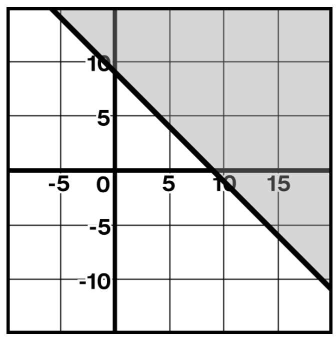
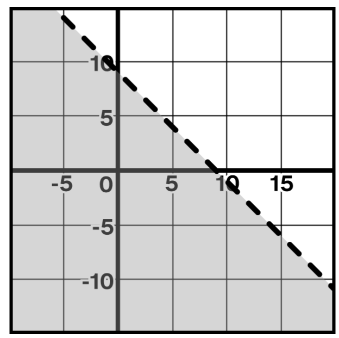
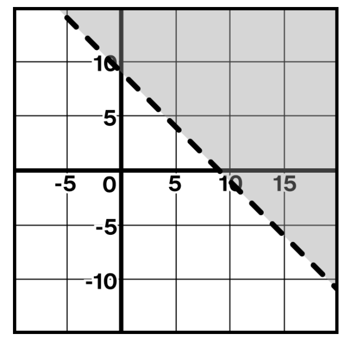
$x = 0$	$x = 10$
$-3(0 - 4) \leq 9$	$-3(10 - 4) \leq 9$
$-3(-4) \leq 9$	$-3(6) \leq 9$
$12 \leq 9$	$-18 \leq 9$

False!	True!

Lessons 13–14

The solutions to a two-variable linear inequality are represented on a graph as a half-plane.

The shaded area represents all of the solutions to each inequality below.

$x + y \leq 9$	$x + y \geq 9$	$x + y < 9$	$x + y > 9$
			
The solid line means the points on the boundary line **are** included in the solutions.		The dashed line means the points on the boundary line **are not** included.	

Lessons 15–16

Solutions to two-variable inequalities on a graph can help make sense of real-life situations.

Angel makes $\$4$ per pound of strawberries they sell and $\$3$ per pound of plums.

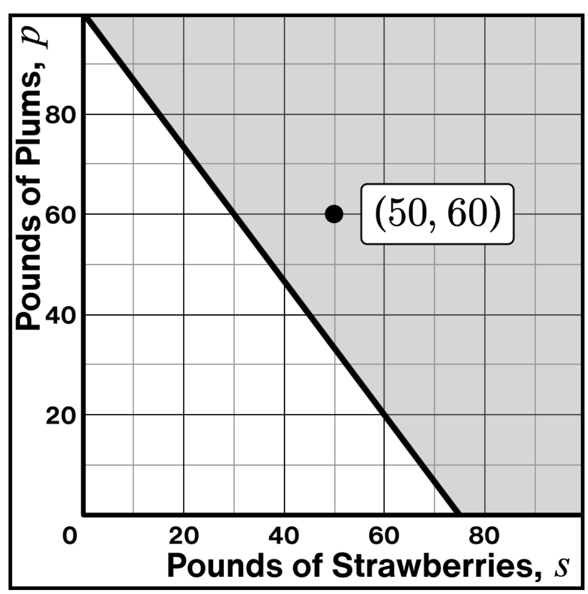
The inequality $4s + 3p \geq 300$ represents the pounds of strawberries, s, and pounds of plums, p, Angel needs to sell to meet their goal of making at least $\$300$.

To determine the solutions to their inequality, graph the corresponding equation and then test some values.

The equation $4s + 3p = 300$ represents what Angel needs to sell to make **exactly** $\$300$.

If Angel sold 50 pounds of strawberries and 60 pounds of plums, they would make $4(50) + 3(60) = 380$ dollars, which meets their goal.

This means that any amount of strawberries and plums in the shaded region would help Angel meet their goal.



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Unit A1.2, Family Resource

Try This at Home

Try After Lesson 2

1. Solve $4(5 - 3x) = 2(x - 4)$.

Emma made an error solving the equation $\frac{1}{2}(x + 4) = -10 + 2x$.

Emma's Work

$$\frac{1}{2}(x + 4) = -10 + 3x$$
$$x + 4 = -20 + 3x$$
$$x + 24 = 3x$$
$$24 = 2x$$
$$12 = x$$

2.1 What is one thing that Emma did well?

2.2 What is one thing that she did incorrectly?

Try After Lesson 4

Here is work to solve $5(2 + 4h) + 2h = -34$.

Step 1:	$5(2 + 4h) + 2h = -34$
Step 2:	$10 + 20h + 2h = -34$
Step 3:	$10 + 22h = -34$
Step 4:	$22h = -44$
Step 5:	$h = -2$

3.1 Write what happens at each step.

3.2 Use substitution to show that $h = -2$ is the correct solution.

Try After Lesson 5

4. How many solutions will the equation $6x - 4 + 2(5x + 2) = 16x$ have?

   Explain how you know.

Try After Lesson 7

5. Adrian is selling tickets for his school's soccer game. It costs \$3 for each child and \$5 for each adult. At the end of the game, Adrian has \$153.

   Select **all** the equations that represent this relationship, where c represents the number of children and a represents the number of adults who attended the game.

- ▢ $5a + 3c = 153$
- ▢ $c = \frac{153-5a}{3}$
- ▢ $a = \frac{153}{5} + \frac{3}{5}c$
- ▢ $c = 30.6 + 0.6a$

desmos

Unit A1.2, Family Resource

Try After Lesson 9

Neel won 20 tickets at their local fair. Each sticker costs 2 tickets and each pencil costs 3 tickets.

6.1 Let s represent the number of stickers Neel gets and p represent the number of pencils.

Write an equation to represent how many of each kind of prize Neel can get.

6.2 Solve your equation for s.

6.3 Write a question your new equation could help you answer.

6.4 Create a graph to represent this situation using either of your equations.

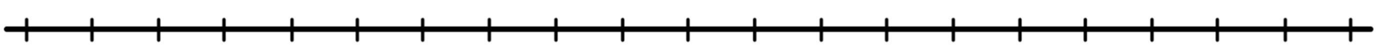
Try After Lesson 12

Here is an inequality: $12 - x < 10$.

7.1 The solution to the equation $12 - x = 10$ is $x = 2$.

Determine the solutions to $12 - x < 10$.

7.2 Graph the solution to $12 - x < 10$ on the number line below.



Jacy's goal is to walk more than $70\,000$ steps this week. For the first 4 days of the week, Jacy walked $8\,020$ steps each day on average.

8.1 If Jacy walks $12\,460$ steps each of the last 3 days, will they reach their goal?
Explain your thinking.

8.2 Write an inequality to represent the number of steps, s, Jacy can walk each of the last 3 days to reach their goal.

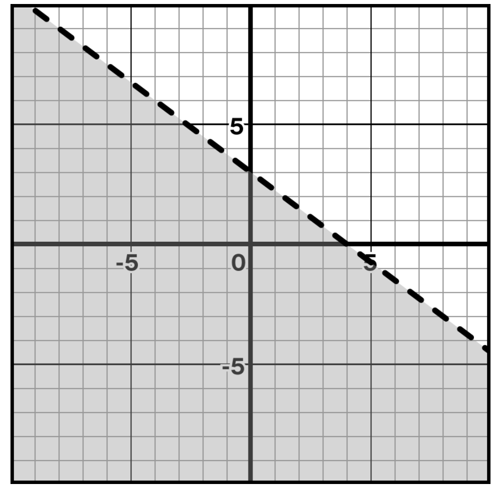
Unit 3
Describing Data

Try After Lesson 14

9.1 Which inequality does this graph represent?

A. $3x + 4y < 12$ B. $3x - 4y < 12$

C. $3x + 4y > 12$ D. $3x - 4y > 12$



9.2 How would the inequality change if the boundary line was solid instead of dashed? Explain how you know.

Try After Lesson 16

A basketball team is trying to raise at least $\$4\,500$ for new travel uniforms. They are planning to hold multiple car wash events at a local gas station. The team pays $\$300$ each event to use the gas station's space, water, and hoses. The team will collect $\$15$ for each car they wash.

10.1 Will the team reach their goal if they have 3 car wash events and wash 345 cars total?

10.2 The team captain wrote the equation $-300g + 15c = 4500$ to represent this situation.

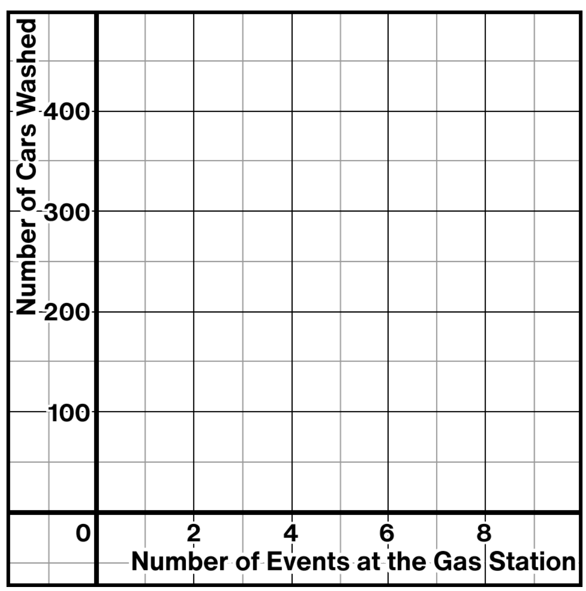
Show or explain what each part of the equation represents.

10.3 Graph the team captain's equation. Make a table if it helps you with your thinking.

10.4 Write an inequality that represents all the combinations of events held at the gas station and the number of cars washed that would raise **at least** $\$4500$ for new travel uniforms.

10.5 Shade in the region on the graph that represents all the solutions to the inequality you wrote.

10.6 Write a question that your graph could help the basketball team answer.



Unit 3 Summary

<table>
<tr><th>Prior Learning</th><th>Algebra 1, Unit 3</th><th>Future Learning</th></tr>
<tr><td>Grade 6
Creating and interpreting data displays (dot plots, histograms, and box plots)
Calculating summary statistics (mean, median, center, spread)
Grade 8
Analyzing scatter plots</td><td>Visualizing one-variable data (dot plots, histograms, box plots)
Summarizing one-variable data (mean and standard deviation, median and IQR, outliers)
Analyzing two-variable data (scatter plots, correlation coefficients, lines of fit, residuals)</td><td>Algebra 1, Unit 6
Exponential models of two-variable data
Algebra 2
Normal distributions
Making inferences with data</td></tr>
</table>

Visualizing One-Variable Data

Lesson 1
Survey data can be classified as *categorical* or *quantitative*.

What is your age? produces *quantitative* data (also called numerical data).

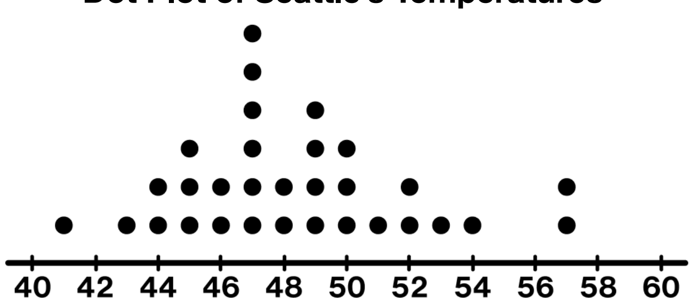
How did you get to work last week? produces *categorical* data.

Lesson 2
A *dot plot* and a *histogram* are two ways to visualize quantitative data.

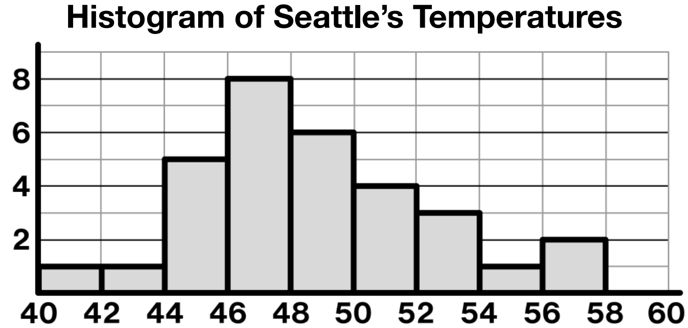
Dot plots show each individual data point. Histograms show the number of data points within each bin, or range of values.

Here are the daily high temperatures (in degrees Fahrenheit) of Seattle in the month of January, 2021.

Dot Plot of Seattle's Temperatures



Histogram of Seattle's Temperatures



desmos

Unit A1.3, Family Resource

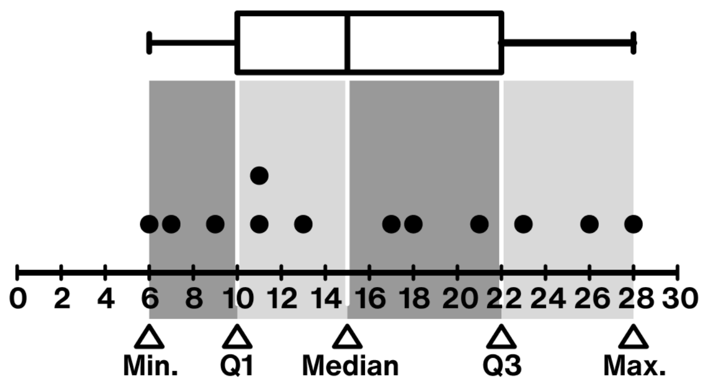
Lesson 3

A *box plot* can be used to visualize a one-variable quantitative data set.

A box plot splits the data into quarters.

The box portion of the box plot contains the middle 50% of the data.

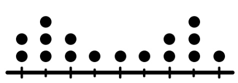
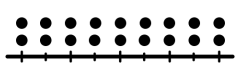
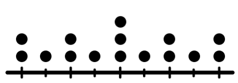
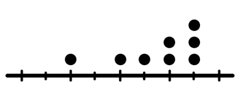
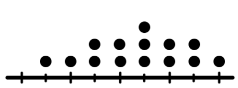
Here are the heights of 12 trees (in feet) in a dot plot and box plot.



In this example, 50% of the trees are between 10 and 22 feet tall, 25% of the trees are between 6 and 10 feet tall, and 25% of the trees are between 22 and 28 feet tall.

Lesson 4

The shapes of data can be described as *bimodal*, *uniform*, *symmetric*, *skewed*, and *bell-shaped*.

Shape Description	Dot Plot	Definition
Bimodal		There are two peaks in the data.
Uniform		Data values are evenly distributed.
Symmetric		The data has a line of symmetry.
Skewed		One side of the data has more values than the other.
Bell-shaped		Most of the data is at the center with fewer points farther from the center.

Summarizing One-Variable Data

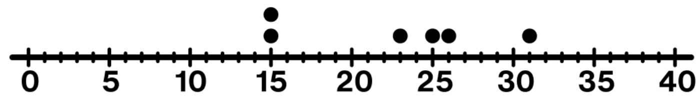
Lesson 5

Here are Oscar's scores from a game.

We can use measures of center to describe his typical score. *Median* and *mean* are two measures of center.

Here is how Oscar calculated each measure of center.

Oscar's Scores



~~15~~, ~~15~~, 23, 25, ~~26~~, ~~31~~

24

The median is 24.

15+25+15+23+26+31=135

135÷6 = 22.5

The mean is 22.5.

Oscar's data is skewed.

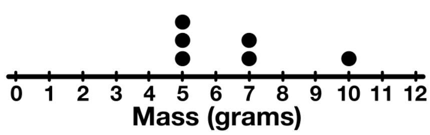
When the shape of data is skewed, the median can be more representative of the typical value than the mean.

Lesson 6

One way to measure the consistency or spread of data is to calculate its *standard deviation*. Data with a larger standard deviation is more spread out than data with a smaller standard deviation.

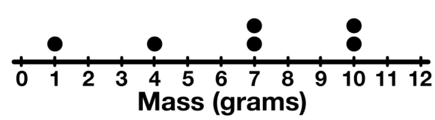
Here are the masses (in grams) of fish in two tanks.

Tank A: $5, 5, 5, 7, 7, 10$



Standard Deviation	
$A = [5, 5, 5, 7, 7, 10]$	
$\mathrm{stdevp}(A)$	≈ 1.8

Tank B: $1, 4, 7, 7, 10, 10$



Standard Deviation	
$B = [1, 4, 7, 7, 10, 10]$	
$\mathrm{stdevp}(B)$	≈ 3.2

Tank A's fish have a more consistent mass than Tank's B fish because Tank A has a smaller standard deviation.

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Unit A1.3, Family Resource

Lesson 7

Mean and *standard deviation* can be used to compare the center and spread of data sets.

Here are the high temperatures (in degrees Fahrenheit) in Desmopolis and Destown last week.

Desmopolis: 65, 73, 80, 82, 79, 68, 71 Destown: 70, 75, 76, 74, 77, 75, 74

City	Mean (°F)	Standard Deviation (°F)
Desmopolis	$A = [65, 73, 80, 82, 79, 68, 71]$ $\operatorname{mean}(A)$ $= 74$	$A = [65, 73, 80, 82, 79, 68, 71]$ $\operatorname{stdevp}(A)$ $= 6$
Destown	$B = [70, 75, 76, 74, 77, 75, 74]$ $\operatorname{mean}(B)$ ≈ 74.42	$B = [70, 75, 76, 74, 77, 75, 74]$ $\operatorname{stdevp}(B)$ ≈ 2.06

The mean compares which city has higher temperatures in general.
The standard deviation compares which city has more consistent temperatures.

Lesson 8

The *interquartile range* (or *IQR*) measures the middle half of a data set, or the distance between the first and third quartiles. IQR is another measure of spread.

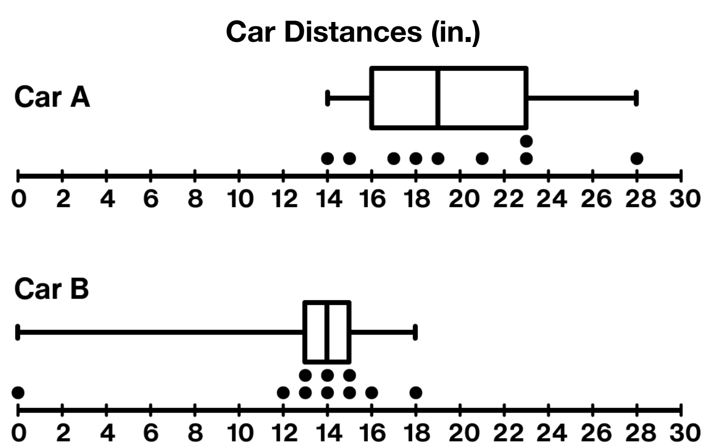
Here are box plots and some statistics of the distances traveled by two toy racecars.

Car A

Q1	Q3	IQR
16	23	7

Car B

Q1	Q3	IQR
13	15	2



Car B is more consistent than Car A because the IQR is smaller.

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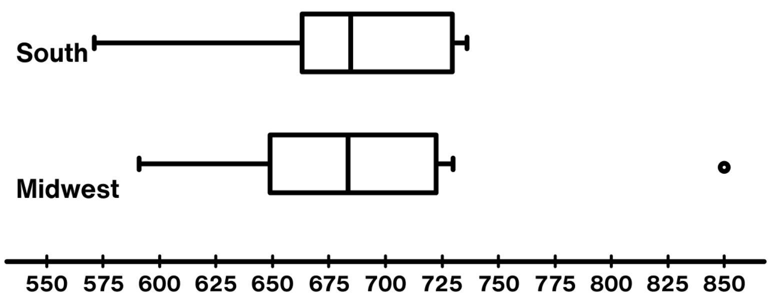
Unit A1.3, Family Resource

Lessons 9–10

Here is the median monthly rent (in dollars) of eight states from the South and Midwest in 2019.



Median Monthly Rent: South

$\$731, \$736, \$728, \$667$
$\$571, \$659, \$672, \$697$

Median Monthly Rent: Midwest

$\$591, \$669, \$682, \$848$
$\$715, \$685, \$629, \$730$

The value $\$848$ from the Midwest rents is called an *outlier* because it is far from other values in the data set. In a box plot, outliers are represented as an open dot.

Measures of center (mean/median) and spread (standard deviation/IQR) can help us interpret and compare data sets. The shape of the data and the outliers can influence which statistics to use.

Region	Mean	Standard Deviation	Median	IQR	Outliers
South	$\$682.63$	$\$51.03$	$\$684.50$	$\$66.50$	none
Midwest	$\$693.63$	$\$71.71$	$\$683.50$	$\$73.50$	$\$848$

The median rent for both regions are alike. The rents in the South are more consistent since the South rents have a smaller IQR of $\$66.50$ compared to the Midwest's IQR of $\$73.50$.

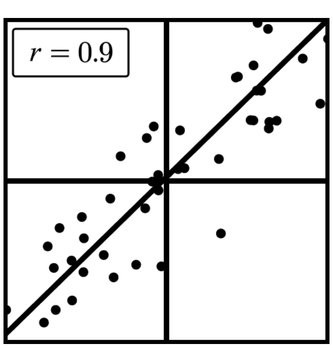
Two-Variable Data

Lesson 11–13

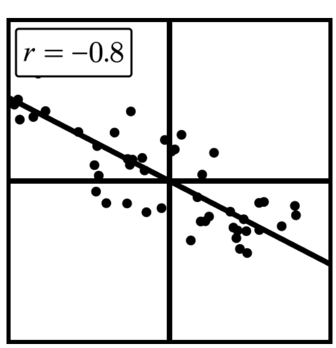
When the points on a scatter plot follow a line, we say there is a *linear association* between x and y and we can draw a line that fits the data.

The r-*value*, also called the *correlation coefficient*, is a number between -1 and 1 that describes the strength (weak, strong) and direction (negative, positive) of the linear association.

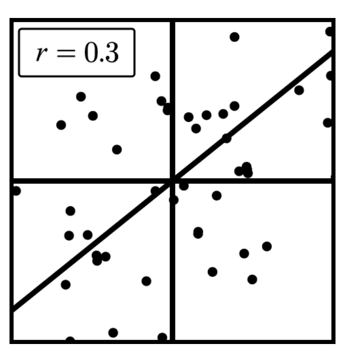
Strong and Positive



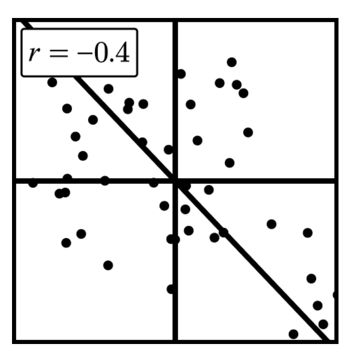
Strong and Negative



Weak and Positive



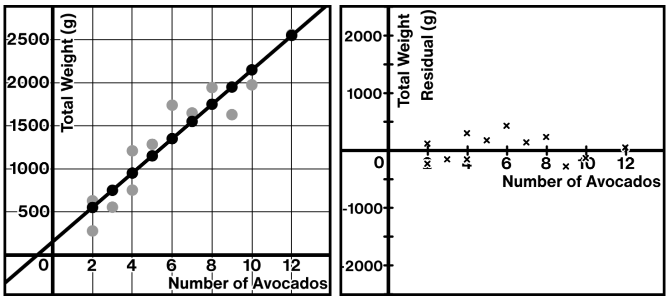
Weak and Negative



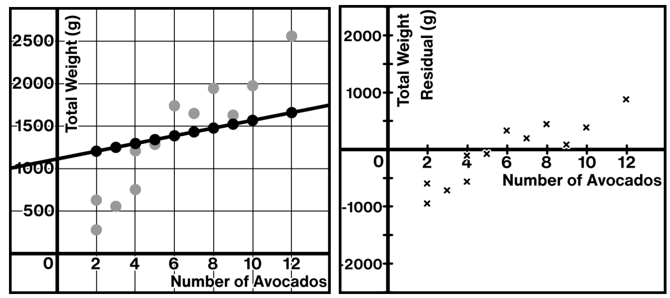
Lesson 14

A *residual* is the difference between the y-value of a data point and the value predicted by the line of best fit. A scatter plot of all the residuals (a *residual plot*) can help us decide if a line fits the data well.

Here are some lines of fit for several orders of avocados along with their residual plots.



This line is a good fit for the data because the residuals are close to the x-axis and there are random points below and above the x-axis.



This line is not a good fit for the data because many of the points are far from the x-axis.

Lesson 15-17

A calculator can compute the *line of best fit* and the correlation coefficient to help describe the relationship (or correlation) between two variables. *Causation* is one type of *correlation*.

In a causal relationship, a change in one variable causes a change in the other variable.

Nyanna noticed a trend at an ice cream shop. She recorded the number of ice cream cones sold and the customers wearing sunglasses one day.

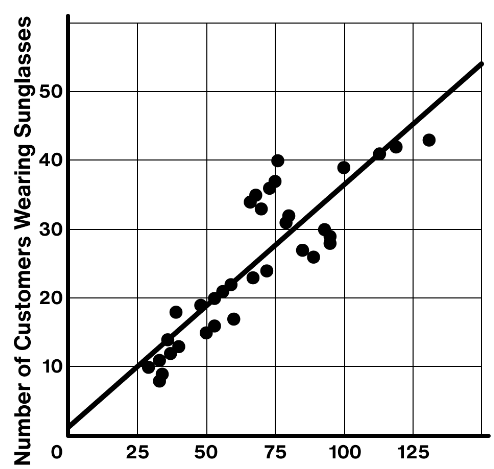
Nyanna used a calculator to generate a line of best fit.

$$y = 0.35x + 1.32$$

We can use Nyanna's model to predict the number of ice cream cones sold if there are 150 people wearing sunglasses.

$$y = 0.35(150) + 1.32$$
$$y = 53.82$$

~ 54 ice cream cones sold



The number of ice cream cones sold does not cause the number of customers wearing sunglasses to increase, so there is no causal relationship between the variables. If it is sunny out, people might be more likely to wear sunglasses and to buy ice cream.

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Unit A1.3, Family Resource

Try This at Home

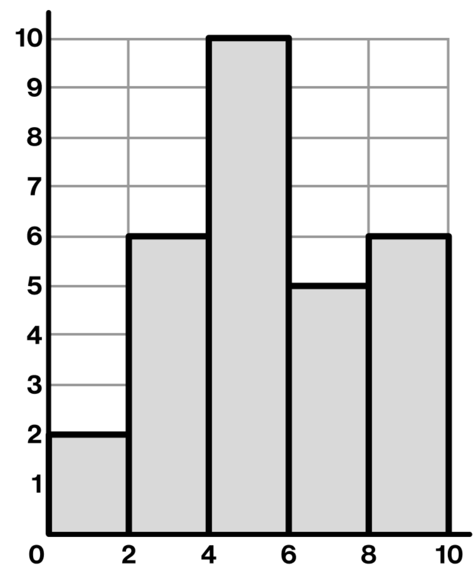
Visualizing One-Variable Data (Lessons 1–4)

Try After Lesson 2

Here is a histogram of how students rated the fall season on a scale from 0–10.

Decide if each statement is true, false, or cannot be determined.

Histogram of Fall Ratings



1.1 There are 29 total ratings.

True | False | Cannot be determined

1.2 The highest rating included was a 9.9.

True | False | Cannot be determined

1.3 The lowest rating was less than 2.

True | False | Cannot be determined

1.4 There are 10 ratings higher than 6.

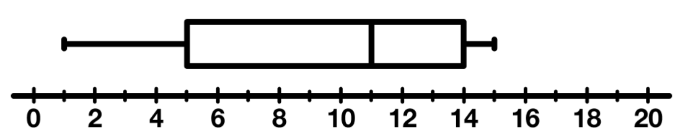
True | False | Cannot be determined

Try After Lesson 3

Ramon plays basketball. He recorded his points for each game in the season.
Use the box plot of his data to identify each statistic.

2.

Min.	Q1	Median	Q3	Max.



Decide if each statement is true, false, or cannot be determined.

3.1 In half of Ramon's games, he scored 11 points or more.

True | False | Cannot be determined

3.2 Ramon scored 0 points in a game.

True | False | Cannot be determined

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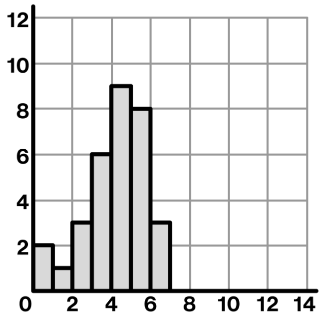
Unit A1.3, Family Resource

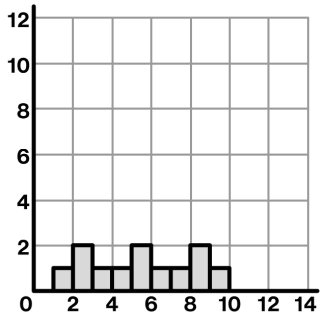
Try After Lesson 4

Match each histogram with the best description of its shape.

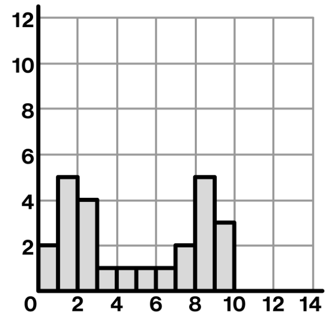
4.1 ______________



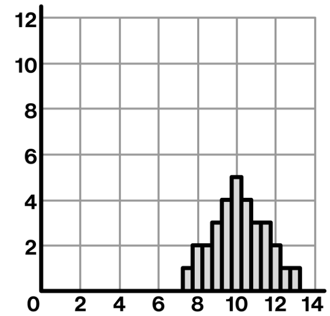
4.2 ______________



4.3 ______________



4.4 ______________



A. Bimodal	B. Bell-shaped	C. Skewed	D. Symmetric

Summarizing One-Variable Data (Lessons 5–10)

Try After Lesson 5

Here are the hourly wages, in dollars, for 9 employees from DesTunes Music.

12	12	13	13	13	15	17	18	19

5. Calculate the mean and median. Use a calculator if it helps with your thinking.

Mean	**Median**	**Which is larger?**

6. Use the Desmos Graphing Calculator (desmos.com/calculator) to create a dot plot or histogram of the data set. How would you describe the shape of the data?

Unit A1.3, Family Resource

Try After Lesson 7

Use the Desmos Calculator to determine the mean and standard deviation of the commute times (in minutes) for each traveler.

7.1 Traveler A: $31, 25, 28, 34, 31, 29, 30$

Mean	Standard Deviation

7.2 Traveler B: $36, 36, 41, 40, 43, 41, 34$

Mean	Standard Deviation

7.3 Which traveler had the most consistent commute time? Use statistics to justify your thinking.

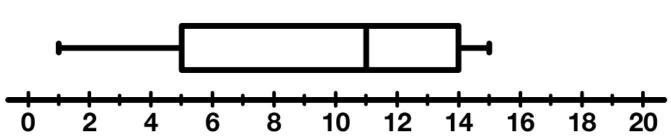
7.4 Which traveler generally had the longer commute time? Use statistics to justify your thinking.

Try After Lesson 8

Two basketball players recorded their points for each game in the season.

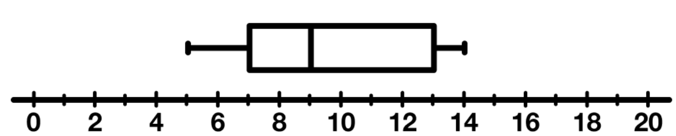
Use the box plots of their data to identify each statistic.

8.1 Ramon



Q1	Q3	IQR	Median

8.2 Neena



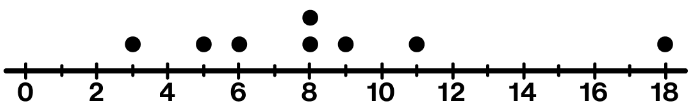
Q1	Q3	IQR	Median

9. Which player was more consistent in their points scored? Explain how you know.

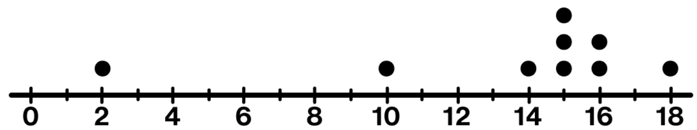
Try After Lesson 10

Here are the number of strikeouts thrown by two pitchers in each game they played this season.

Pitcher A: $3,\ 5,\ 6,\ 8,\ 8,\ 11,\ 18$



Pitcher B: $2,\ 10,\ 14,\ 15,\ 15,\ 15,\ 16,\ 16,\ 18$



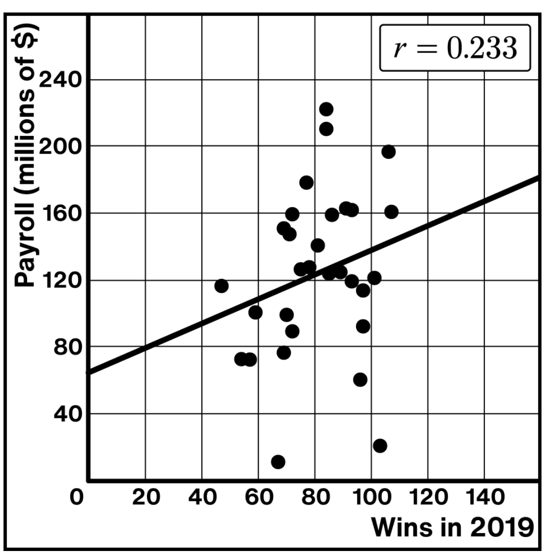
10. Complete the table with the statistics for Pitcher B.
Use the Desmos Calculator if it helps with your thinking.

Player	Mean	Standard Deviation	Median	IQR	Outliers
Pitcher A	8.43	4.56	8	6	None
Pitcher B					

11. Compare Pitcher A's and Pitcher B's number of strikeouts.
Use statistics about center and spread to support your ideas.

Two-Variable Data (Lessons 11–17)

Try After Lesson 12



12. Lucy was curious about the relationship between money and wins in professional baseball.

She found data about:

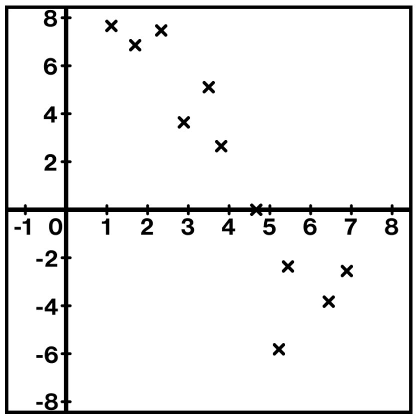
- Payroll (in millions of dollars).
- Wins in 2019.

The r-value is ________. This means . . .

Try After Lesson 14

Here are residual plots for lines that are not shown. Describe how you think each line fits the data.

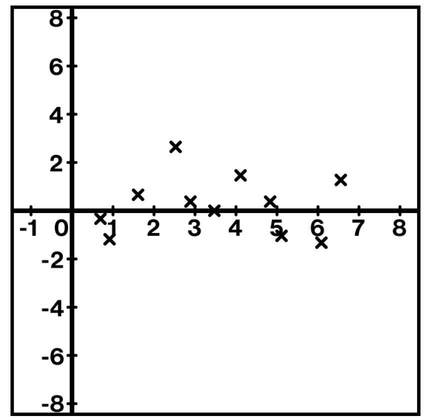
13.1



Circle one:
The line fits the data well / not well.

Explain your thinking.

13.2



Circle one:
The line fits the data well / not well.

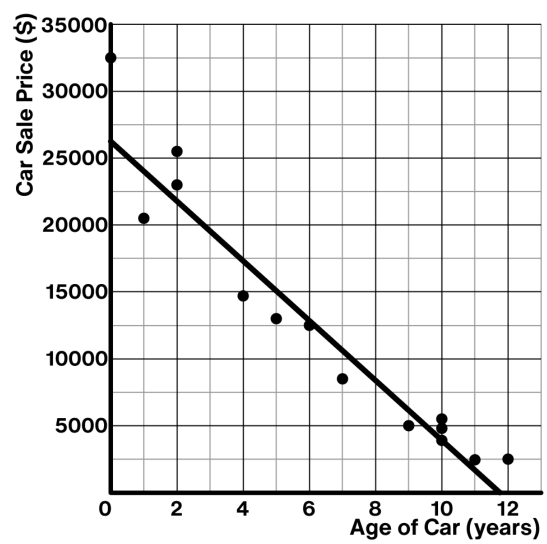
Explain your thinking.

Try After Lesson 17

Kwasi was curious about the relationship between the ages of cars and their values. He found data on the ages of several cars and their sale prices.

Here is the line of best fit equation:

$$y = -\ 2\,270.38x + 26\,886.7$$



14. What does the model predict the price would be for a car that is 8 years old?

15. Do you think one of the variables causes the other? If not, what else could be affecting the relationship?

    Explain your thinking.

Unit 4
Describing Functions

Unit 4 Summary

Prior Learning	Algebra 1, Unit 4	Future Learning
Math 6–8 and Algebra 1, Units 1 and 2 • Reasoning with absolute value • Calculating slope • Defining a function • Creating and analyzing graphs • Modeling relationships	• Function notation • Key features of functions • Piecewise-defined and absolute value functions	Algebra 1, Units 6 and 7 • Exponential & quadratic functions Algebra 2 • Transformations of functions • Polynomial, rational, logarithmic, and trigonometric functions

Function Notation

Lesson 1

A *function* is a rule that assigns exactly one output to each possible input.

When determining if a rule is a function, a table can be used to organize inputs and outputs. If one input has multiple possible outputs, then the rule is not a function.

In **Rule H**, each input has exactly one output, so it's a function.

In **Rule J**, inputs have multiple possible outputs, so it's not a function.

Rule H takes any measurement in meters and converts it to centimeters.

Input	Output
3 m	300 cm
2.6 m	260 cm
5.5 m	550 cm
3 m	300 cm

Rule J takes whole numbers from 1 to 15 and outputs a word of that length.

Input	Output
5	watch
3	are
3	the
1	a

Lessons 2–3

Function notation is a way of writing the inputs and outputs of a function.

For example, suppose we made a function for determining the price of a medium pizza.

$m(t) = 15.5 + 1.5t$ represents the cost of a medium pizza with t toppings.

What is the value of $m(4)$?

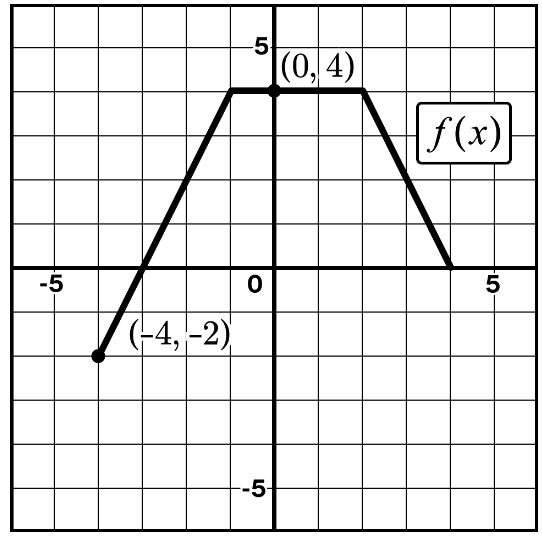
$$m(4) = 15.5 + 1.5(4)$$
$$m(4) = 21.5$$

The statement $m(4) = 21.5$ means the price of a medium pizza with 4 toppings is $\$21.50$.

Lesson 4

Any input-output pair for a function can be represented on the coordinate plane.

Statement	Coordinate Pair
$f(-4) = -2$	$(-4, -2)$
$f(0) = 4$	$(0, 4)$



We can also compare function statements.

$f(-2) = f(4)$ means the function has the same output when the input is -2 and when the input is 4.

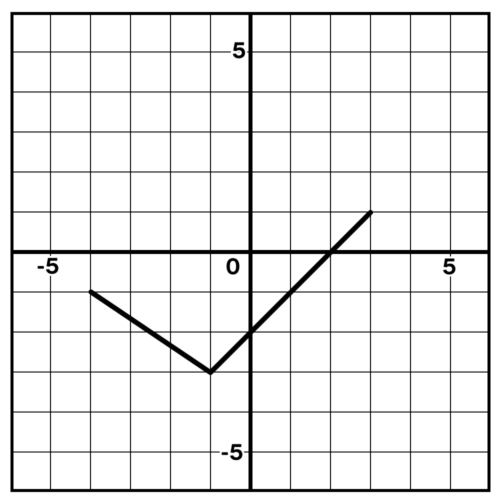
$f(-1) > f(3)$ means the function's output is greater when the input is -1 than when the input is 3.

Key Features of Functions

Lessons 5–6

The terms *maximum*, *minimum*, *positive*, *negative*, *increasing*, and *decreasing* can be used to describe parts of a graph.

Key Features	When
Minimum: Coordinates of the lowest point of the graph	$(-1, -3)$
Maximum: Coordinates of the highest point of the graph	$(3, 1)$
Positive: When the function has positive outputs. The graph is above the x-axis.	$x > 2$
Negative: When the function has negative outputs. The graph is below the x-axis.	$x < 2$
Increasing: When the function's outputs increase as the inputs increase; graph is upward-sloping, left to right	$x > -1$
Decreasing: When the function's outputs decrease as the inputs increase; graph is downward-sloping, left to right	$x < -1$

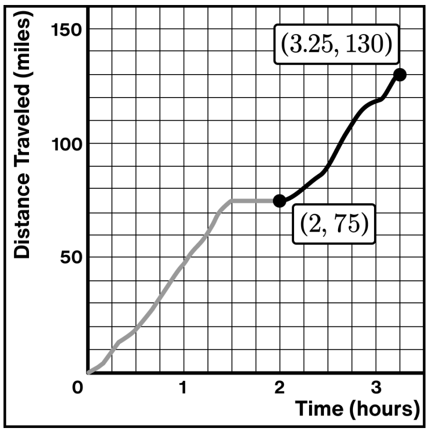


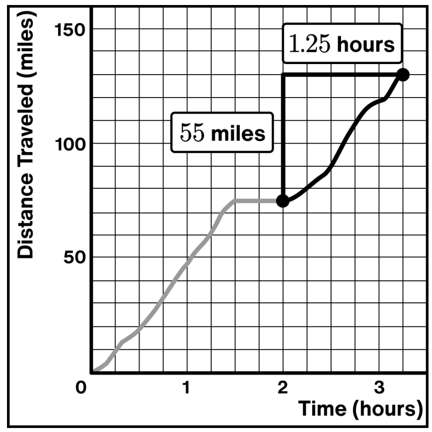
Unit A1.4, Family Resource

Lesson 7

The *average rate of change* is equivalent to the slope of the line between two points.

Let's look at an interval of Arjun's car trip below. To determine the average rate of change between 2 to 3.25 hours, divide the change in distance (55 miles) by the change in time (1.25 hours).





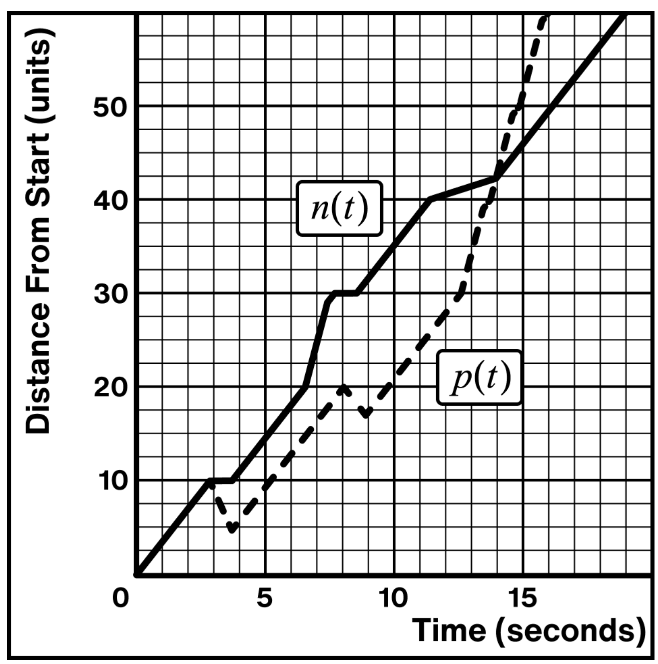
$$\frac{55 \text{ miles}}{1.25 \text{ hours}} = \frac{44 \text{ miles}}{\text{per hour}}$$

Lesson 8

When analyzing two or more functions, you can compare the key features and behavior of different parts of their graphs.

For example, Nekeisha and Polina raced their spaceships. Functions $n(t)$ and $p(t)$ give their spaceships' distances from the starting line after t seconds.

Statement	Meaning
$n(3) = p(3)$	At 3 seconds, Nekeisha and Polina both traveled the same distance.
$n(16) < p(16)$	At 16 seconds, Nekeisha was behind Polina.
$n(6) > p(6)$	At 6 seconds, Nekeisha was ahead of Polina.
$n(14) = p(14)$	At 14 seconds, Nekeisha and Polina both traveled the same distance.



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Unit A1.4, Family Resource

Lesson 9

The *domain* of a function is the set of all possible input values. The *range* of a function is the set of all possible output values.

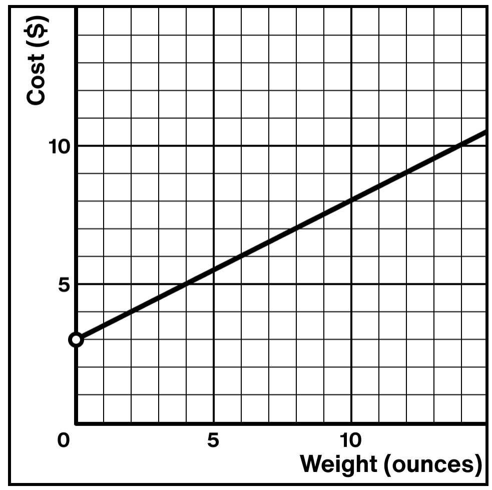
$f(w) = 3 + 0.5w$ represents the cost of a frozen yogurt that weighs w ounces.

Let's look at a frozen yogurt shop's pricing model. When determining the domain, consider if a variety of inputs are possible, such as negative numbers, positive numbers, fractions, large values, and zero.

The domain of $f(w)$ is all numbers greater than 0. It is not possible to have negative ounces and it is not possible to make an order of 0 ounces.

When determining the range, consider a variety of outputs and use the situation or graph to help you make sense of the possibilities.

The range of $f(w)$ is all numbers greater than 3. It is not possible to have negative costs and it is not possible to get a cost of $\$3$ since that would mean a customer purchased a yogurt weighing 0 ounces.

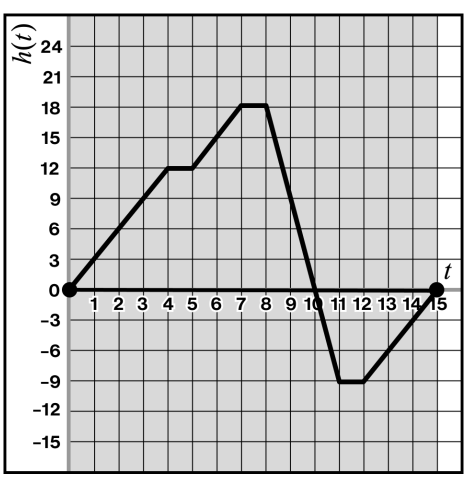


Lessons 10–11

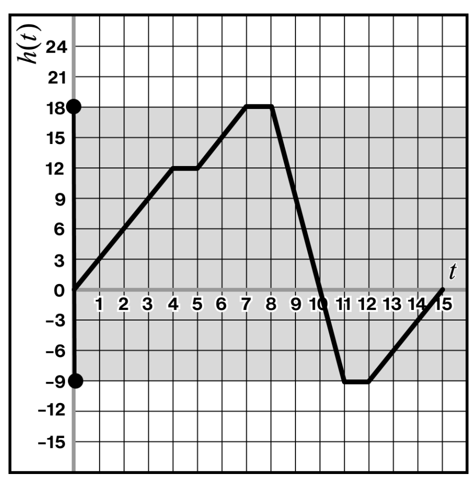
The domain and range of functions can be described using a *compound inequality*, which is two or more inequalities joined together.

Let's look at a guest's elevator ride at a hotel. The graph shows $h(t)$, the height of the elevator in meters, t seconds into the guest's ride.

The domain of $h(t)$ can be described using the compound inequality $0 \leq t \leq 15$, which describes all the numbers from 0 to 15.



The range of $h(t)$ can be described using the compound inequality $-9 \leq h(t) \leq 18$, which describes all the numbers from -9 to 18.

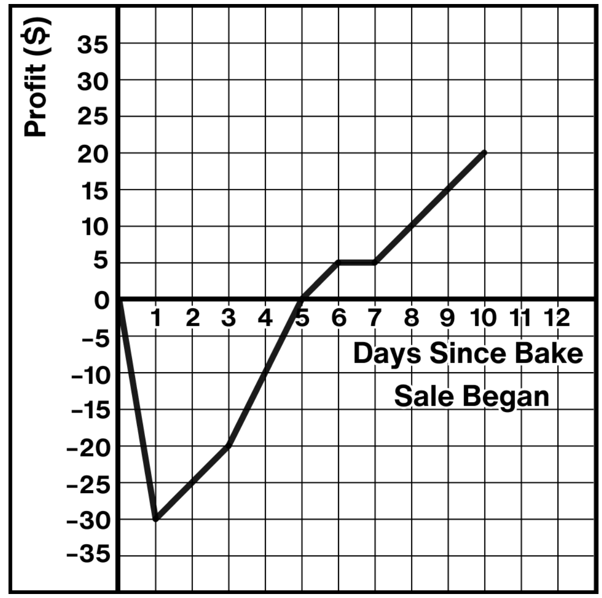


Lesson 12

A function's graph can be described using key features, which can be interpreted when provided a context.

Let's look at Kayleen's bake sale experience at her school. Kayleen decided to make cakes for her school's bake sale and tracked her profit.

- The *maximum* at $(10, 20)$ represents when Kayleen made the most profit.
- The *negative* interval from $0 < x < 5$ represents the days she has not made her money back.
- The *increasing* interval from $1 < x < 5$ represents the days she increased her profits.



Piecewise-Defined and Absolute Value Functions

Lessons 13–14

A *piecewise-defined function* is a function in which different rules apply to different intervals in its domain.

At Omar's Farm, the function $f(x)$ represents the price of a pumpkin with a weight of x pounds. Pumpkins 10 pounds or less cost $\$2$ per pound, and pumpkins more than 10 pounds cost $\$24$.

When $0 \le x \le 10$, $f(x) = 2x$.

When $x > 10$, $f(x) = 24$.

$f(4)$ means: What is the price of a 4-pound pumpkin?

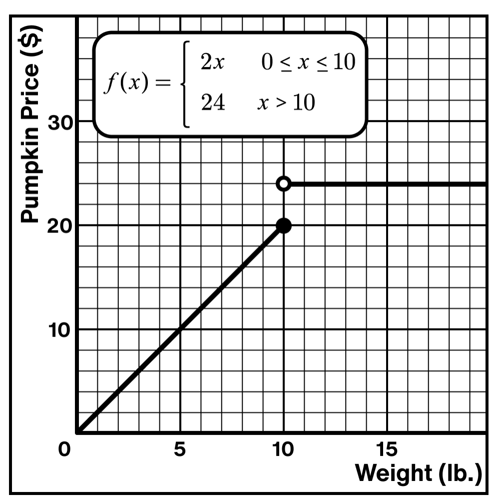
$\$8$

$f(15)$ means: What is the price of a 15-pound pumpkin?

$\$24$

$f(10)$ means: What is the price of a 10-pound pumpkin?

$\$20$



Unit A1.4, Family Resource

Lessons 15–16

The output of an *absolute value function* is how far away the input is from a given value.

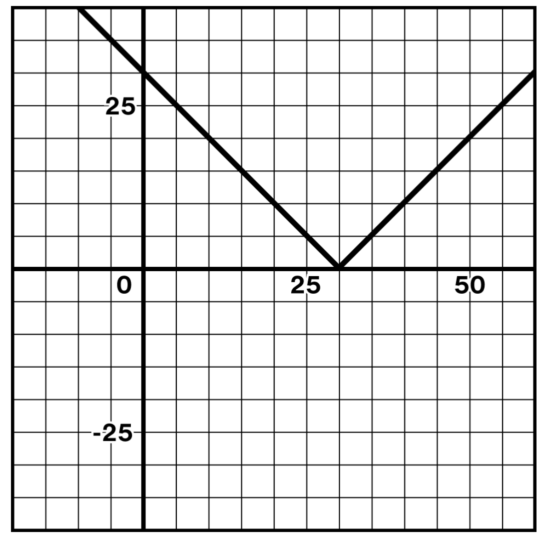
For example, there is a jar of 30 marbles at a carnival. Several people guessed the number of marbles in the jar.

The function $f(x) = |x - 30|$ represents how far a person's guess, x, is from 30.

What is the value of $f(25)$? What does it mean?

$$\begin{aligned} f(25) &= |25 - 30| \\ &= |-5| \\ &= 5 \end{aligned}$$

A person who guessed 25 marbles was 5 marbles away the actual number of marbles.



Identifying the minimum or making a table can be helpful in making a graph or writing an equation of an absolute value function.

Lesson 17

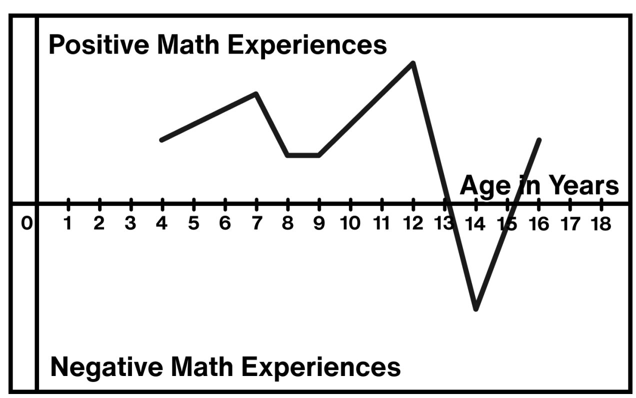
Storytelling is a powerful way to learn more about others and to reflect on one's own journey. Graphing a story can allow for interesting self-discoveries and deeper discussions.

Let's look at Joel's math story.

Joel is an 11th grade student and a child of military parents. His family is required to move every few years. He graphed his math experiences, $f(x)$, as a function of his age, x.

We can use Joel's graph to ask questions like:

- *What made your math experiences at age 12 so positive?*
- *What was your earliest math experience?*



Try This at Home

Try After Lesson 1

1. Circle the rule that is **not** a function.

Rule A takes any day of the week and outputs the next day.

Input	Output
Monday	Tuesday
Friday	Saturday
Sunday	Monday
Thursday	Friday

Rule B takes any letter and returns its order in the alphabet.

Input	Output
a	1
c	3
d	4
c	3

Rule C takes any value and multiplies it by 10.

Input	Output
2	20
10	100
0.3	30
0	0

Rule D takes any value and multiplies it by 1 or -1.

Input	Output
2	-2
2	2
5	-5
-3	3

Try After Lessons 2–3

At a rental car company, customers decide how many days they want to rent the car.
$r(x) = 50x + 75$ represents the cost of a rental car, where x represents the number of days.

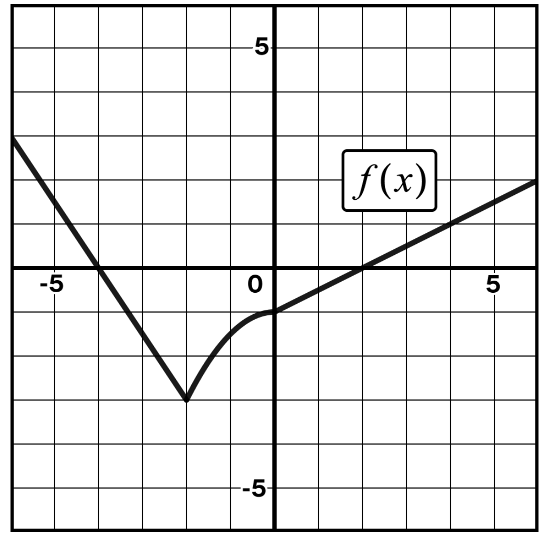
2.1 What is the value of $r(2)$?

2.2 What does $r(4) = 275$ mean in the context of the rental car?

Unit A1.4, Family Resource

Try After Lesson 4

3.1 Evaluate the function notation statements.



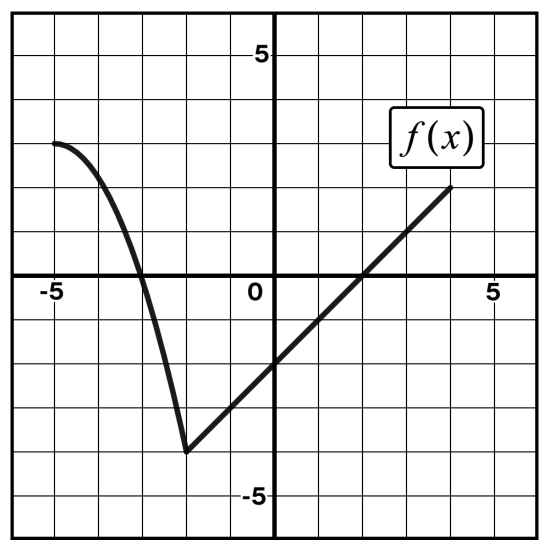
- $f(-4) =$
- $f(-2) =$
- $f(0) =$
- $f(4) =$

3.2 Select **all** the true statements.

- ☐ $f(-4) = f(2)$
- ☐ $f(-2) = f(-3)$
- ☐ $f(-4) > f(0)$
- ☐ $f(6) < f(-6)$
- ☐ $f(-4) > f(4)$

Try After Lessons 5–6

Circle **all** the descriptions that apply to the specified interval of $f(x)$.



4.1 $x < -3$

positive negative increasing decreasing

4.2 $x > 2$

positive negative increasing decreasing

5.1 What is the maximum? ____________

5.2 What is the minimum? ____________

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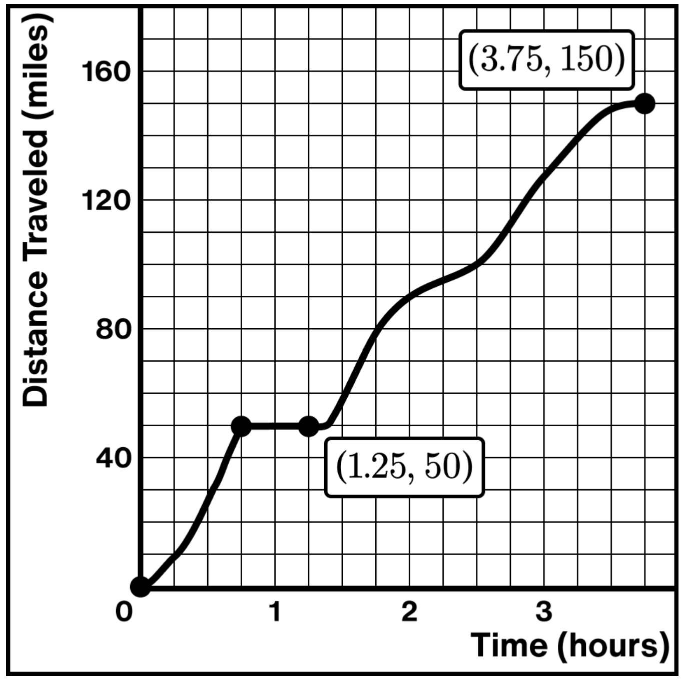
Unit A1.4, Family Resource

Try After Lesson 7

Oscar took the train to attend his friend's birthday. This graph represents his trip.

6. Determine Oscar's average rate of change from 1.25 to 3.75 hours.

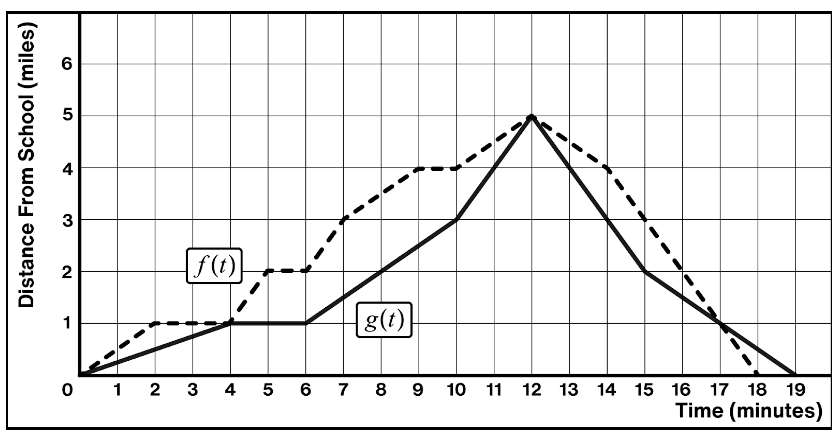


Try After Lesson 8

A school has two buses that take different routes to drop students off. They leave at the same time. $f(t)$ and $g(t)$ represent the distance of each bus from school (in miles) after t minutes.

7.1 Select **all** the true statements.

- ☐ $f(6) = g(6)$
- ☐ $f(10) > g(10)$
- ☐ $f(17) = g(17)$
- ☐ $g(5) = 1$
- ☐ $f(18) > g(18)$



7.2 Write one value of t where $f(t) = g(t)$.

desmos

Unit A1.4, Family Resource

Try After Lesson 9

A local mechanic sells a tire replacement service where they can replace up to 4 tires. The total cost is $\$50$ for labor plus an additional $\$100$ for each tire. The function $r(t) = 50 + 100t$ represents the total cost of service for t tires.

8.1 Select **all** the numbers that are in the domain of $r(t)$.

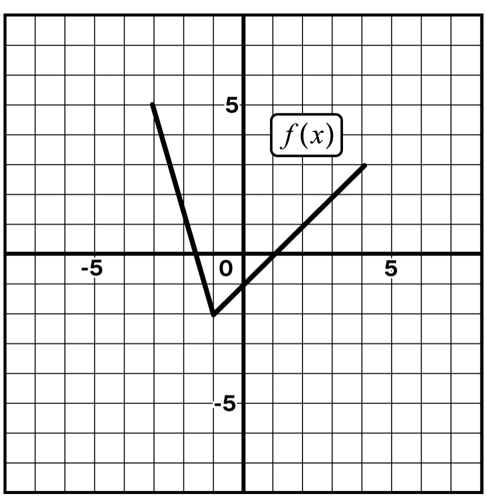
☐ -2 ☐ 0 ☐ 1 ☐ 4 ☐ 100

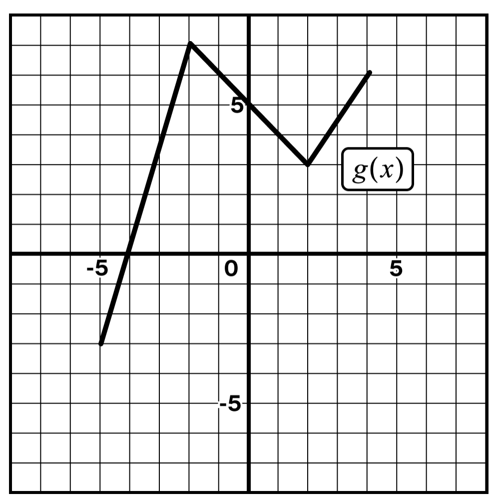
8.2 Describe the domain of $r(t)$.

Try After Lessons 10–11

Complete the compound inequalities to describe the domain and range of each function.

9.1 Domain	9.2 Range	10.1 Domain	10.2 Range
$\_\_\_ \leq x \leq \_\_\_$	$\_\_\_ \leq f(x) \leq \_\_\_$	$\_\_\_ \leq x \leq \_\_\_$	$\_\_\_ \leq g(x) \leq \_\_\_$



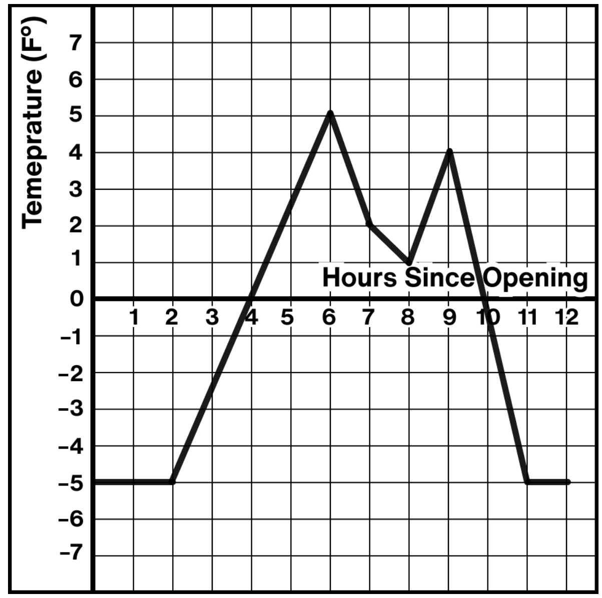


Try After Lesson 12

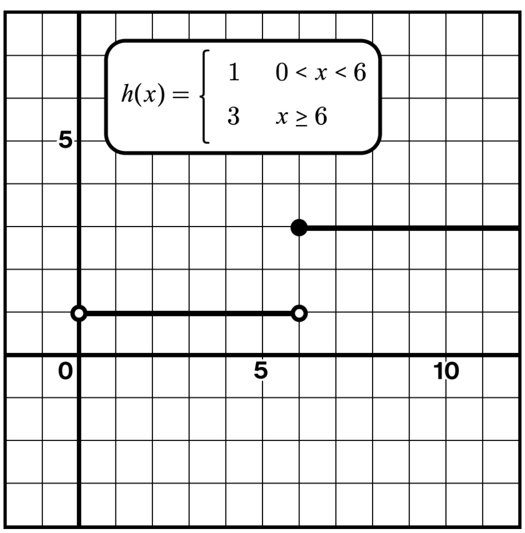
A frozen yogurt shop stores its yogurt in a freezer. The graph shows $f(t)$, the freezer's temperature in degrees Fahrenheit after t hours since the shop's opening.

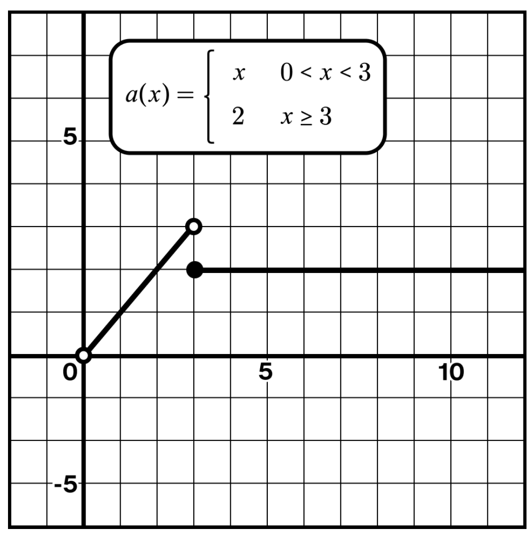
11. Complete the table with interpretations of each term in this context.

Term	Meaning
Increasing interval	
Maximum	
Domain	
Negative interval	



Try After Lessons 13–14





12.1 What is $h(2)$?

12.2 What is $h(8)$?

13.1 What is $a(1)$?

13.2 What is $a(3)$?

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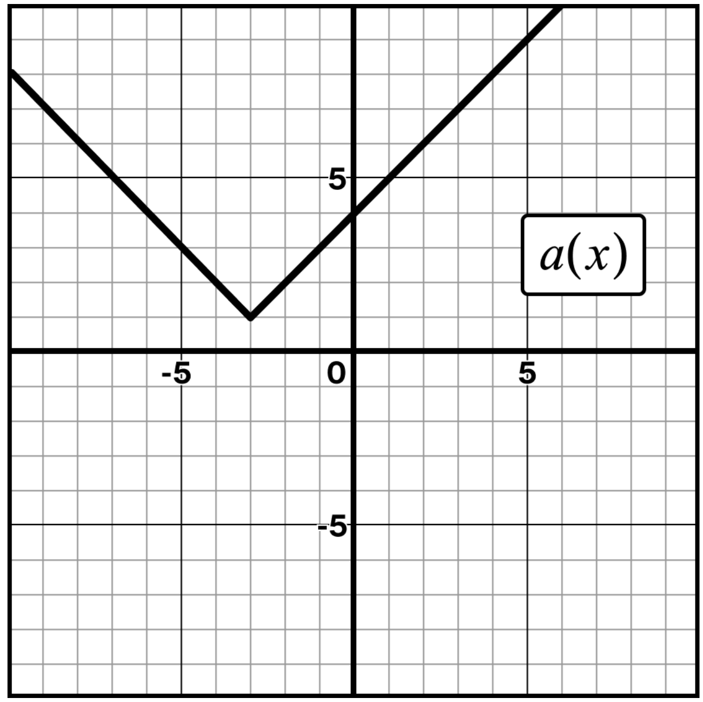
Unit A1.4, Family Resource

Try After Lessons 15–16

$$a(x) = |x + 3| + 1$$



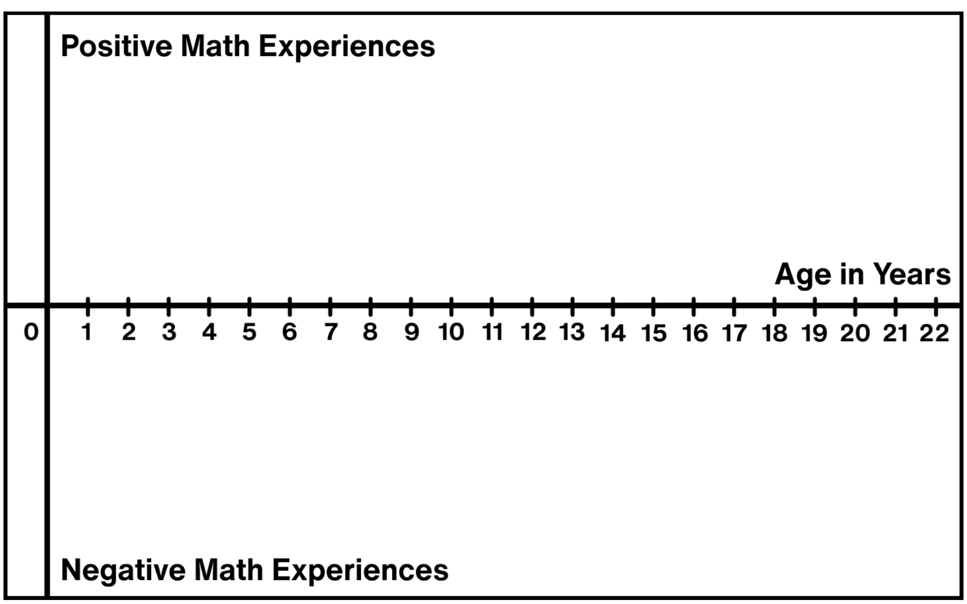
14.1 What is $a(-3)$?

Where do you see $a(-3)$ on the graph?

14.2 What is $a(-13)$?

Try After Lesson 17

15. Graph your own math experience and share your graph and story with your student.



Unit 5
Systems of Linear Equations & Inequalities

Unit 5 Summary

Prior Learning	Algebra 1, Unit 5	Future Learning
Math 6–8 And Algebra 1, Units 1 and 2 • Solving equations • Graphing linear equations • Solving systems of linear equations graphically and symbolically • Graphing and writing linear inequalities	• Solving systems of equations using graphing, substitution, elimination • Solving and graphing systems of inequalities • Representing situations with systems of linear equations and inequalities	Algebra 1, Unit 6 • Writing and solving exponential equations Algebra 1, Unit 8 • Solving systems of linear and quadratic equations

Systems of Equations

Lessons 1–2

A *system of equations* is two or more equations that use the same variables. The *solution to a system of equations* is a set of values that makes both equations true.

$$\begin{aligned} 2x + y &= 30 \\ x + y &= 23 \end{aligned}$$

Elimination is one method of solving systems of equations where you add or subtract the equations to create a new equation with fewer variables (also called eliminating a variable). These are two examples of systems of equations that have been solved using elimination.

$$\begin{array}{r} 2x + y = 30 \\ -\ (x + y = 23) \\ \hline x + 0 = 7 \\ x = 7 \\ \\ (7) + y = 23 \\ y = 16 \end{array}$$

The solution to this system is $x = 7, y = 16$.

$$\begin{array}{r} -2x + y = 9 \\ +\ (8x - y = 3) \\ \hline 6x + 0 = 12 \\ x = 2 \\ \\ -2(2) + y = 9 \\ y = 13 \end{array}$$

The solution to this system is $x = 2, y = 13$.

Unit A1.5, Family Resource

Lesson 3

When adding or subtracting the equations does not eliminate a variable, you can write an equivalent equation by multiplying one or more of the equations by a number before adding or subtracting.

In this example, the second equation is multiplied by –3 so that x can be eliminated by addition.

$$9x - 4y = 2$$
$$-3\,(3x + y = 10)$$

$$\begin{array}{r} 9x - 4y = 2 \\ +\ \ -9x - 3y = -30 \\ \hline 0 - 7y = -28 \\ \boxed{y = 4} \end{array}$$

$$3x + (4) = 10$$
$$3x = 6$$
$$\boxed{x = 2}$$

Lesson 4

Substitution is a method of solving systems of equations where you replace a variable with an expression of equal value.

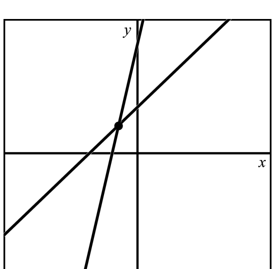
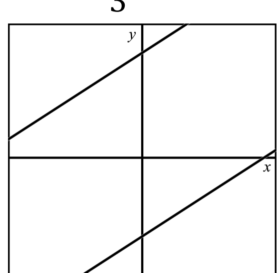
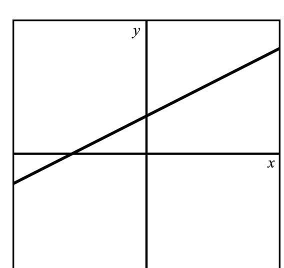
Here is an example of a system solved by substitution.

The solution to this system is $x = 3$, $y = -6$.

$$y = -4x + 6 \qquad y = 3x - 15$$
$$-4x + 6 = 3x - 15$$
$$-7x = -21$$
$$\boxed{x = 3}$$
$$y = 3(3) - 15$$
$$\boxed{y = -6}$$

Lesson 5

The *solution* to a system of equations on a graph is the point where the lines intersect. A system of linear equations can have one solution, no solutions, and infinite solutions. Here are three examples:

One Solution	No Solutions	Infinite Solutions
$y = 2x + \frac{1}{4}$	$y = \frac{2}{3}x + 10$	$y = \frac{1}{2}x + 3$
$y = 4x + 7$	$y = \frac{2}{3}x - 7$	$2y = x + 6$
		

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Lessons 6–7

Systems of linear equations can represent constraints in real-world situations.

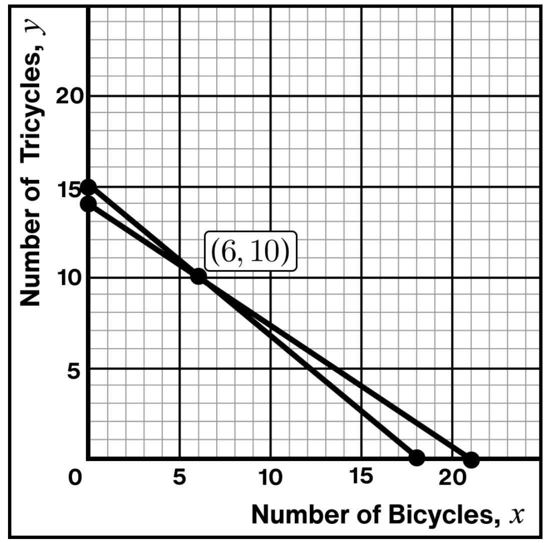
Suppose a bike shop owner wants to know how many bikes they can make given these constraints:

- The shop can make 2-wheel bicycles and 3-wheel tricycles.
- The shop has 42 wheels and enough materials to make 16 bikes total.

The constraints can be represented by this system and its graph, where x is the number of bicycles and y is the number of tricycles the shop will build.

$$x + y = 16$$
$$2x + 3y = 42$$

The solution to this system is $(6,\ 10)$, which means that the bike shop can make 6 bicycles and 10 tricycles.



Lesson 8

Elimination or substitution can be used to solve any system of linear equations.

There are strategic reasons to choose one method over the other.

Here are two examples.

$$\begin{array}{r} 3x + 4y = 3 \\ + \; -3x + 3y = 18 \\ \hline 7y = 21 \\ y = 3 \end{array}$$

$$3x + 4(3) = 3$$
$$x = -3 \qquad \boxed{(3, -3)}$$

Elimination is used here because the equations are in standard form and the x-variable can be eliminated by adding the equations.

$$y = 3x + 6$$
$$2x + 2y = 20$$

$$2x + 2y = 20$$
$$2x + 2(3x+6) = 20$$
$$2x + 6x + 12 = 20$$
$$8x + 12 = 20$$
$$8x = 8$$
$$x = 1$$

$$y = 3(1) + 6$$
$$y = 9$$
$$\boxed{(1, 9)}$$

Substitution is used here because one of the equations has an isolated variable that can be substituted into the other.

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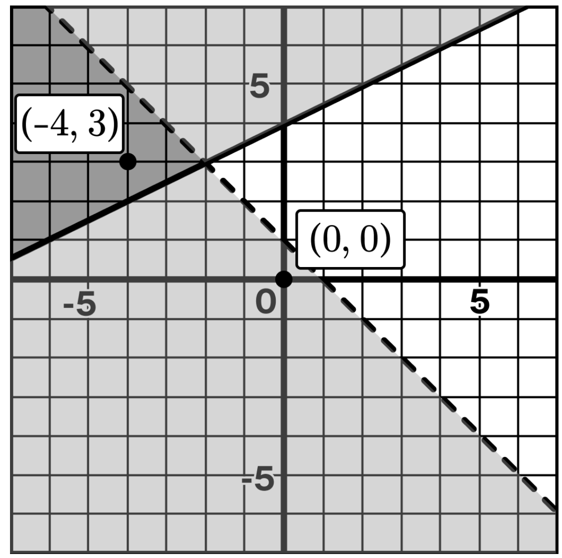
Systems of Inequalities

Lessons 9–10

The *solutions to a system of inequalities* are all the points that make both inequalities true. The solutions can be seen where the graphs overlap, called the *solution region*.

This graph shows the solutions to this system of inequalities.

$$x + y < 1$$
$$y \geq \frac{1}{2}x + 4$$



- $(-4, 3)$ **is** a solution to the system of inequalities because it is in the solution region on the graph and makes both inequalities true when tested.
- $(0, 0)$ is **not** a solution because it is not in the solution region and only makes one inequality true.

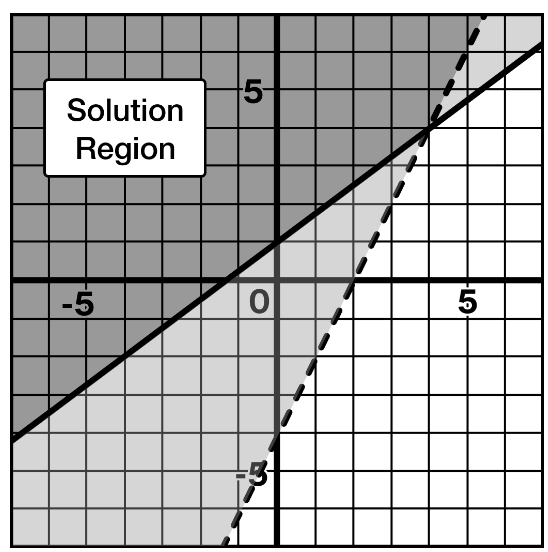
Lesson 11

Many strategies can be used to graph the boundary lines of inequalities. Some strategies include:

- Plot the y-intercept and then use the slope to determine other points. This can be especially useful for equations written in slope-intercept form ($y = mx + b$).
- Plot and connect the x- and y-intercepts. This can be especially useful for equations written in standard form ($Ax + By = C$).

This graph shows the solutions to this system of inequalities.

$$y \geq \frac{3}{4}x + 1$$
$$2x - y < 4$$



The solid line used when graphing $y \geq \frac{3}{4}x + 1$ means the points on the boundary line **are** included in the solutions.

The dashed line used when graphing $2x - y < 4$ means the points on the boundary line are **not** included.

Lesson 12

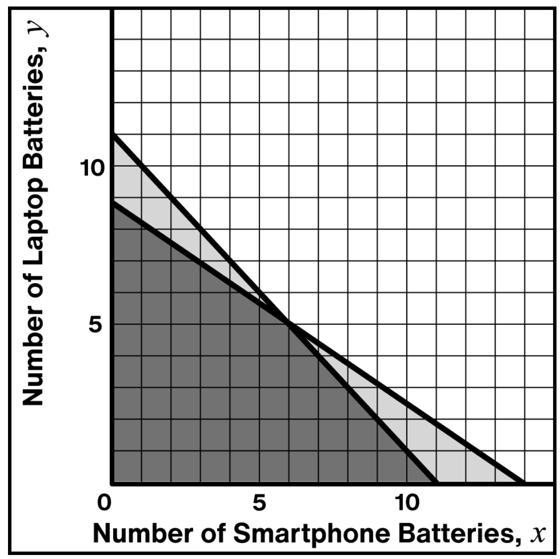
Systems of inequalities can help solve problems involving real-world constraints.

Suppose a battery recycling shop has $\$700$ to give to customers in exchange for used smartphones and laptops. To decide how many of each battery they can exchange, the shop owner must consider these constraints:

- The shop earns $\$50$ for each smartphone battery, x, and $\$80$ for each laptop battery, y.
- The shop can process up to 11 batteries.

The constraints can be represented by this system of inequalities and its graph:

$$50x + 80y \leq 700$$
$$x + y \leq 11$$



Any point in the solution region represents a combination of smartphone and laptop batteries that satisfy the shop's constraints.

For example, the point $(5,\ 4)$ is a solution that means the shop can process 5 smartphone batteries and 4 laptop batteries given the $\$700$ and 11-battery constraints.

Try This at Home

Try After Lesson 2

Circle whether adding, subtracting, or either can be used to eliminate a variable.

1.1 $p + 3q = 14$
$p + 2q = 10$
Adding Subtracting Either

1.2 $5x + 7y = 64$
$0.5x - 7y = -9$
Adding Subtracting Either

1.3 $5x + 4y = 40$
$5x - 4y = 20$
Adding Subtracting Either

1.4 Pick one of the examples above and solve using elimination.

Try After Lesson 3

Solve each system of equations.

2.1 $-4x + 3y = 23$
$x - y = -7$

2.2 $2x + 3y = 12$
$3x - 9y = 18$

Try After Lesson 4

Solve each system of equations.

3.1 $x = 4$ $2x - 4y = 20$

3.2 $y = 6x + 11$ $2x - 3y = 7$

Try After Lesson 5

Determine the number of solutions each system of equations has.

4.1 $y = 3x - 4$
$4y = 12x - 16$
No solutions One solution Infinite solutions

4.2 $y = -x + 13$
$y = \frac{1}{2}x + 2$
No solutions One solution Infinite solutions

4.3 $y = 5x + 11$
$y = 5x - 9$
No solutions One solution Infinite solutions

Try After Lesson 7

The knitting club sold 40 scarves and hats at a winter festival and made $\$700$ from the sales. They charged $\$18$ for each scarf and $\$14$ for each hat.

5.1 If s represents the number of scarves sold and h represents the number of hats sold, which system of equations represents the constraints in this situation?

A. $40s + h = 700$
$18s + 14h = 700$

B. $18s + 14h = 40$
$s + h = 700$

C. $s + h = 40$
$18s + 14h = 700$

D. $40(s + h) = 700$
$18s = 14h$

5.2 Solve the system of equations you chose and interpret the solution in context.

Try After Lesson 8

Which method would you prefer to use to solve each system of equations? Explain your choice.

6.1 $7x - 4y = -13$
$7x + 4y = 20$

Elimination Substitution Either

6.2 $y = 3x$
$y = 9x - 30$

Elimination Substitution Either

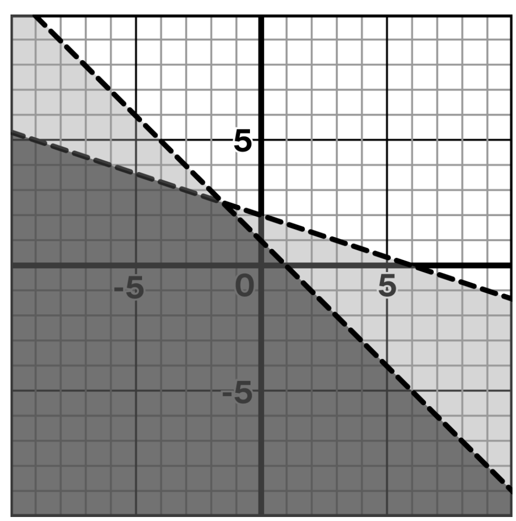
6.3 $2x - 4y = 10$
$x + 5y = 40$

Elimination Substitution Either

Try After Lesson 10

This is the graph of this system of inequalities:

$$x + 3y < 6$$
$$x + y < 1$$



7.1 Is the point $(0, 0)$ in the solution region?

Yes No

7.2 Is the point $(6, -3)$ in the solution region?

Yes No

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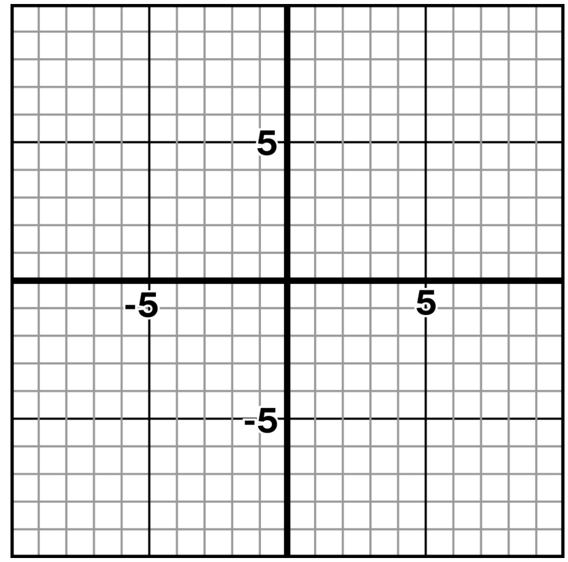
Try After Lesson 11

8. Graph this system of inequalities. Highlight the solution region.

$$y > 4x + 8$$
$$-4x + 3y \leq 12$$

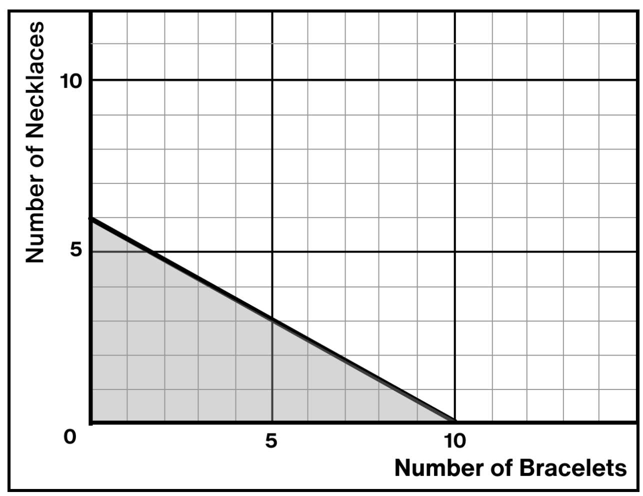


Try After Lesson 12

Natalia is planning to buy bracelets and necklaces as gifts for her friends.
Bracelets, b, cost $3 each and necklaces, n, cost $5 each. She can spend no more than $30 and needs at least 7 gifts.

This graph shows one of the inequalities that represents the constraints:

$$3b + 5n \leq 30$$



9.1 Write the second inequality needed to represent the constraints.

9.2 Graph the inequality you wrote on the same set of axes.

9.3 What are two possible combinations of bracelets and necklaces that Natalia could buy?

9.4 Explain how you can tell from the graph that Natalia **cannot** buy 1 bracelet and 6 necklaces.

Unit 6
Exponential Functions

Unit 6 Summary

Prior Learning	Algebra 1, Unit 6	Future Learning
Math 6–8 • Percentages • Properties of exponents Algebra 1, Unit 1 • Linear and exponential relationships Algebra 1, Unit 4 • Function notation and key features	• Comparing linear and exponential functions • Exponential growth and decay by percentages • Compounding interest and modeling data	Algebra 1, Units 7 and 8 • Quadratic functions and equations Algebra 2 • Solving exponential equations and noninteger inputs • Compounding continuously and e • Logarithmic functions

Comparing Linear and Exponential Functions (Lessons 1–5)

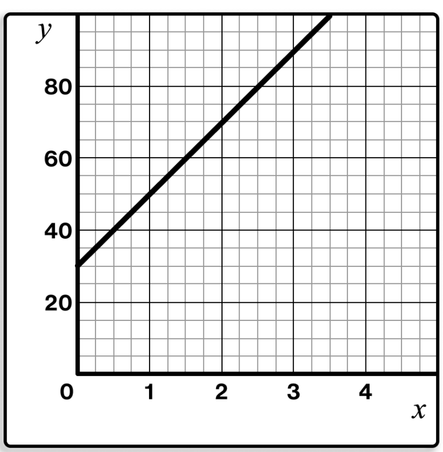
Lessons 1–3

Here is an example of a *linear* function and an *exponential* function. Linear functions have a constant rate of change. The rate of change for this linear relationship is 20 because the y-values increase by 20 as the x-values increase by 1. Exponential functions have a growth factor. The growth factor for this exponential relationship is 2 because the y-values grow by a factor of 2 as the x-values increase by 1.

Linear

$y = 30 + 20x$

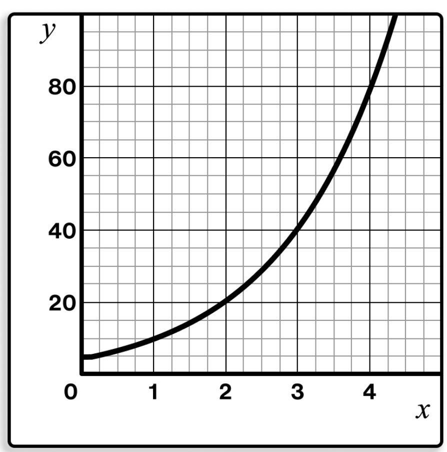
x	y
0	30
1	50
2	70
3	90



Exponential

$y = 5 \cdot (2)^x$

x	y
0	5
1	10
2	20
3	40



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Lesson 4

Here are examples of *simple interest* and *compound interest*.

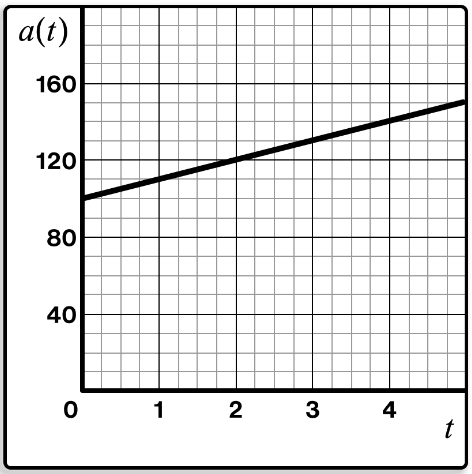
Account A: Simple Interest

t	$a(t)$
0	100
1	110
2	120
3	130

$+10$ $+10$ $+10$

$a(t) = 100 + 10t$

Account A starts with $\$100$ and $\$10$ is added each year.



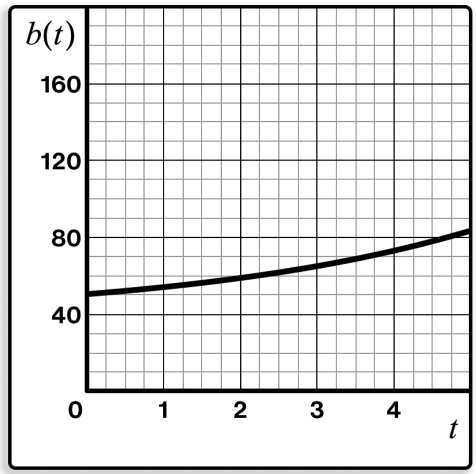
Account B: Compound Interest

t	$b(t)$
0	50
1	55
2	60.5
3	66.55

$\times 1.1$ $\times 1.1$ $\times 1.1$

$b(t) = 50 \cdot (1.1)^t$

Account B starts with $\$50$ and increases by 10% each year.



Simple interest is a linear relationship because it changes by a constant difference.
Compound interest is an exponential relationship because it changes by a constant growth factor.

Lesson 5

Exponential functions can be used to model exponential behavior. In 2000, scientists started measuring the volume of a coral reef. The function $v(t) = 10 \cdot \left(\frac{1}{2}\right)^t$ models the volume of the coral reef in cubic meters t years after 2000.

$v(0)$ models the volume of the coral reef in 2000.

$v(0) = 10 \cdot 1$, or 10, because any value to the zero power is one. $\frac{1}{2}^{0} = 1$.

$v(-1)$ models the volume of the coral reef in 1999 (1 year before they started measuring).

$v(-1) = 10 \cdot 2$, or 20, because for negative inputs, you can rewrite the growth factor. For example, $\left(\frac{1}{2}\right)^{-1}$ can be rewritten as 2, then multiplied.

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Exponential Growth and Decay (Lessons 6–9)

Lessons 6

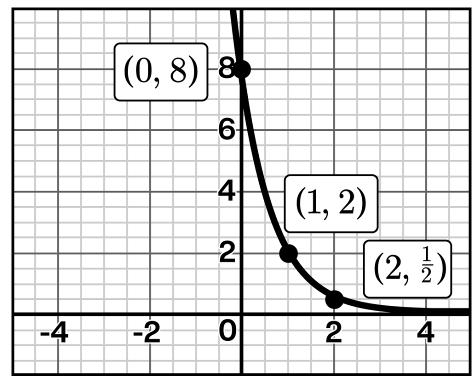
Exponential functions can be written in the form $f(x) = a \cdot b^x$, where:

- The a-value is the y-intercept or initial value.
- The b-value is the growth factor.

In this example, the y-intercept is $(0,\ 8)$, so $a = 8$.

The b-value or growth factor is $\frac{1}{4}$ since the y-values change by a factor of $\frac{1}{4}$ as the x-values increase by 1.

The equation of this graph is $f(x) = 8 \cdot (\frac{1}{4})^x$.



Lessons 7–9

We can use the growth factor, b, in an exponential equation like $y = a \cdot b^x$ to determine the type of exponential relationship: growth or decay.

- $b > 1$ is an example of exponential **growth**.
- $0 < b < 1$ is an example of exponential **decay**.

Consider an algae bloom in a lake that covers 100 square meters. Recognizing whether the algae bloom is growing or decaying and by what percentage each day can help you write an equation that models the area of the algae bloom after x days.

Description	Growth or Decay?	Equation
Doubles every day	Growth	$y = 100(2)^x$
Increases by 5% every day	Growth	$y = 100(1 + 0.05)^x$ or $y = 100(1.05)^x$
Decreases by 3% every day	Decay	$y = 100(1 - 0.03)^x$ or $y = 100(0.97)^x$

Compound Interest and Modeling Data (Lessons 10–14)

Lessons 10–11
Compound interest can be earned at different time intervals, such as daily, monthly, and annually.

Here are three equivalent expressions that will calculate the total value of a $\$1\,000$ loan with a 1% monthly interest rate after 2 years, with no additional payments.

$1000 \cdot (1.01)^{24}$	$1000 \cdot (1.01^{12})^{2}$	$1000 \cdot (1.12683)^{2}$

The third expression can help you see that the interest rate per year is about 12.7%.

This table shows the total value of a $\$1\,000$ loan with a 15% annual interest rate for different compounding intervals.

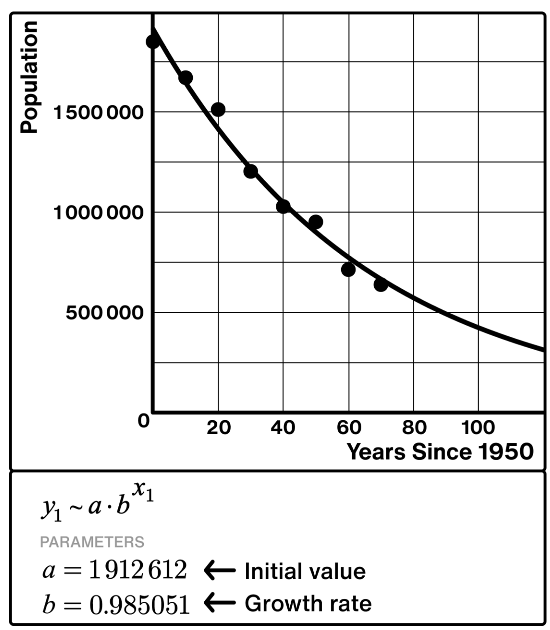
Interest	Owed In	Compounded Monthly	Compounded Quarterly	Compounded Annually
15% annually	5 years	$1000(1+\frac{0.15}{12})^{12\bullet 5}$ $\approx \$2\,107.18$	$1000(1+\frac{0.15}{4})^{4\bullet 5}$ $\approx \$2\,088.15$	$1000(1+0.15)^{5}$ $\approx \$2\,011.36$

The formula $P(1+\frac{r}{n})^{nt}$ is sometimes used to calculate the total amount for an account with compound interest, where r is the interest rate, t is the number of years, n is the number of compounding intervals, and P is the initial investment or loan.

Lessons 12–14
We can use linear and exponential functions to model many real-world situations.

The population of Detroit from 1950–2020 can be modeled with the closest fit by the exponential function $f(x) = 1\,912\,612 \cdot (0.985)^{x}$, where x represents the number of years since 1950. You can use a graphing calculator to generate this function.

Models can help us make predictions. For example, $f(80) \approx 570\,854$, which tells us the model predicts that the population of Detroit will be about $570\,854$ people in 2030.



Try This at Home

Try After Lesson 3

Determine whether each table shows a linear function, exponential function, or something else. Circle your choice.

1.1

x	y
3	5
4	15
5	45

Linear / Exponential / Something else

1.2

x	y
1	1
2	11
3	111

Linear / Exponential / Something else

1.3

x	y
2	25
3	50
4	75

Linear / Exponential / Something else

Try After Lesson 4

Adah invests \$75 in an account that earns 5% simple interest yearly. The function $a(t) = 75 + 3.75t$ models the account balance after t years.	Jamir invests \$75 in an account that earns 3% compound interest yearly. The function $j(t) = 75(1.03)^t$ models the account balance after t years.

Determine the balance of each account at:

2.1 10 years Adah: ________________ Jamir: ________________

2.2 12 years Adah: ________________ Jamir: ________________

Determine how many years it will take for each account to reach a balance of:

2.3 \$100 Adah: ________________ Jamir: ________________

2.4 \$200 Adah: ________________ Jamir: ________________

Try After Lesson 5

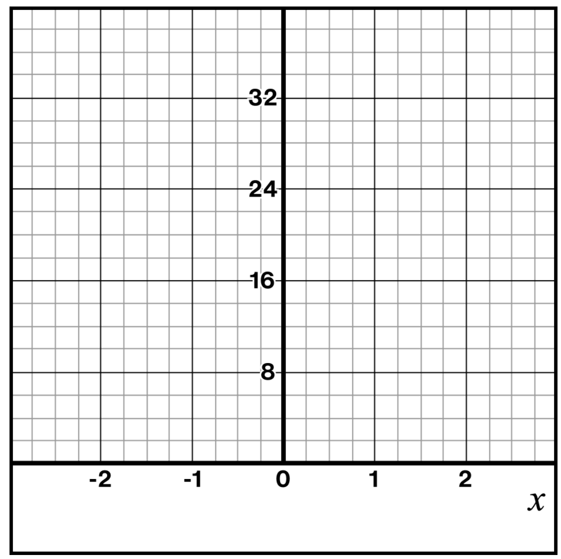
Marc's video channel had 18 subscribers in 2020. The function $m(x) = 18(\frac{3}{2})^x$ represents his total subscriber count x years after 2020.

3.1 Complete the table.

x	$m(x)$
-2	
-1	
0	
1	

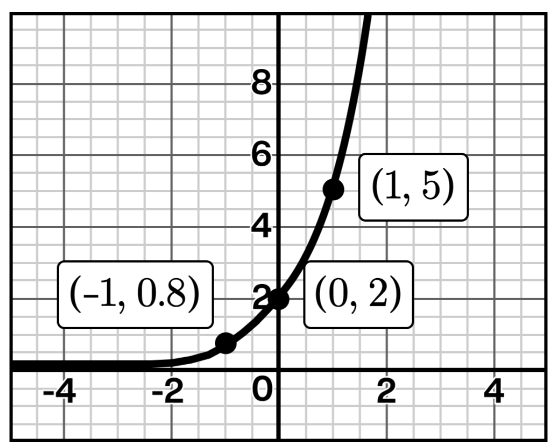
3.2 Explain what $m(-1)$ tells you about the situation.

3.3 Sketch the graph of $m(x)$.



Try After Lesson 6

4. Write the exponential equation that represents this graph. Use the table if it helps with your thinking.



x	y

$y =$ ____________________

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Try After Lesson 9

The value of a Desmon card collection increases 4% every year. In 2020, the price of the collection was valued at $\$500$.

5. Circle the equation that matches the situation.
   Use y to represent the cost of the collection and x to represent time in years since 2020.

A. $y = 500(0.4)^x$ B. $y = 500(0.04)^x$ C. $y = 500(1.4)^x$ D. $y = 500(1.04)^x$

A laptop battery can currently stay on for 480 minutes. The battery's capacity is decreasing by 8% each year.

6.1 Write a function that represents the battery's capacity, $b(x)$, after x years.

6.2 Use the function you wrote to determine how long the laptop battery will stay on after 5 years.

Try After Lesson 11

A bank offers a $\$100$ loan with a monthly interest rate of 1.5%.

7. Select **all** the expressions that can be used to calculate the balance after 5 years. Assume that no additional payments, deposits, or withdrawals are made.

☐ $100(1.015)^5$ ☐ $100(1.015^{12})^5$ ☐ $100(1.015)^{60}$ ☐ $100(1.19562)^5$

A bank offers different interest rates for their checking accounts.

- **Option A:** 2% annual interest rate compounded daily.
- **Option B:** 3% annual interest rate compounded semi-annually.

8. If you deposit $\$900$, which option will give you a greater balance in 4 years?
   Show your reasoning.

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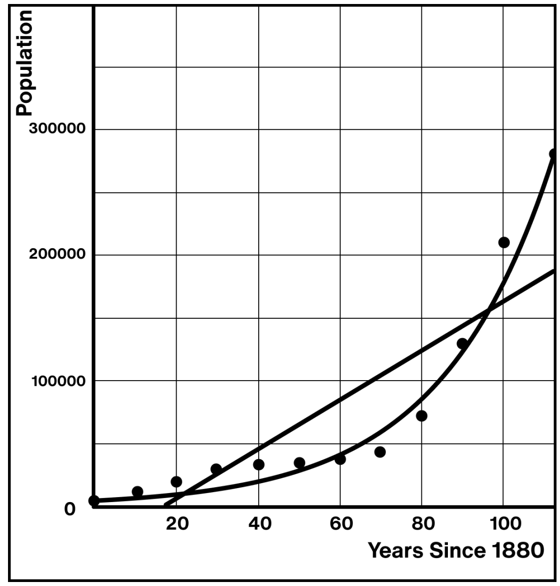
Try After Lesson 14

Aki was curious about the population changes in Colorado. They generated this line and exponential curve of best fit.



$$y_1 \sim mx_1 + b$$

PARAMETERS

$m = 2075.25$

$b = -38082.20$

$$y_1 \sim a \cdot b^{x_1}$$

PARAMETERS

$a = 3991.89$

$b = 1.03967$

Describe what the slope and growth factor means in each.

9.1 Which model do you think better fits the data for the population of Colorado from 1880–1990? (Circle one.)

Linear / Exponential

Use Aki's graphs to estimate the population of Colorado in 1945 (65 years after 1980).

9.2 **Linear model prediction**:

9.3 **Exponential model prediction**:

Unit 7
Quadratic Functions

Unit 7 Summary

Prior Learning

Grades 6–8
- Order of operations
- Modeling with linear functions

Algebra 1, Unit 4
- Function notation
- Key features of functions, including domain and range

Algebra 1, Unit 7

- Quadratic functions in tables and graphs
- Forms of quadratic functions: factored, standard, and vertex
- Graphing and building quadratic functions using key features

Future Learning

Algebra 1, Unit 8
- Factoring
- Completing the square
- Quadratic formula

Algebra 2
- Polynomial functions
- Transformations of other functions

Introduction to Quadratic Functions (Lessons 1–7)

Lessons 1–2

Let's look at the relationship between the step number and the number of tiles in this pattern.

You might notice that the number of tiles is made up of a large square plus one more tile.

This means figure n would have $n^2 + 1$ tiles.

This is an example of a new type of relationship called a *quadratic relationship*.

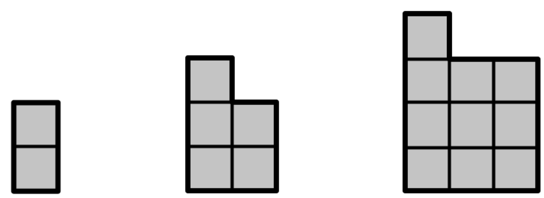


Figure 1 **Figure** 2 **Figure** 3

Lesson 3

Do quadratic relationships have a constant difference? **No.** Is there a constant ratio? **No.**

So, what *is* the pattern?

Even though the first difference is not constant, the *second difference* is constant.

In this example, the second difference is 2.

Figure	Number of Tiles
1	2
2	5
3	10
4	17

$+3$ $+5$ $+7$

$+2$ $+2$

Lessons 4–5

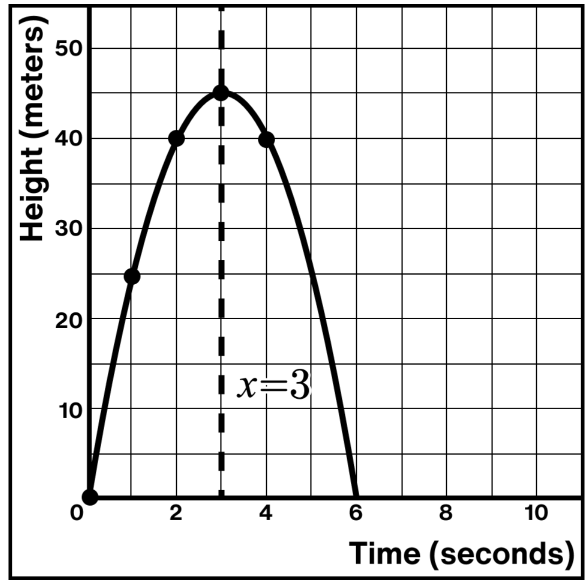
Constant second differences and symmetry can help us make predictions about quadratics.

This table and graph show the height of a stomp rocket over time.

Time (seconds)	Height (meters)
0	0
1	25
2	40
3	45
4	40

$+25$ $+15$ $+5$ -5

-10 -10 -10



After 3 seconds, its height begins to decrease.

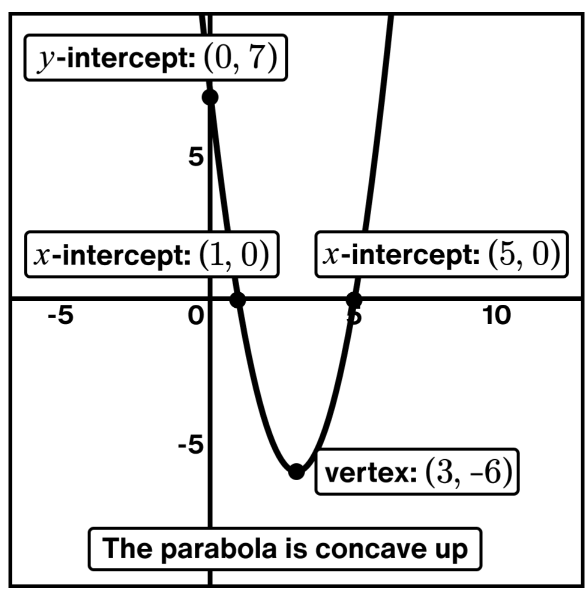
$x = 3$ is called the *line of symmetry* because if you fold the graph along this line, you get two identical halves.

Lessons 6–7

The graph of a quadratic function is called a *parabola.*

Here are some key features of parabolas:

- The x-*intercepts* have a y-value of 0.
- The y-*intercept* has an x-value of 0.
- The *vertex* is the turning point of a parabola, and also its minimum or maximum.
- A parabola is *concave up* if it opens upward and has a minimum.
- A parabola is *concave down* if it opens downward and has a maximum.



Standard Form and Factored Form (Lessons 8–13)

Lessons 8–9

Quadratic equations come in many forms.

Standard Form	Factored Form
Examples:	Examples:
$f(x) = ax^2 + bx + c$	$f(x) = a(x - m)(x - n)$
$y = 2x^2 + 5x + 3$	$y = (5x + 2)(3x - 1)$
$h(x) = x^2 + 3x$	$g(x) = x(x + 10)$

Lesson 10

Here is the same function written in factored form and standard form.

Factored form: $m(x) = (x + 3)(2x - 2)$

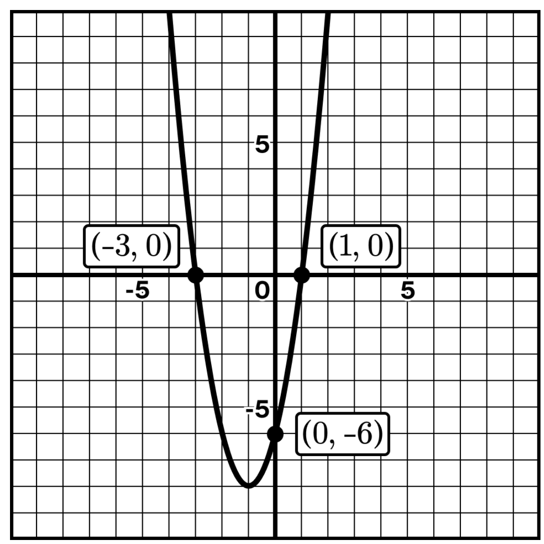
Factored form can tell you about the x-intercepts.

−3 and 1 each make one factor of $m(x)$ equal to 0.

Standard form: $m(x) = 2x^2 + 4x - 6$

Standard form can tell you about the y-intercept.

When $x = 0$, all terms of $m(x)$ are equal to 0 except −6.



Lesson 11

In the parabola above, the x-intercepts are at $(-3,\ 0)$ and $(1,\ 0)$. The vertex is in the middle.

The middle of −3 and 1 is −1, so the x-value of the vertex is $x = -1$.

A table can help determine the y-value when $x = -1$.

x	$(x + 3)$	$(2x - 2)$	$(x + 3)(2x - 2)$
-1	2	-4	-8

The vertex of the parabola is $(-1,\ -8)$.

Lessons 12–13

Here are equations of three different parabolas.

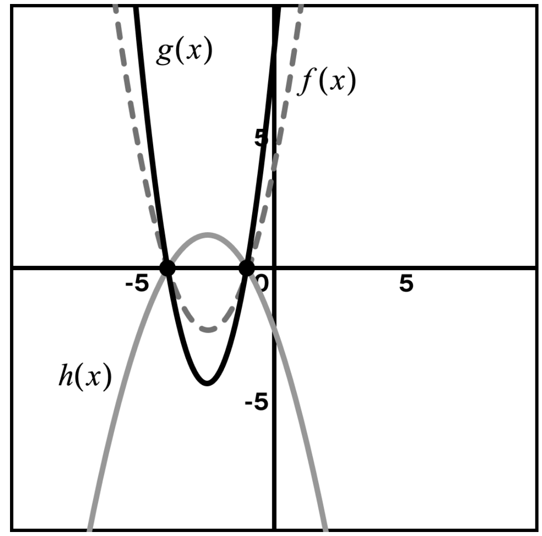
$$f(x) = (x + 4)(x + 1)$$

$$g(x) = 2(x + 4)(x + 1)$$

$$h(x) = -0.5(x + 4)(x + 1)$$

The multiplier (or a-value) in each equation is different.

This changes the vertical stretch of the parabola.



Vertex Form (Lessons 14–17)

Lessons 14–15

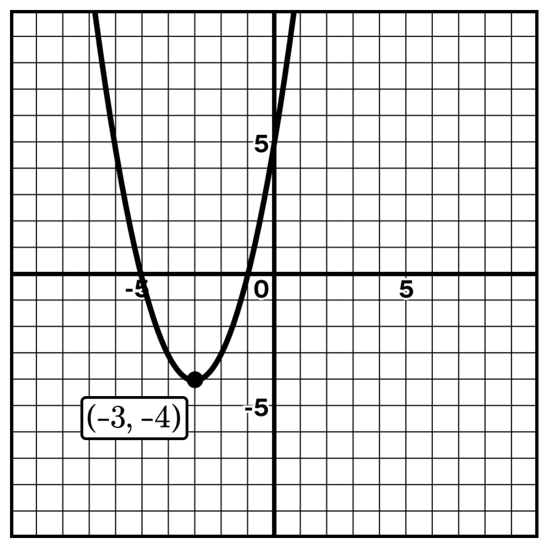
Another form of quadratic functions is *vertex form.*

Example: $f(x) = (x + 3)^2 - 4$

The vertex and minimum of $f(x)$ is $(-3, -4)$.

All other x-values will create a larger y-value.

x	$x + 3$	$(x + 3)^2$	$(x + 3)^2 - 4$
-3	0	0	-4



Lessons 16–17

Now we know three different forms of quadratic equations.

These three functions all represent the parabola above.

Standard Form	Factored Form	Vertex Form
$f(x) = x^2 + 6x + 5$	$f(x) = (x + 1)(x + 5)$	$f(x) = (x + 3)^2 - 4$

Try This at Home

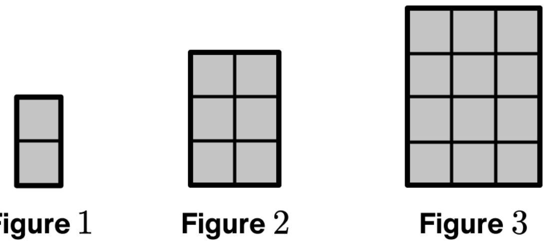
Introduction to Quadratic Functions (Lessons 1–7)

Try After Lesson 3

Here is a visual pattern.

1.1 Draw or describe what figure 4 and figure 10 will look like.

1.2 Is the relationship between figure number and number of tiles linear, quadratic, exponential, or something else? Explain how you know.



Try After Lesson 5

The height of a falling object over time is a quadratic relationship.

This table shows the height of a ball dropped from the top of the Leaning Tower of Pisa.

2.1 What was the initial height of the ball?

2.2 Predict the height of the ball after 3 seconds.

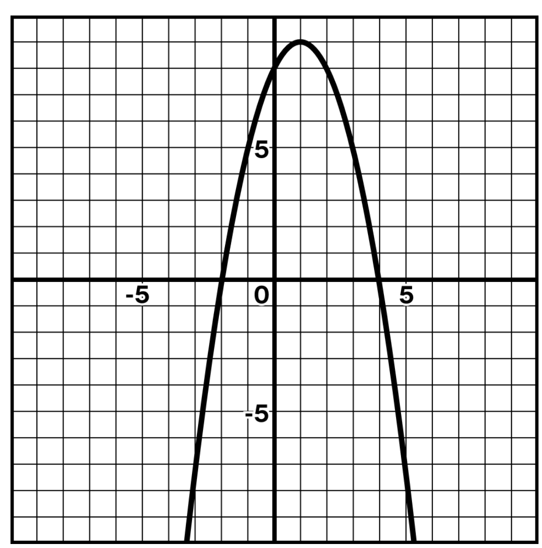
2.3 Will the ball hit the ground before or after 4 seconds? Explain how you know.

Time (sec.)	Height (ft.)
0	190
1	174
2	126

Try After Lesson 7

3. Identify the key features of this parabola.

Vertex	
x-intercept	$(-2,\ 0)$
x-intercept	
y-intercept	
Line of symmetry	$x =$
Concave up/down	



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Try After Lesson 10

Here is the same function written in two forms.

Factored form: $g(x) = (2x - 3)(x + 2)$ **Standard form:** $g(x) = 2x^2 + x - 6$

4.1 Use either form to determine $g(2)$ and $g(-3)$.

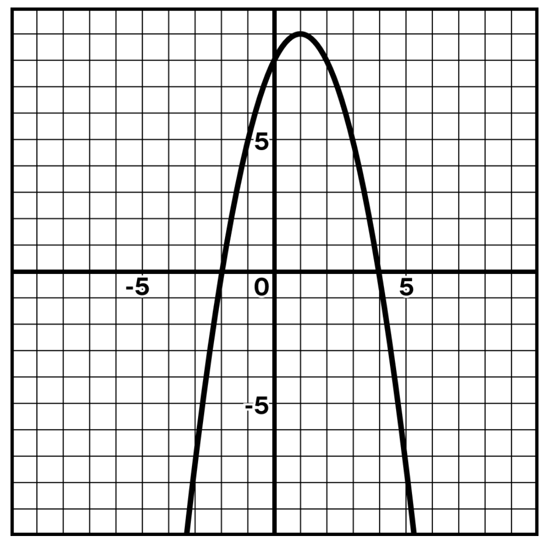
4.2 Use either form to determine the x-intercept(s) and y-intercept of g.

Try After Lesson 12

5.1 Write an equation to match the graph on the right.

5.2 On the same graph, sketch $y = (x + 1)(2x - 6)$.

Include the x-intercept(s) and vertex.



Try After Lesson 15

6.1 Here is a function: $f(x) = 2(x - 7)^2 - 1$. Determine the vertex of f.

6.2 Write an equation of a parabola that has a vertex at $(-4, 5)$.
Use graphing technology to check your equation.

Try After Lesson 17

7. Select **all** the equations that represent this graph.

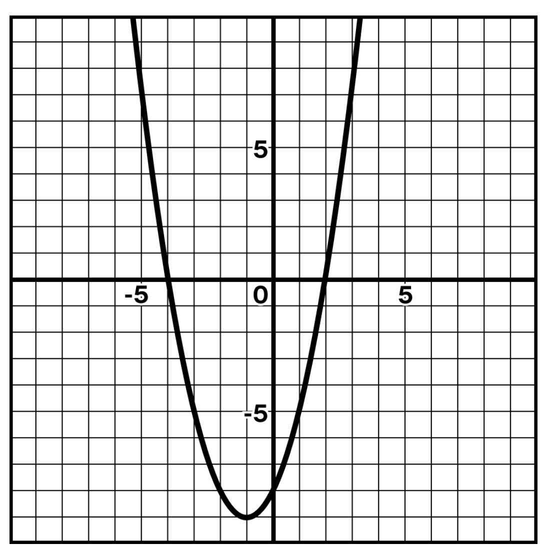
✓ $y = (x - 1)^2 - 9$

✓ $y = (x + 1)^2 - 9$

✓ $y = x^2 + 2x - 8$

✓ $y = -(x - 2)(x + 4)$

✓ $y = (x - 2)(x + 4)$



Unit 8 Summary

Prior Learning	Algebra 1, Unit 8	Future Learning
Math 6–8, Algebra 1, Unit 2 and Unit 5 • Equivalent expressions • Operations with numbers • Solving equations • Evaluating expressions	• Multiplying and factoring to write equivalent quadratic expressions • Solving quadratic equations using the zero-product property, completing the square, and the quadratic formula • Connecting solving to graphing	Algebra 2 • Solving for imaginary solutions • Solving polynomials of higher degrees • Graphing polynomial functions

Multiplying and Factoring (Lessons 1–6)

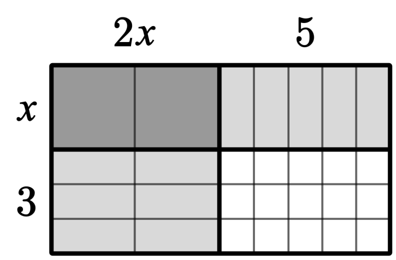
Lessons 1–2

An area model can help us rewrite a factored-form quadratic expression, which looks like $a(x - m)(x - n)$, as a standard-form quadratic expression, which looks like $ax^2 + bx + c$.

In an area model, the side lengths represent the factors in a factored-form expression, while the inside tiles add up to the expression in standard form.

In this area model example, there are four sections that make up the total area.

Factored Form	Standard Form
$(x + 3)(2x + 5)$	$2x^2 + 11x + 15$



- $(x)(2x) = 2x^2$
- $(x)(5) = 5x$
- $(3)(2x) = 6x$
- $(3)(5) = 15$

By combining the like terms, you can rewrite the expression in standard form.

In this example, $2x^2 + 5x + 6x + 15$ combines to make $2x^2 + 11x + 15$.

Lessons 3–4

An area model can also help rewrite a standard-form quadratic expression in factored form.

Let's look at how to use an area model to factor $3x^2 + 14x + 15$.

1. Write ax^2 in the top-left corner and c in the bottom-right corner. In this example, a is 3 and c is 15.

	$3x$	
x	$3x^2$	
		15

2. Try different values for the side lengths until the inside tiles add up to the expression in standard form.

   In this example, $3x + 3$ and $x + 5$ are *not* the correct side lengths, because they would make the total area $3x^2 + 18x + 15$, not $3x^2 + 14x + 15$.
   The correct side lengths are $3x + 5$ and $x + 3$.

	$3x$	$\cancel{3}$
x	$3x^2$	$\cancel{3x}$
$\cancel{5}$	$\cancel{15x}$	15

3. Combine the side lengths to rewrite the expression in factored form. In this example, the expression is $(3x + 5)(x + 3)$.

	$3x$	5
x	$3x^2$	$5x$
3	$9x$	15

Lessons 5–6

The *zero-product property* says that if the product of two or more factors is 0, then at least one of the factors is 0.

To determine the x**-intercepts** of quadratic functions using the zero-product property, first factor the quadratic function and then set each factor equal to 0.

For example, since $f(x)$ is already in factored form, set $x - 3 = 0$ and $x + 9 = 0$. That means the x-intercepts are $(3, 0)$ and $(-9, 0)$.

A
$$f(x) = (x - 3)(x + 9)$$

To determine the **solutions** of quadratic equations using the zero-product property, set the equation equal to 0, factor it, and then set each factor equal to 0.

For example, you can subtract 33 from both sides of equation B to get $x^2 + 8x - 33 = 0$. This can be factored as $(x + 11)(x - 3) = 0$ to get $x = 3$ and $x = -11$.

B
$$x^2 + 8x = 33$$

Solving Equations and Completing the Square (Lessons 7–12)

Lesson 7
The structure of a quadratic equation can help determine the number of solutions.

If $(x + \_\_)^2$ equals a negative number, there are no solutions.

For example, there are no values for x that would make $(x + 0)^2$ equal -25, since a number multiplied by itself is always positive.

No Solution

$$x^2 = -25$$

If $(x + \_\_)^2$ equals 0, there is only one solution.

For example, if $(x - 3)(x - 3) = 0$, then $x - 3 = 0$ must be true because of the zero-product property. That means the only possible solution is $x = 3$.

One Solution

$$(x - 3)(x - 3) = 0$$

If $(x + \_\_)^2$ equals a positive number, there are two solutions.

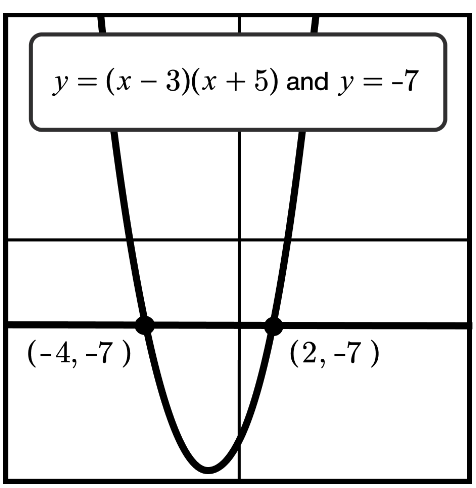
For example, $(x + 4)^2 = 1$ has two solutions because there are two numbers, 1 and -1, that can be squared to get 1. We can solve for $(x + 4) = 1$ and $(x + 4) = -1$ to get both solutions.

Two Solutions

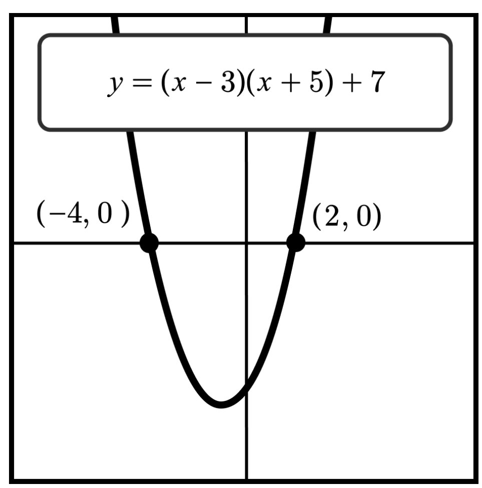
$$(x + 4)^2 = 1$$

Lesson 8
The graph of a quadratic equation can help determine the solutions.

Let's look at how a graph can help solve the equation $(x - 3)(x + 5) = -7$ to get the solutions $x = -4$ and $x = 2$.



You can graph both sides of an equation as two separate graphs, then find the x-coordinates where the graphs intersect.



You can rearrange the equation so that it equals 0, then graph the equation. The solutions will be at the x-intercepts.

Lesson 9

Quadratic equations can be solved by taking the *square root*.

You can take the square root ($\sqrt{}$) of an equation in the form $(x + \_\_\_\_)^2 = \_\_\_\_$, or rewrite an equation in that form.

When you take the square root of a number, you get two solutions, a positive number and a negative number. This can be expressed using the *plus/minus symbol* ($\pm$).

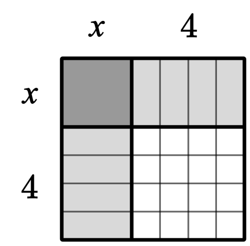
$$(x - 1)^2 = 36$$
$$x - 1 = \pm\sqrt{36}$$
$$x - 1 = \pm 6$$
$$x = 1 \pm 6$$
$$x = 7 \text{ and } x = -5$$

$$2(x + 4)^2 = 12$$
$$(x + 4)^2 = 6$$
$$x + 4 = \pm\sqrt{6}$$
$$x = -4 \pm \sqrt{6}$$

Lessons 10–12

A quadratic expression is a *perfect square* if it can be represented as something multiplied by itself.

For example, $(x + 4)^2$ and its equivalent expression, $x^2 + 8x + 16$, are both perfect squares.



Completing the square is the process of rewriting a quadratic expression or equation to include a perfect square. This can make it easier to solve certain equations.

To rewrite a quadratic equation as a perfect square, you can add a constant value to both sides.

1. In this example, we can rewrite $x^2 + 12x$ as a perfect square by halving 12 to get 6, then squaring 6 to get 36.
2. Adding 36 to both sides results in a equivalent equation that can be rewritten to look like $(x + \_\_\_)^2 = \_\_\_$.
3. Now we can solve by taking the square root.

$$x^2 + 12x = 24$$
$$x^2 + 12x + 36 = 24 + 36$$
$$(x+6)^2 = 60$$
$$x + 6 = \pm\sqrt{60}$$
$$x = 6 \pm\sqrt{60}$$

Completing the square can also be used to rewrite functions in vertex form.

The vertex form reveals the vertex of the quadratic, which in this example is $(3, 8)$.

$$f(x) = x^2 - 6x + 17$$
$$(x^2 - 6x + 9) - 9 + 17$$
$$(x - 3)^2 - 9 + 17$$
$$(x - 3)^2 + 8$$

The Quadratic Formula and More (Lessons 13–17)

Lessons 13–15

The solutions to any quadratic equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

This formula is known as the quadratic formula.

Here is an example of how the quadratic formula can be used to solve the equation $x^2 - 4x - 12 = 0$.

$$x = \frac{-(-4) \pm \sqrt{(-4)^2 - 4(1)(-12)}}{2(1)}$$

$$x = \frac{4 \pm \sqrt{64}}{2}$$

$$x = \frac{4 \pm 8}{2}$$

$x = -2$ and $x = 6$

You can write your own quadratic equations to represent situations and solve problems about them.

For example, the function $h(t) = -1.5t^2 + 12t + 8$ represents the height, in meters, of a stomp rocket t seconds after it has been launched.

Since we know that the rocket will touch the ground when the height is 0, we can set $h(t)$ equal to 0 to write the equation $0 = -1.5t^2 + 12t + 8$. Solving this equation will let us know when the rocket will touch the ground.

Using the quadratic equation gives us the solutions $x \approx -0.619$ and $x \approx 8.619$. Since it is not possible for time to be negative in this situation, the only possible solution is that the rocket touches the ground at 8.619 seconds.

Lessons 16–17

A *system of equations* contains two or more equations. These can include linear and quadratic equations.

The solutions of a system of equations are the points of intersection on the graph.

1. When solving a system, you can use substitution or elimination to create an equation with only one variable.
2. Then you can solve this equation by factoring and using the zero-product property, solving with square roots, completing the square, or using the quadratic formula.
3. After determining the value of one variable, substitute that value into one of the equations to determine the value of the other variable.

$y = 2x - 7 \qquad y = x^2 - 5x + 3$

$2x - 7 = x^2 - 5x + 3$

$0 = x^2 - 7x + 10$

$0 = (x - 5)(x - 2)$

$\boxed{x = 5} \qquad \boxed{x = 2}$

$y = 2(5) - 7 \qquad y = 2(2) - 7$

$\boxed{y = 3} \qquad \boxed{y = -3}$

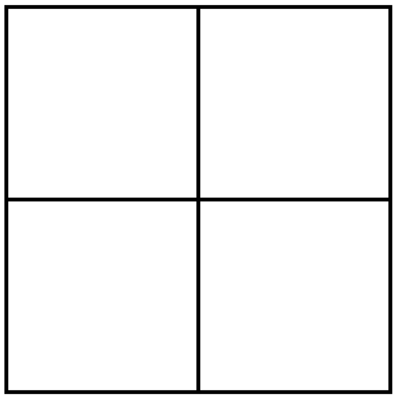
Points of intersection:

$\boxed{(5, 3)} \qquad \boxed{(2, -3)}$

Try This at Home

Try After Lesson 2

1. Rewrite $(2x - 5)(x + 3)$ in standard form. Use the diagram if it helps with your thinking.



Standard form: ______________________

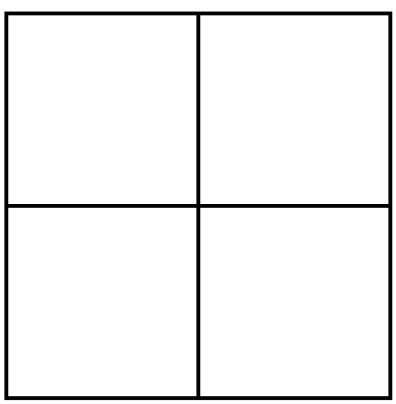
2. Identify the a-, b-, and c-values for the expression $5x^2 + 9x - 15$.

$a =$ ____ $b =$ ____ $c =$ ____

Try After Lesson 4

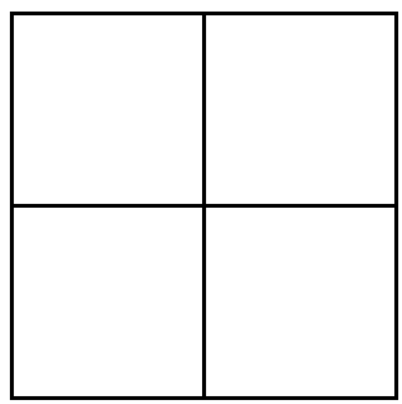
Factor the expressions. Use the diagrams if they help with your thinking.

3.1 $x^2 + 10x + 21$



Factored form: ______________________

3.2 $2x^2 + 3x - 27$



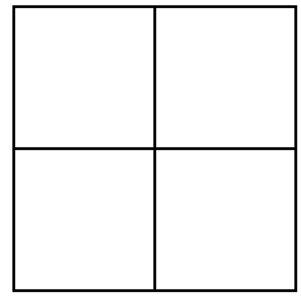
Factored form: ______________________

Try After Lesson 6

4. Determine the x-intercepts of the quadratic function $h(x) = (x - 1)(x + 6)$.

x-intercepts: ______________________

5. Determine the solutions to the quadratic equation $x^2 - 6x = 40$. Use the diagram if it helps with your thinking.



Solution: ______________________

Try After Lesson 7

Circle whether each equation has no solutions, one solution, or two solutions. Solve the equation if there are solutions.

6.1 $x^2 + 10 = 110$

No solutions One solution Two solutions

$x =$ ________________

6.2 $(x - 8)^2 = 0$

No solutions One solution Two solutions

$x =$ ________________

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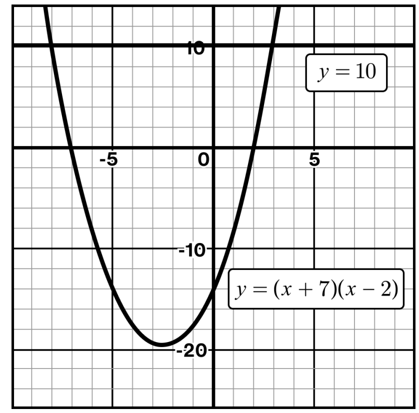
Try After Lesson 8

Here is the graph that Peter made to solve $(x + 7)(x - 2) = 10$.

7. Determine the solution(s) to this equation.



Try After Lesson 9

Determine the exact solution(s) to each equation.

8.1 $(x + 3)^2 = 7$

$x =$ ____________________

8.2 $10(x - 5)^2 = 10$

$x =$ ____________________

Try After Lesson 12

9. Fill in the blank to make $x^2 - 14x +$ _____ a perfect square.

10. Solve $x^2 + 10x = 2$ by completing the square.

$x =$ ________________

11. Rewrite $x^2 + 8x + 10$ in vertex form.

Try After Lesson 15

Use the quadratic formula to solve the equation.

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

12. $0 = x^2 + 6x + 3$ $\quad a =$ ____ $b =$ ____ $c =$ ____

$$x = \frac{\quad \pm \sqrt{\qquad\qquad}}{\qquad\qquad}$$

The function $h(t) = -2.5t^2 + 6t - 8$ represents the height, in meters, of a stomp rocket t seconds after it has been launched.

13.1 Write an equation that can be solved to determine when the rocket will return to its original height of -8 meters.

13.2 How long will it take for the rocket to return to its original height of -8 meters?

Try After Lesson 17

14. Solve this system of equations:

$y = x^2 - 3x$

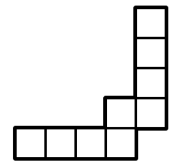
$y = 2x - 6$

Solutions

Solutions:

1.1 See the sketch on the right.

Figure 4



1.2 9 tiles

1.3 21 tiles

1.4 $1 + 2n$ tiles (or equivalent)

2.1 $\$40$. The point $(0, 40)$ means that when you have taken 0 rides, the card has $\$40$ on it.

2.2 10 rides

Explanations vary. If I want to know when there is $\$15$ left on the card, look for where on the graph the y-value is 15. This happens at the point $(10, 15)$, which means that there is $\$15$ on the card when you have taken 10 rides.

3.1 Neither

3.2 Exponential

3.3 Linear

4.1 $b = 10 \cdot 8^t$ (or equivalent)

t is the number of hours, and b is the number of bacteria cells.

4.2 After 6 hours, there will be $2\ 621\ 440$ bacteria cells.

$b = 10 \cdot 8^6$

$b = 10 \cdot 262\ 144$

$b = 2\ 621\ 440$

4.3 *Responses vary.*

5.1 *Responses vary.* If the relationship were linear, there would be a constant difference. $60 - 30 = 30$ and $30 - 15 = 15$, so there is no constant difference. There is a constant ratio because $\frac{30}{60} = \frac{1}{2}$ and $\frac{15}{30} = \frac{1}{2}$.

5.2 $y = 60 \cdot \left(\frac{1}{2}\right)^x$ (or equivalent)

Solutions:

1. $x = 2$. *See one example to the right.*

$$4(5-3x)=2(x-4)$$
$$20-12x=2x-8$$
$$+12x \quad +12x$$
$$20=14x-8$$
$$+8 \quad +8$$
$$\frac{28}{14}=\frac{14x}{14}$$
$$2=x$$

2.1 *Responses vary.*
One thing Emma did well . . .

- Tried to undo the $\frac{1}{2}$ as the first step.
- Added 2 to both sides to make $-20 + 20 = 0$.
- Subtract x from both sides to make $x - x = 0$.

2.2 *Responses vary.* One thing Emma did incorrectly was forget to divide the $3x$ term by $\frac{1}{2}$.

3.1 *Responses vary.*

Step 1: This is the original problem.

Step 2: Multiply 5 by $2 + 4h$.

Step 3: Add $20h$ and $2h$.

Step 4: Subtract 10 from both sides to make $10 - 10 = 0$.

Step 5: Divide by 22 to make $22 \div 22 = 1$.

3.2 See the work on the right.

$$5(2+4(-2))+2(-2) \stackrel{?}{=} -34$$
$$5(2+(-8)) + (-4) \stackrel{?}{=} -34$$
$$5(-6) + (-4) \stackrel{?}{=} -34$$
$$-30 + (-4) \stackrel{?}{=} -34$$
$$-34 = -34 \checkmark$$

4. Infinite solutions

Explanations vary. The equation has an infinite number of solutions because after distributing the multiplication by 2 and combining like terms, the equation becomes $16x = 16x$. Because the left and right sides of the equation are identical, any value of x will make the equation true.

5. ✓ $5a + 3c = 153$

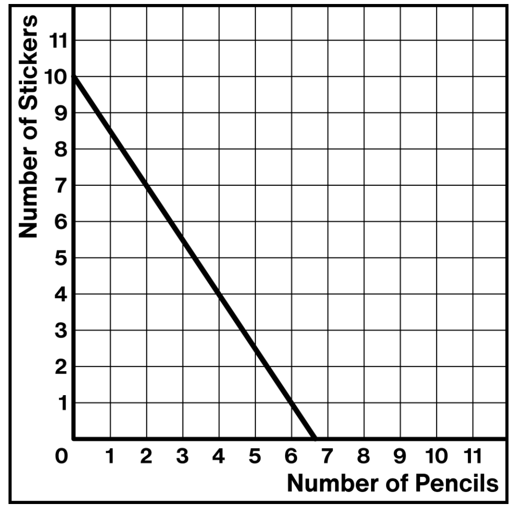
✓ $c = \frac{153-5a}{3}$

6.1 $2s + 3p = 20$

6.2 $s = \frac{20-3p}{2}$

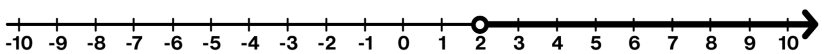
6.3 If Neel gets 2 pencils, how many stickers can he get with his remaining tickets?

6.4 See the graph on the right.



7.1 $x > 2$

7.2



8.1 No, they will not reach their goal.

Explanations vary. The inequality $4(8\,020) + 3(12\,460) > 70\,000$ is false, so they will not reach their goal.

8.2 *Equations vary.* $4(8020) + 3s > 70000$ (or equivalent)

9.1 A. $3x + 4y < 12$

9.2 If the boundary line were solid, we would want to include solutions where $3x + 4y = 12$. Therefore, the inequality would be $3x + 4y \leq 12$.

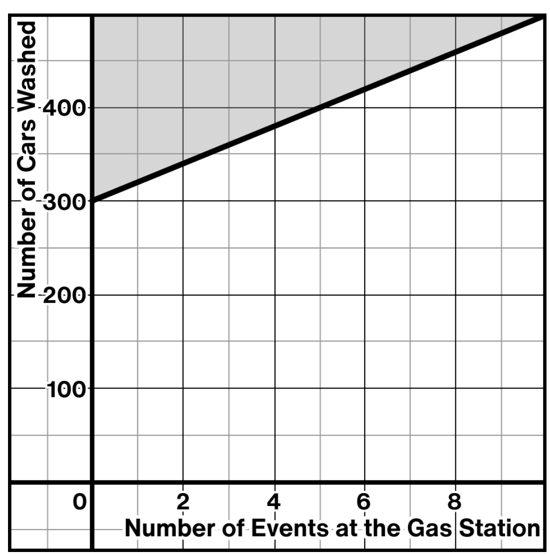
10.1 No.

10.2 *Responses vary.* The $-300g$ represents the number of dollars that the team has to pay the gas station per event. The $15c$ represents the money the team will collect from washing c cars. The $4\,500$ represents the money the team is trying to raise.

10.3 See the line in the graph on the right.

10.4 $-300g + 15c \geq 4500$

10.5 See the graph on the right.



10.6 If the basketball team decides to host 4 car wash events, how many cars will they need to wash to raise enough money for their uniforms?

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Solutions:

1.1 True 1.2 Cannot be determined 1.3 True 1.4 False

2.

Min.	Q1	Median	Q3	Max.
1	5	11	14	15

3.1 True 3.2 False

4.1 Skewed 4.2 Symmetric 4.3 Bimodal 4.4 Bell-shaped

5.

Mean	Median	Which is larger?
$A=[12, 12, 13, 13, 13, 15, 17, 18, 19]$ $\text{mean}(A)$ ≈ 14.67	$A=[12, 12, 13, 13, 13, 15, 17, 18, 19]$ $\text{median}(A)$ $=13$	Mean

6. Skewed

7.1

Mean	Standard Deviation
$A=[31, 25, 28, 34, 31, 29, 30]$ $\text{mean}(A)$ ≈ 29.71	$A=[31, 25, 28, 34, 31, 29, 30]$ $\text{stdevp}(A)$ ≈ 2.6

7.2

Mean	Standard Deviation
$B=[36, 36, 41, 40, 43, 41, 34]$ $\text{mean}(B)$ ≈ 38.71	$B=[36, 36, 41, 40, 43, 41, 34]$ $\text{stdevp}(B)$ ≈ 3.1

7.3 *Responses vary.* I think Traveler A has the most consistent commute time because their standard deviation is smaller.

7.4 *Responses vary.* I think Traveler B has the longer commute time because their mean travel time is longer.

8.1

Q1	Q3	IQR	Median
5	14	9	11

8.2

Q1	Q3	IQR	Median
7	13	6	9

9. *Explanations vary.* Player B was more consistent because their IQR was smaller.

10.

Player	Mean	Standard Deviation	Median	IQR	Outliers
Pitcher A	8.43	4.56	8	6	none
Pitcher B	13.44	4.52	15	4	2

11. *Responses vary*. Pitcher B had more strikeouts in general because the mean and median of their strikeouts was higher. Pitcher B is also more consistent because their IQR is lower.

12. The r-value is 0.233. This means there is a positive and weak relationship between wins in 2019 and payroll in millions of dollars.

13.1 The line fits the data not well. *Explanations vary*. The residual plot has points that are very far from the x-axis. Also, the residual values go from positive to negative, and are not random.

13.2 The line fits the data well. *Explanations vary.* The residual plot has points that are close to the x-axis and has random points above and below the x-axis.

14. $y = -2\,270.38(8) + 26\,886.7$
$y = 8\,723.66$
The predicted price will be $\$8\,723.66$

15. *Explanations vary.* Yes. I believe that the age of the car causes the price to go down because an older car is more likely to have more mileage or other mechanical issues.

Unit A1.4, Family Resource

Solutions:

1. D

2.1 $r(2) = 175$

2.2 *Responses vary.* It costs $\$275$ to rent a car for 4 days.

3.1
- $f(-4) = 0$
- $f(-2) = -3$
- $f(0) = -1$
- $f(4) = 1$

3.2 ✓ $f(-4) = f(2)$ ✓ $f(-4) > f(0)$ ✓ $f(6) < f(-6)$

4.1 positive, decreasing

4.2 positive, increasing

5.1 $(-5, 3)$

5.2 $(-2, -4)$

6. 40 miles per hour

7.1 ✓ $f(10) > g(10)$

✓ $f(17) = g(17)$

✓ $g(5) = 1$

7.2 *Possible correct solutions.*
$t = 0$
$t = 4$
$t = 12$
$t = 17$

8.1 ✓ 1
✓ 4

8.2 Whole numbers from 1 to 4.

9.1 Domain	9.2 Range
$-3 \le x \le 4$	$-2 \le f(x) \le 5$

10.1 Domain	10.2 Range
$-5 \le x \le 4$	$-3 \le g(x) \le 7$

11. *Responses vary.*

Increasing interval: Temperature in the freezer is rising.

Maximum: Freezer reaches its highest temperature.

Domain: The shop was open for 12 hours.

Negative interval: Freezer temperature was below 0°F.

12.1 $h(2) = 1$

12.2 $h(8) = 3$

13.1 $a(1) = 1$

13.2 $a(3) = 2$

14.1 $a(-3) = 1$

14.2 $a(-13) = 11$

Responses vary. At the point $(-3, 1)$.

15. *Responses vary.*

desmos

Unit A1.5, Family Resource

Solutions:

1.1 *Responses vary.* Subtracting or either. $p = 2,\ q = 4$

1.2 *Responses vary.* Adding or either. $x = 10,\ y = 2$

1.3 *Responses vary.* Either. $x = 6,\ y = 2.5$

1.4 See solutions above.

2.1 $x = -2,\ y = 5$

2.2 $x = 6,\ y = 0$

3.1 $x = 4,\ y = -3$

3.2 $x = -2.5,\ y = -4$

4.1 Infinite solutions

4.2 One solution

4.3 No solutions

5.1 C. $s + h = 40$

$18s + 14h = 700$

5.2 The knitting club sold 35 scarves and 5 hats.

6.1 *Responses vary.* I would solve this system of equations by elimination because both equations are in standard form, and I can add the two equations to eliminate y,

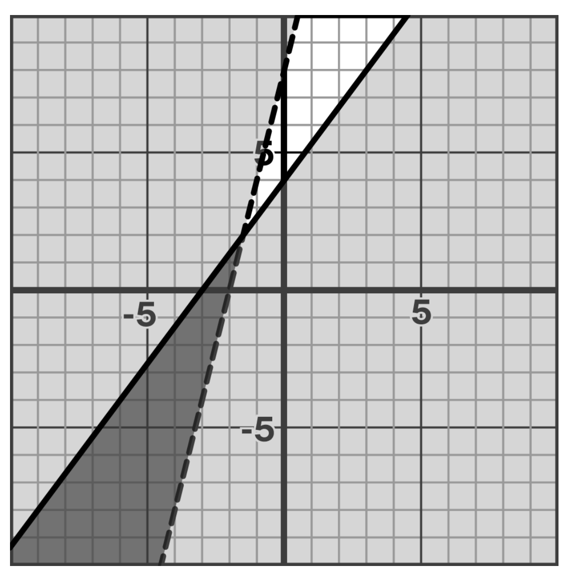
6.2 *Responses vary.* I would solve this system of equations by substitution because I can substitute $3x$ for y and get $3x = 9x - 30$.

6.3 *Responses vary.* I would solve this system either way. I could multiply the second equation by −2 and add the equations or subtract $5y$ from both sides and substitute $-5y + 40$ for x.

7.1 Yes

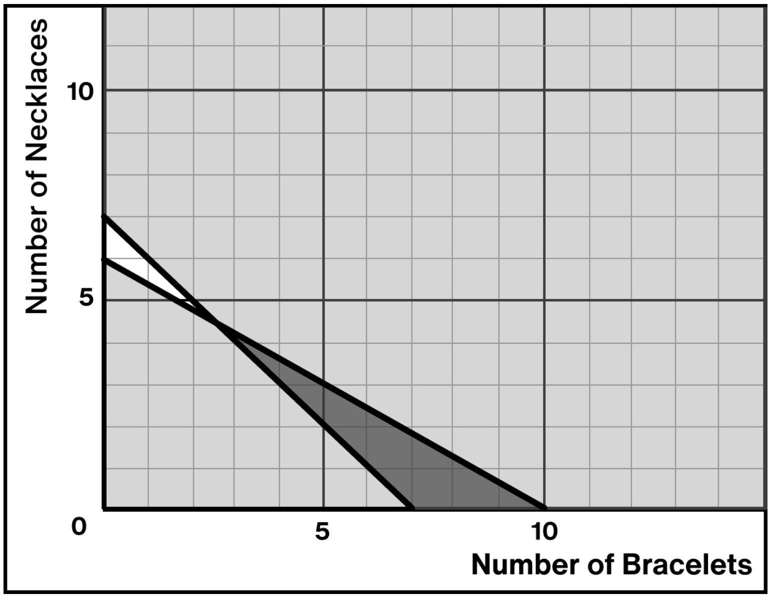
7.2 No

8. See the graph on the right.



9.1 $b + n \geq 7$

9.2



9.3 *Responses vary.* 5 bracelets and 3 necklaces, 6 bracelets and 1 necklace.

9.4 *Responses vary.* The point $(2, 5)$ is on the solid boundary line of one inequality and not within the solution region at all for the other. This means the combination only meets one constraint. It is the right amount of gift items, but too expensive.

Solutions:

1.1 Exponential 1.2 Something else 1.3 Linear

2.1 Adah: $\$112.50$, Jamir: $\$100.79$

2.2 Adah: $\$120$, Jamir: $\$106.93$

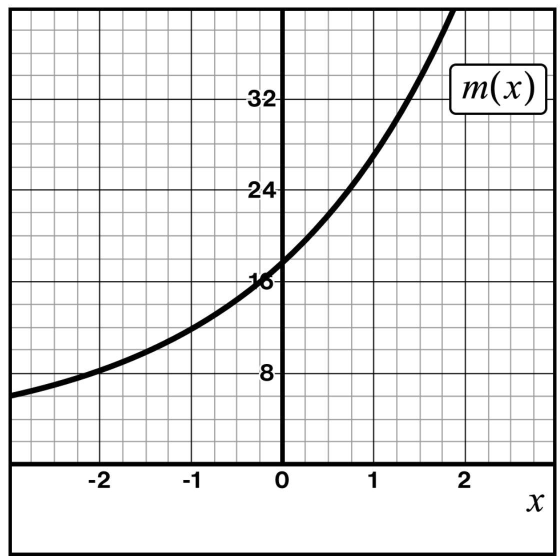
2.3 Adah: 6.67 years, Jamir: 9.73 years

2.4 Adah: 33.33 years, Jamir: 33.18 years

3.1

x	$m(x)$
-2	8
-1	12
0	18
1	27

3.2



3.3 *Responses vary.* $m(-1)$ represents the total subscriber count in the year 2019, which makes sense.

4. $y = 2(\frac{5}{2})^x$ (or equivalent)

5. **D.** $y = 500(1.04)^x$

6.1 $b(x) = 480(1 - 0.08)^x$ (or equivalent)

6.2 ≈ 316 minutes

7. $100(1.015^{12})^5$, $100(1.015)^{60}$, $100(1.19562)^5$

8. Option B. *Responses vary.*

   Option A's balance would be $900(1 + \frac{0.02}{365})^{4 \cdot 365} \approx \$974.96$.

   Option B's balance would be $900(1 + \frac{0.03}{2})^{4 \cdot 2} \approx \$1\,013.84$.

   So Option B would have a greater balance.

9.1 *Responses vary*. The exponential model fits better because the data follows the shape of an exponential curve more closely than it follow a line.

9.2 Linear model prediction: $\sim 100\,000$ people

9.3 Exponential model prediction: $\sim 50\,000$ people

Solutions:

1.1 See drawings on the right.

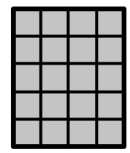


Figure 4

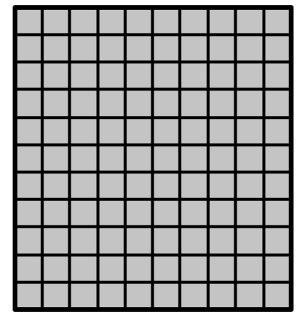


Figure 10

1.2 Quadratic.
Explanations vary. Each figure is made up of a square that is n by n plus n tile(s) on top. Quadratic relationships involve squaring a number. Also, there is a constant second difference of 4.

2.1 190 feet

2.2 46 feet

2.3 Before 4 seconds.
Explanations vary. $190 - 174 = 16$ and $174 - 126 = 48$. Therefore, the constant second difference is 32 feet. This means that between 2 and 3 seconds, the ball will drop $48 + 32 = 80$ feet. $126 - 80 = 46$, so the height after 3 seconds is 46 feet. Between 3 and 4 seconds, the ball would drop $80 + 32 = 112$ feet. Since the ball is only at 46 feet, it will hit the ground before 4 seconds are up.

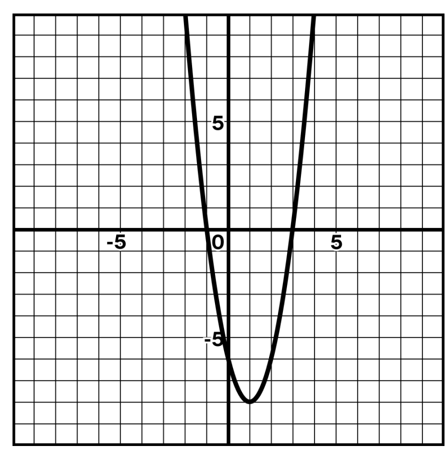
3. **Vertex:** $(1, 9)$, x**-intercept:** $(4, 0)$, y**-intercept:** $(0, 8)$, **Line of symmetry** $x = 1$, **Concave up/down:** Concave down

4.1 $g(2) = 4$ and $g(-3) = 9$

4.2 x**-intercepts:** $(1.5, 0)$ and $(-2, 0)$, y**-intercept:** $(0, -6)$

5.1 *Equations vary.* $y = -(x + 2)(x - 4)$ (or equivalent)

5.2 See the sketch on the right.



6.1 $(7, -1)$

6.2 *Equations vary.*
$y = 2(x + 4)^2 + 5,\ y = -3(x + 4)^2 + 5$

7. ✓ $y = (x + 1)^2 - 9$
✓ $y = x^2 + 2x - 8$
✓ $y = (x - 2)(x + 4)$

desmos

Unit A1.8, Family Resource

Solutions:

1. $2x^2 + x - 15$

2. $a = 5 \quad b = 9 \quad c = -15$

3.1 $(x + 7)(x + 3)$ **(or equivalent)**

3.2 $(2x + 9)(x - 3)$ **(or equivalent)**

4. $(-6, 0)$ and $(1, 0)$

5. $x = -4$ and $x = 10$

6.1 Two solutions, $x = -10$ and $x = 10$

6.2 One solution, $x = 8$

7. $x = -8$ and $x = 3$

8.1 $x = -3 \pm \sqrt{7}$

8.2 $x = 5 \pm 1$

9. $x^2 - 14x + 49$

10. $x^2 + 10x + 25 = 2 + 25$
$(x + 5)^2 = 27$
$x + 5 = \pm\sqrt{27}$
$x = -5 \pm \sqrt{27}$

11. $(x + 4)^2 - 6$

12. $a = 1 \quad b = 6 \quad c = 3$

$$x = \frac{-(6) \pm \sqrt{(6)^2 - 4(1)(3)}}{2(1)}$$

$$x = \frac{-6 \pm \sqrt{24}}{2}$$

13.1 $-8 = -2.5t^2 + 6t - 8$ (or equivalent)

13.2 2.4 seconds

14. $(2, -2)$ and $(3, 0)$