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NEW YORK STATE NUMERACY INITIATIVE

AN OVERVIEW OF THE NYSED NUMERACY BRIEFS

*FEATURING A PRESENTATION FROM
DR. DEBORAH LOEWENBERG BALL*

MAY 13, 2025

9:00 AM – 3:00 PM

**EASTERN SUFFOLK BOCES
INSTRUCTIONAL SUPPORT CENTER AT SEQUOYA
750 WAVERLY AVENUE, HOLTSVILLE**

AGENDA

Time	Activity	Lead
9:00 – 9:20 AM	Welcome and Opening Remarks	NYSED
9:20 – 9:30 AM	Big Picture Overview (<i>Brief Roadmap</i>)	Dr. Deborah Loewenberg Ball
9:30 – 9:50 AM	Introduction of Brief 1 (<i>The Research Base for Mathematics Teaching and Learning</i>), and Brief 2 (<i>Debunking Myths about Mathematics Teaching and Learning</i>)	Dr. Deborah Loewenberg Ball
9:50 – 10:35 AM	On-Site Discussion and Reflection on Briefs 1 and 2	On-Site Facilitators
10:35 – 10:55 AM	Introduction of Brief 3 (<i>High-Leverage Mathematical Content</i>), and Brief 4 (<i>High-Leverage Instructional Practices</i>)	Dr. Deborah Loewenberg Ball
10:55 AM – 11:45 AM	On-Site Discussion and Reflection on Briefs 3 and 4	On-Site Facilitators
11:45 AM – 12:45 PM	Lunch	
12:45 – 1:15 PM	Introduction of Brief 5 (<i>Mathematics Assessment of and for Student Learning</i>), Brief 6 (<i>The Role and Challenges of Using Representations</i>), and 7 (<i>Understanding, Using, and Modifying Curriculum Materials</i>)	Dr. Deborah Loewenberg Ball
1:15 – 2:15 PM	On-Site Discussion and Reflection on Briefs 5, 6, and 7	On-Site Facilitators
2:15 – 2:25 PM	Introduction of Brief 8 (<i>The Role of Leadership</i>)	Dr. Deborah Loewenberg Ball
2:25 – 2:45 PM	On-Site Discussion and Reflection on Brief 8	On-Site Facilitators
2:45 – 2:50 PM	Numeracy Leadership Moves	NYSED
2:50 – 3:00 PM	Next Steps and Closing Remarks	NYSED

Briefs Roadmap: Supporting Evidence-Based P–12 Mathematics Teaching Practice

Produced for the New York State Education Department
by Deborah Loewenberg Ball and TeachingWorks at the University of Michigan

Understanding Evidence-Based Practices
for Teaching Mathematics

Using Evidence-Based Practices
for Teaching Mathematics

Key Ideas and Myths

What does the research
tell us about mathematics
teaching and learning?

High-Leverage Content and Practices

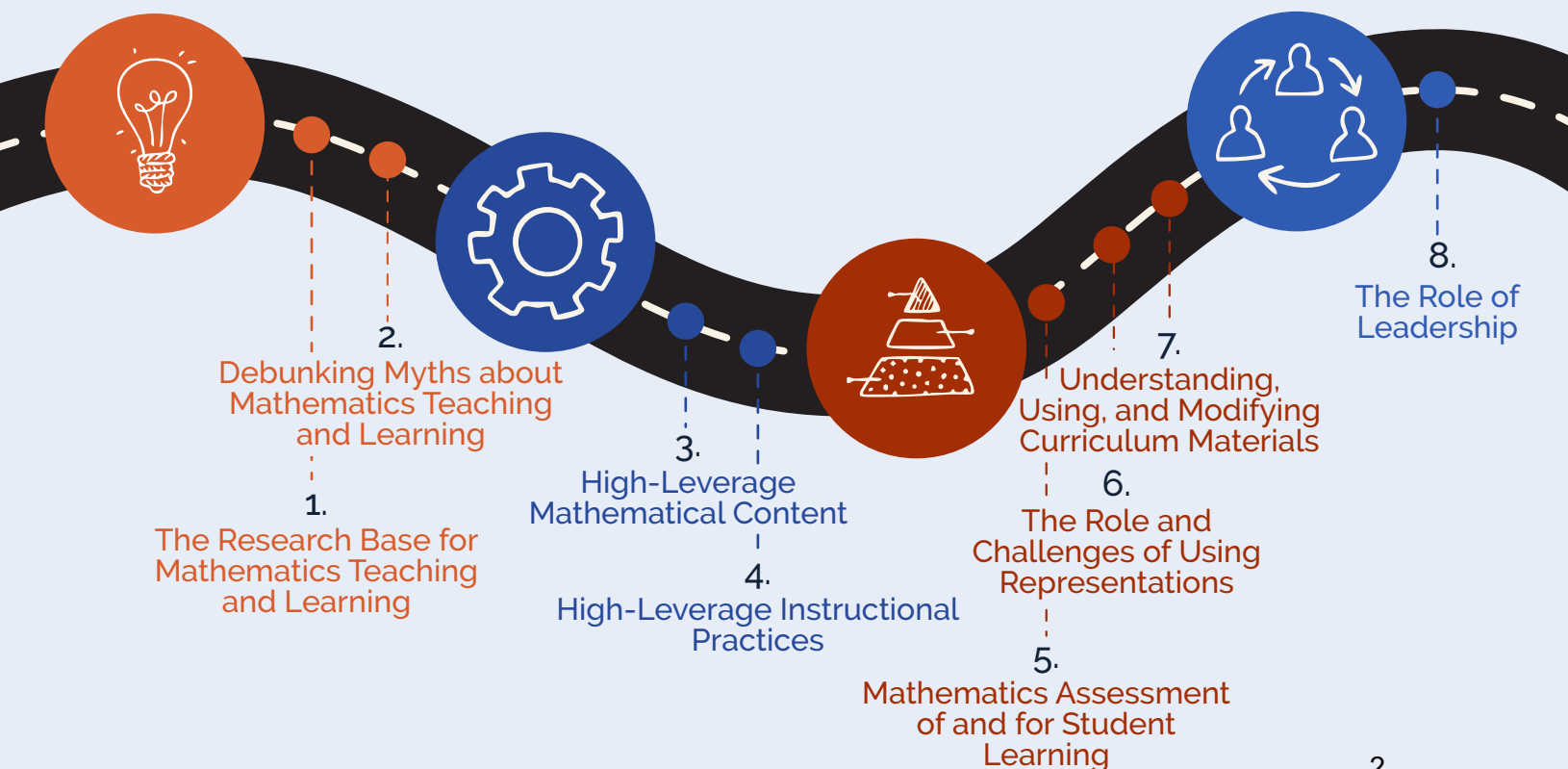
What do teachers need
to know and be able to
do to teach mathematics?

Key Evidence-Based Instructional Practices

How can teachers best use
the resources they are given
to teach mathematics?

Leading for Mathematical Proficiency

How can leaders
support
evidence-based
mathematics teaching?



Brief 1:

The Research Base for Mathematics Teaching and Learning

Supporting Evidence-Based P–12 Mathematics Teaching Practice

Produced for the New York State Education Department
by Deborah Loewenberg Ball and TeachingWorks at the University of Michigan

Background

Across decades, scholars, researchers, and practitioners from multiple fields have produced substantial peer-reviewed, evidence-based research to ground high-quality mathematics teaching and learning. No single field can provide all the knowledge needed to inform mathematics teaching and learning. Instead, researchers from various disciplines—mathematics, teacher education, curriculum design and use, sociology, ethnic studies, developmental psychology, cognitive science, education policy, and psychometrics—have all contributed to these important resources.

This research, which draws on different kinds and methods of inquiry (e.g., controlled experiments, descriptive and narrative inquiry, conceptual analysis, and ethnography), offers valuable insight and information to inform practice and policy. It includes information on particular topics and skills and explores what is involved for young people to develop proficiency with fundamental mathematical ideas, procedures, reasoning, and problem solving. It has contributed to understanding the role of representations and manipulatives, how particular instructional practices and classroom environments impact learning, and how to support growth in students' mathematical competence and positive mathematical identities. Some research also reveals taken-for-granted practices or curricular designs that harm or impede students' development and suggests strategies for replacing these with approaches that support positive growth in students' mathematical competence and identity.

Key Ideas

Key Idea #1: Skillful mathematics teaching makes use of instructional practices that develop all strands of mathematical proficiency.

In 2001, the National Research Council brought together top education researchers to form the

Mathematics Learning Study Committee, which produced a seminal report synthesizing research on K–8* mathematics teaching and learning, *Adding It Up: Helping Children Learn Mathematics*. The report lays out five interwoven mathematical proficiencies necessary for student success in mathematics:

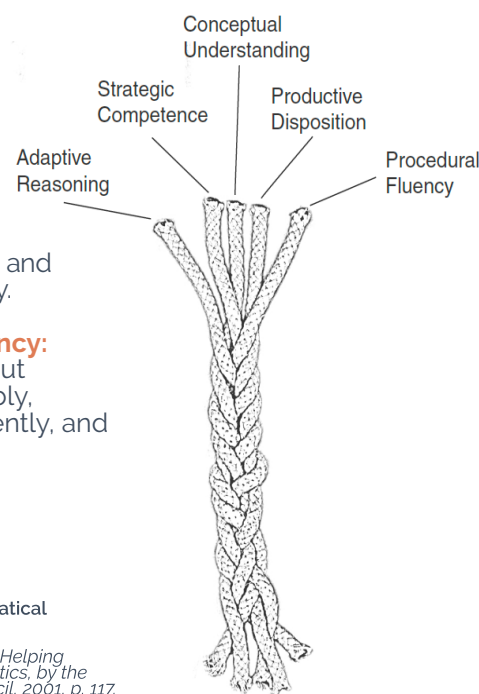
Adaptive Reasoning: Capacity for logical thought, reflection, explanation, and justification.

Strategic Competence: Ability to formulate, represent, and solve mathematical problems.

Conceptual Understanding: Comprehension of mathematical concepts, operations, and relations.

Productive Disposition: Habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy.

Procedural Fluency: Skill in carrying out procedures flexibly, accurately, efficiently, and appropriately.



The Strands of Mathematical Proficiency

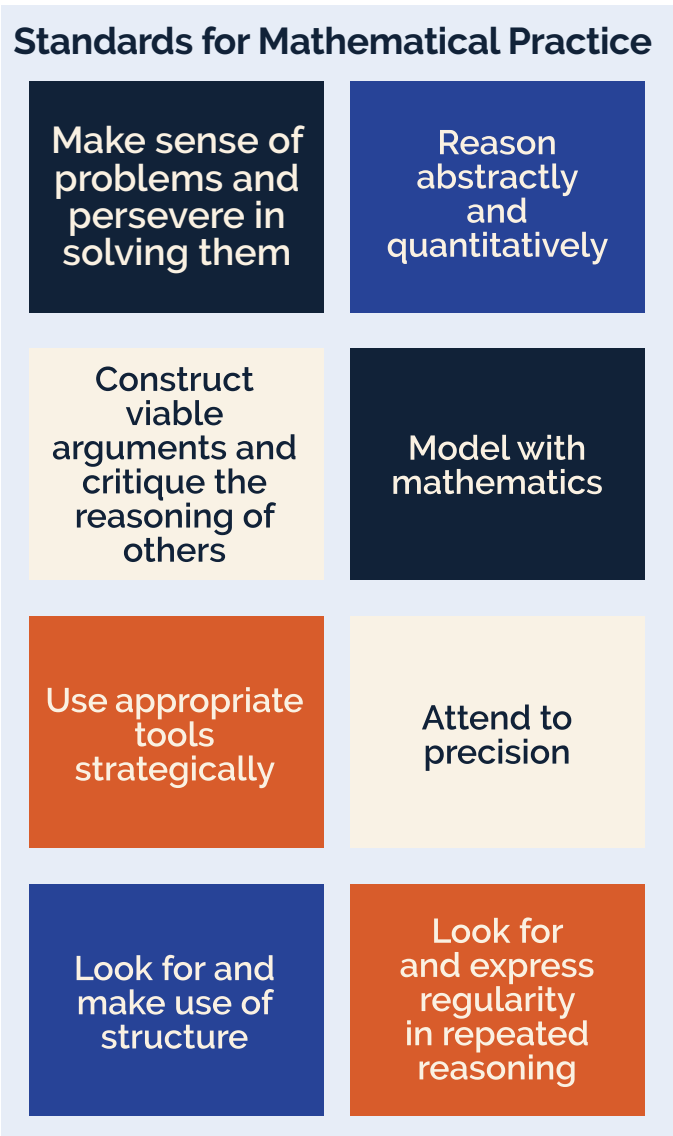
Note: From *Adding It Up: Helping Children Learn Mathematics*, by the National Research Council, 2001, p. 117.

Copyright 2001 by the National Academy of Science

*Although this report specifically addressed K–8 mathematics, extensive research also exists around secondary mathematics teaching and learning, particularly around the teaching of functions, geometry, and statistics.

For a more detailed description of the strands, along with examples of mathematical tasks that call on each strand, see "Chapter 4: The Strands of Mathematical Proficiency" from *Adding It Up: Helping Children Learn Mathematics* (2001).

Skillful teaching that supports student learning and development makes use of instructional practices that support the building of these strands of mathematical proficiency. In addition to designing instruction around curriculum materials that provide opportunity for students to build these proficiencies (see Brief #7), teachers should utilize practices that are specifically supportive of developing these strands and are linked to student learning in mathematics (e.g., eliciting student thinking, leading discussions, setting up and managing small group work, and modeling and explaining content). Each of these instructional practices has a strong research base showing improved student outcomes with consistent use (see Brief #4). These instructional practices, when enacted skillfully, make space for students to engage in ways that develop the strands of mathematical proficiency and the Standards for Mathematical Practice (New York State Education Department, 2019).



Key Idea #2: Research-based learning trajectories provide teachers with information about students' progress toward fluency and conceptual understanding.

Research on student learning of different concepts, topics, and skills has highlighted the core elements of trajectories that develop fluency with mathematics. Conceptual understanding of high-leverage mathematical ideas (see Brief #3) and structures must be intertwined with deep sensemaking and practice in their use and application. Take, for example, the fundamental system of place value numeration. Developing competence with computation includes being able to calculate, interpret the meaning of a result, determine the reasonableness of an answer, and judge the extent of precision required for a particular situation (e.g., finding the number of vehicles needed for a field trip or determining the profit margin for a product). This goal requires that students have experience with making sense of numbers of different magnitudes. They must be able to understand, reason with, and use the structure of the base ten place value system, both with concrete and pictorial representations and in expanded and compressed symbolic form, and to calculate efficiently and flexibly with combinations of single digits. A sequence of instruction must balance and connect these different understandings and skills for students to develop comprehensive command of the place value system's structure and its application across a wide range of problems and situations. If students can name that 5 is in the ten-thousands place in 254,378, or in the thousandths place in 0.345, but do not have a sense of how big or small these quantities are, then the sheer naming of places lacks the robust knowledge needed for mathematical competence. Research-informed trajectories are crucial for ensuring that the different aspects of understanding, skill, and reasoning are connected so that students' competence has the depth needed for sustained proficiency.

Key Idea #3: Research on mathematics teaching and learning highlights the importance of culturally responsive and sustaining learning environments.

Studies over the last 30 years make plain the importance of attending to and using students' cultural and community knowledge to connect their mathematical development meaningfully with resources they have (Paris & Alim, 2017; Zavala & Aguirre, 2024). These resources are in experiences they have in their families and communities, in the languages they speak, and in their common practices. Three interlocking domains of experience and resources are vital to authentic connections.

Domains of Experience and Resources Vital to Authentic Connections



Content

Cultural knowledge, activities, practices, histories, people



Practices

Discourse patterns, ways of knowing what there is to know, values



Language

Multilingualism, translanguaging, mathematical assets of different languages

Content

The domain labeled “content” supplies resources for situating and understanding the areas in which mathematics is used and the kind of mathematical practice used and known across cultural and community contexts. For example, learning about the highly developed base 20 place value system used in Indigenous mathematics allows students to connect ways of counting and numerating used by elders in their community. Students also learn to appreciate what a place value system of numeration does and that there are reasons to structure such systems based on different needs. It is also crucial to broaden students’ encounters with who makes and uses mathematical knowledge by systematically including members of non-dominant groups. One example of this is the series *Journeys of Black Mathematicians*. Exposing students to mathematical activity and thinking engaged in different cultures may highlight that there is mathematical knowledge that is both similar to and distinct from the Western canon. There are many important reasons to broaden the sources and resources for what and who students learn to label as “mathematical,” including for members of historically marginalized groups to see people who look like them and to honor and preserve knowledge too often ignored, unknown, or treated as “different” or “exotic.” The use of broader cultural and community knowledge enables all students to encounter and learn mathematics as a wide domain of human thought, inquiry, and development.

Practices

The domain labeled “practices” highlights the ways of interacting and knowing—the discourse patterns and what is valued—that provide guidance for the construction of inclusive learning environments. So much of the way we focus on mathematical topics in school neglects the central role of mathematical reasoning and representing. Engaging students in mathematical practices provides resources for developing norms and ways of working that draw on students’ experiences in their communities. It also can support all students to develop a wide range of skills of interaction, discourse, and collaboration that are specific to mathematical activity but also extend

beyond it. Drawing on, for example, the practices of working collectively that are common in many minoritized communities can enable students to use ways of working with others with which they are experienced, and contributes to enriching the learning environment and the skills for all students. Attending to a broader set of cultural practices and ways of knowing—embracing norms and patterns of non-dominant communities—also enlarges the resources for learning by expanding who contributes to the class work. Relationships with students’ families and communities are vital for learning more about students’ cultural resources, practices, and knowledge. Joining in community activities and learning more about traditions and common activities afford educators many opportunities for learning about how mathematics and mathematical reasoning are used, how people respect and work together, and what are valued ways of knowing.

Language

The domain labeled “language” highlights the importance of attending to the languages spoken by students. Students who are multilingual bring skills and resources from across the language systems and vocabulary that they know (Baker-Bell, 2020). Many will engage in translanguaging—or using the vocabulary, grammar, and linguistic structures from more than one language to communicate. Teachers support multilingual learners when they understand the assets of multilingualism as well as the challenges that learners may face. Teachers can draw on these assets and address these challenges by using instructional practices that support multilingual learners’ development (e.g., using multiple representations, building clear correspondences between language and pictorial or physical representations), by encouraging students to work with peers, and by helping students draw on strengths across their languages as they develop mathematically.

Key Idea #4: Research on mathematics teaching and learning shows that teachers’ own relationship to and understanding of mathematics is crucial for supporting positive student development.

All of these key areas depend on teachers’ understanding of and ability to hear, notice, and communicate mathematically. This requires that teachers, pre-K through high school, are rooted in deep mathematical understanding in order to set them up for the mathematical flexibility and imagination needed to explain mathematical ideas. Teachers that have such specialized understanding and a sense of themselves as people who know and enjoy mathematics can guide young learners to do the same (see Briefs #3 and #6). This means that professional learning must take this need seriously, offering both curriculum-based professional development and ongoing opportunities for teachers to develop as mathematical knowers, situated for the purposes of lighting up mathematics for their students (see Brief #8).



Key Take-Aways

1. Different fields contribute to the research base for mathematics teaching and learning. These include research on teaching, curriculum and its use, mathematics as a discipline, sociology, ethnic studies, developmental psychology, cognitive science, education policy, psychometrics, and teacher education. No single field can provide all the knowledge needed to inform mathematics teaching and learning.
2. Skillful teaching that supports student learning and development makes use of instructional practices that support the building of five strands of mathematical proficiency: adaptive reasoning, strategic competence, conceptual understanding, productive disposition, and procedural fluency.
3. It is important to attend to and use students' cultural and community knowledge to connect their mathematical development meaningfully with resources they have. Three interlocking domains of experience and resources are vital to authentic connections: content, practices, and language.
4. Teachers are crucial for students' development; investing in their professional learning through curriculum-based professional development, ongoing opportunities to develop as mathematical knowers, and job-embedded professional learning that focus on mathematics instruction is crucial. These opportunities should be tailored to teachers' career stage.



Reflect & Analyze:

1. Is there research you have found particularly useful or compelling and that has informed your work? Is there research that you are curious to learn more about?
2. Are there questions you have or challenges you face that you wonder whether there is research that could provide insight or guidance?



Key References

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The resources included in this brief are designed to provide helpful information. Resources are provided for instructional use purposes only and do not constitute NYSED endorsement of any vendor, author, or other sources. To the best of our knowledge, the resources provided are true and complete.

BRIEF #1: The Research Base for Mathematics Teaching and Learning

DISCUSSION PROMPTS AND QUESTIONS

DIRECTIONS

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Read the questions and jot down your initial thoughts. Take a moment to reflect on each.

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Consider what key points you'd like to share with the larger group, if time allows.

General Discussion Questions ***(for use with any of the numeracy briefs)***

- Identify the strengths in your district, school, or classroom that align with the brief(s).
- Identify existing challenges or barriers in your district, school, or classroom that may hinder implementation of the ideas presented in the brief(s).
- Offer suggestions for regional collaboration to support the ideas presented in the brief(s).

BREIF #1 QUESTIONS	REFLECTION / NOTES
Is there research you have found particularly useful or compelling and that has informed your work? Is there research that you are curious to learn more about?	
Are there questions you have or challenges you face that you wonder whether there is research that could provide insight or guidance?	

Brief 2:

Debunking Myths about Mathematics Teaching and Learning

Supporting Evidence-Based P–12 Mathematics Teaching Practice

Produced for the New York State Education Department
by Deborah Loewenberg Ball and TeachingWorks at the University of Michigan

Background

Mathematics teaching and learning have been the focus of debate for decades, beginning long before the “math wars” of the late 20th century. In 2006, the National Mathematics Advisory Panel was created to examine the body of scientific evidence related to mathematics teaching and learning, releasing a culminating report *Foundations of Success: The Final Report of the National Mathematics Advisory Panel* in 2008. In the nearly 20 years since the creation of that panel, the gathering of scientific evidence continues, but myths about math teaching and learning endure.

Underlying the myths and misunderstandings are persistent disagreements about the purposes and goals of mathematics education, and about how to support young people’s mathematics learning. False dichotomies characterize these disagreements. For example, is the goal to develop procedural skill and fluency OR to build conceptual understanding and number sense? Is the purpose for students to solve problems efficiently and quickly OR to develop fluency, flexibility, and communication skills? Should instruction always begin with direct instruction and then move to application OR does robust mathematics learning also require opportunities for investigation and inquiry? Research demonstrates that all of these are necessary and comprise intersecting, not dichotomous, goals of mathematics education. Below we identify and debunk common myths that impede progress in mathematics education.

Common Myths

Myth #1: “Ability” grouping improves success.

Separating students into categories to organize tailored instruction is often referred to as “ability grouping.” Although students may have reached different levels of skill and understanding, this does not mean that they are more or less capable. And because students can be skillful with and understand different aspects of mathematics, it is important not to apply fixed labels. Labeling often also has negative effects on children as they come to see themselves as “smart” or not.

The research on grouping students by levels of achievement is mixed: some studies show gains for students in all groups; some show increasing gaps between groups; and some show no changes. Meanwhile, other studies show gains for all students in mixed-ability classrooms when compared to ability-grouped classrooms. So, what is happening here?

In mixed ability classrooms, students can have the opportunity to learn from a wide variety of classmates and develop skill with mathematical argumentation while engaging with grade-level content. These same things can happen in ability-grouped classrooms, but they often do not. In a large study of ability grouping, teachers noted that the “lower” groups are often seen as less capable and are excluded from exposure to higher-level work or all grade-level standards (Mazenod et al., 2018). This can lead to a widening of achievement gaps between the groups. This has also been shown to have a negative effect on students’ mathematical identities, leading them to see themselves as “not good at math.”

However, studies have also shown that flexible grouping, where students can move into groups to target their instructional needs, can be designed in ways that increase student learning. The key is that these groups are both focused on a student’s learning of a particular concept or process and that the groupings are truly flexible, meaning that groups will change depending on the learning target and on a student’s need.



Myth #2: Most students struggle in mathematics.

Unfortunately, many adults complete school with a negative view of mathematics. It is commonplace to say one is “not a math person.” Because of this, a false assumption is that young children, too, are scared of math and regularly struggle to learn it. This is further reinforced when students are labeled “below” or “behind” as a result of poor performance on timed tests. Responding by reducing cognitive demand when a student assumes they “are not a math person” or are labeled as “below” or “behind” results in the constraint of deeper learning. Remedial help can prevent students from using and retaining important content and skills resulting in students falling further behind. Research shows that when students are instead supported to solve challenging problems that enable them to learn and use the concepts and skills they need, they are more likely to thrive mathematically.

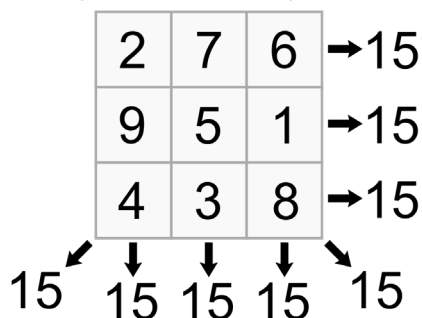
Myth #3: Mathematical competence means being efficient and accurate.

There is a long-held belief that being “good” at mathematics means being fast and correct. However, professional mathematicians do not work fast and often take false paths as they solve problems. Researchers agree that mathematical competence is broader than speed and accuracy alone. Procedural fluency (efficiency, flexibility, and accuracy) is interwoven with conceptual understanding, strategic competence, adaptive reasoning, and a productive disposition (National Research Council, 2001) (see Brief #1). This means that students should be supported to demonstrate mathematical competence in a wide variety of ways including: (1) choosing and justifying a strategy to solve a particular task; (2) providing the reasoning or proving a result; (3) asking questions about core mathematical ideas; and (4) explaining concepts.

Myth #4: Developing competence requires isolated repetition and drills.

Procedural fluency supports student success in mathematics by reducing students’ cognitive load in the course of problem solving. However, myths persist that the sole way to build such fluency is through repeated speed drills and timed tests. While these may provide repeated opportunity to build fact fluency, they also frequently cause students stress and to see themselves as not “good at math.” Practice for fluency can also be accomplished using problems that require repeated use of facts to solve a more complex problem—for example, magic square (see Figure 1) or magic

Figure 1: Example of a Magic Square



triangle tasks. These tasks require repeated practice with facts, but also require reasoning and strategic thinking, accomplishing multiple mathematical goals simultaneously. The student-facing goal isn’t to practice their facts, but students have repeated practice opportunities in the course of solving the problem.

Myth #5: The most effective teaching method is providing step-by-step procedures for solving problems.

Research in mathematics learning, particularly in psychology and special education, demonstrates that detailed, step-by-step instruction can support students to learn mathematical procedures. Much of this research is focused specifically on students with disabilities and utilizes assessments of procedural knowledge to measure change. Although explicit instruction can be helpful for the teaching and learning of procedures, it is important that it be grounded in meaning and understanding of why the procedures work. It is also not the “best” way to teach or learn mathematical thinking and reasoning. This requires supporting students to grapple with mathematical tasks that demand reasoning and justification, clarify and refine their reasoning with others through discourse, and demonstrate justification of their reasoning. Standards for mathematical competence include constructing, analyzing, and critiquing the completeness of mathematical arguments.

Myth #6: “Inquiry” approaches leave students to learn on their own instead of teaching them mathematics.

Misconceptions about inquiry and discovery approaches to teaching and learning mathematics persist, framing inquiry-based instruction as posing problems and leaving students to create mathematics on their own. In reality, effective inquiry approaches are highly designed and scaffolded to support both student investigation of mathematical ideas and teacher facilitated and directed summarization and instruction. Well-designed and implemented inquiry or problem-based lessons begin with an accessible mathematics task that requires students to engage in conjectures related to foundational grade-level standards. The teacher facilitates student understanding of the problem or task, structures opportunities for students to engage in conjecture, justification, and argumentation, and supports students to make sense of the conceptual

and procedural ideas raised by the task, often through questioning and sometimes through explicit instruction. Studies have found that inquiry methods enhance mathematical problem solving and reasoning without reducing procedural knowledge. Inquiry-based teaching and learning has also been found to have positive longitudinal impacts on student learning (Hmelo-Silver et al., 2007). Thus, inquiry approaches support the teaching and learning of mathematical thinking and reasoning.

Myth #7: Success in calculus is the ultimate goal of school mathematics

Many high schools have taken the view that student success in calculus is the goal for all students. Two paths of reasoning have pushed schools in this direction. First, schools reasoned that the goal of schooling was to prepare students for all possible future pathways, including college. Second, analyses had shown that about two-thirds of community college students and more than one-third of four-year college students were required to take remedial coursework and that the majority of these students dropped out without degrees. Many schools reasoned that if they could get students through calculus, the required course for many undergraduate degrees, students would be prepared for college if they chose that pathway, would no longer need remediation, and

would complete desired degrees. However, a study at the City University of New York (CUNY) showed that the issue may not be in students' high school pathway, but rather in college pathways. Across the seven-year study, students without challenges in basic arithmetic bypassed remedial college algebra courses, taking statistics instead. These students were more likely to earn their college mathematics credit and complete their degree. Many universities are now reconsidering their mathematics pathways, identifying the appropriate mathematics courses for degrees rather than requiring calculus for all.

How does this shift in college pathways impact schools? Schools may still hold preparing students for all future pathways as a goal, but that no longer means that calculus is the target for all students. Courses in statistics, quantitative reasoning, and using and understanding data may be more beneficial targets for many students. Regardless of the course pathways, these courses should focus on mathematical thinking, justification, fluency, and communication in order to best prepare students for all future pathways.



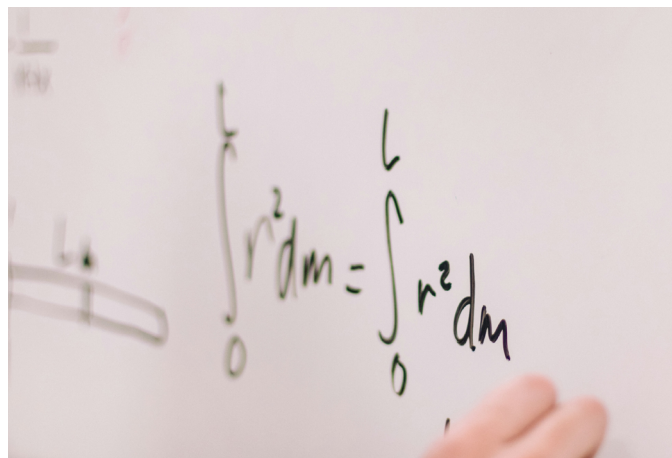
Key Take-Aways

1. Meeting students where they are at is important, but static ability-grouped classrooms often do not support the learning of students.
2. Inquiry and explicit instruction can be effective for student learning, particularly when used in combination with careful thinking about the learning goals and when each method is most appropriate.
3. There are many competencies that are critical for learners of mathematics, not just fluency. Additionally, fluency can be built in tandem with problem-solving skills.
4. Supporting students to do challenging mathematical work can help to avoid cycles of failure and the development of negative math identities.



Reflect & Analyze:

1. Consider which of these myths exist in your district, school, classroom, or community. How will you work to be a mathematics myth-buster?
2. Are there any myths about which you remain unconvinced? How might you learn more to help you build a nuanced view of the issue?



Key References

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BRIEF #2: Debunking Myths about Mathematics Teaching and Learning

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Are there any myths about which you remain unconvinced? How might you learn more to help you build a nuanced view of the issue?	



Brief 3: High-Leverage Mathematical Content

Supporting Evidence-Based P–12 Mathematics Teaching Practice

Produced for the New York State Education Department
by Deborah Loewenberg Ball and TeachingWorks at the University of Michigan



All young [people] must learn to think mathematically, and they must think mathematically to learn.

(National Research Council, 2001, p. 1)

What is meant by “high-leverage mathematical content”?

High-leverage mathematical content comprises mathematical concepts, skills, and practices essential for mathematical competence. It focuses on what it takes to be able to use mathematics to solve problems and to reason about questions and situations across a range of settings. Mathematics includes arithmetic, algebraic computation, and estimation, but it also encompasses a vast domain of human thinking and ideas, developed over centuries in diverse cultures and contexts and for a wide range of uses. Historically, the U.S. school curriculum began by focusing on the measurement and quantitative skills needed for commerce. It has expanded over decades to include many more domains, such as probability (the math involved in figuring out the likelihood of a particular outcome) and discrete mathematics (for example, the math involved in finding the number of ways that a set of objects can be rearranged), and more areas of application (for example, engineering).

The curriculum in this country has been described as a “mile wide and an inch deep,” a critique of the lack of depth and focus needed to develop in young people the capacity for mathematical problem solving and thinking. It is also fragmented, with bits of a particular mathematical strand showing up each year and

different areas separated in the chapters of instructional materials. Identifying key ideas and practices as high-leverage mathematical content and building connections among them is essential to supporting the development of mathematical competence. The [New York State Next Generation Mathematics Learning Standards](#) contribute to this goal by providing coherence links within the standards document to help teachers build connections across grade levels.



Spotlight on Practice

Curriculum materials often do not make connections across grades and mathematical concepts. Teachers can help to build these connections through their instructional choices.

Ms. Natasha teaches fourth grade and is helping her students understand equivalent fractions. The guidance given by her instructional materials focuses on shading rectangles—for example, shading $\frac{2}{8}$ of a rectangle to show that it covers the same area as $\frac{1}{4}$.



Because she wants to support her students to make connections, Ms. Natasha asks her students for other examples of things they have learned where it was useful to represent something in equivalent form. Several students recall that in carrying out multi-digit subtraction, they learned to write equivalent forms of a number to make it possible to subtract, as in this example, where 81 is written as 7 tens and 11 ones, which is the same value as 8 tens and 1 one:

$$\begin{array}{r} 7 \text{ } 11 \\ 81 \\ - 38 \\ \hline \end{array}$$

What makes mathematical content “high-leverage?”

High-leverage mathematical content centers on the ideas and concepts that are foundational across grades, in the field of mathematics, and for deep capacity to think and solve problems. It is a combination of mathematical knowledge and mathematical practices, focused on core ideas and ways of thinking, understood in ways that make it possible to use mathematics flexibly to reason, think critically, and solve problems. High-leverage mathematical content is:

- **Inclusive of concepts, procedures, skills, language, and practices.**
- **Fundamental across the early grades and through high school.**
- **Foundational to mathematical reasoning and thinking.**
- **Crucial for accessing and mastering more advanced knowledge and skills in mathematics.**

The idea of high-leverage mathematical content is centered on students, and what they need to learn so that they can use the tools and practices of mathematics and see themselves as capable to do so.

Spotlight on Practice

Mathematics problems that provide opportunities to develop concepts, fluency, skill, and reasoning can support students to practice using mathematics, and to learn crucial practices.

The 8's Problem is an example of such a problem:

Write a string of 8's and insert + signs between them to equal 1000. You can use as many 8's as you want, and you can also put them together like this: 88 (but not like this: $8/8$). How can this be done? How many different ways are there to do this?

One solution is to add 125 8's because $125 \times 8 = 1000$. But this is just one of the solutions.

Working on this problem involves computation practice; use of concepts of addition, subtraction, multiplication, and division; seeing and using mathematical structure; methods of recording; skills for persevering; and practices of explaining and reasoning. Comparing the mathematical opportunities of this problem with a common computational practice worksheet highlights the way that high-leverage mathematical content can be developed.

(Adapted from Gelfand & Shen, 1993).

One example of high-leverage content across the grades

Elementary

In elementary grades, students learn the foundational ideas underlying fractions concepts. They learn to think flexibly about fractions as numbers (e.g., $1/2$ is one way to represent the number 0.5 and is at the same point on the number line). They also can use fractions to name part-whole relationships ($1/2$ can be represented by a whole divided into two equal pieces with one shaded) and as relationships (e.g., $1/2$ is equivalent to $2/4$ because the relationship between the numerator and denominator is the same in both fractions). These ideas are fundamental to developing ratio and proportional reasoning in the middle grades, often serving as a “gatekeeper” to middle school mathematics.

Middle School

In the middle grades, students extend the foundational concepts of fractions to build their understanding of ratio and proportion. They learn to think flexibly about a ratio representing the relationship between two quantities, understanding that these are often not part-whole relationships and are often different units of measurement altogether (e.g., people and work-hours). They understand proportions as equivalent ratios, utilizing their strategies for finding equivalent fractions to solve proportion problems and learning new strategies for situations where these don't apply. Understanding of ratio and proportion is fundamental to the concept of rates of change in the secondary grades and is a fundamental part of everyday mathematical sensemaking about information.

High School

In late middle school and throughout high school, students extend the foundational concepts of ratio to build understanding of rates of change and their impact on functions. They examine how the relationship between two variables is represented in graphs, tables, and equations. They learn to identify when rates of change are constant or varied and how to identify a best-fit-function that describes the relationship between the variables. These fundamental understandings lay the foundation for both practical applications of mathematics and calculus, which focuses on how functions change.

Why is it important to identify “high-leverage mathematical content?”

The school curriculum has been expanding over decades, sometimes adding more content without subtracting anything. Everything seems important and pacing guides can often undermine the need for deep understanding and capacity with core mathematical ideas. Instructional materials have become longer and more complex over time, with many learning goals and a plethora of additional resources for the teacher and for students. This diffuseness is not easy to navigate. Topics that are related are treated as different. Take, for example, the use of area models with multiplication in fourth grade and geometric representations of binomial multiplication in high school algebra. In the fourth grade, the product of two numbers is represented as the area of a rectangle with those side lengths. Those side lengths are broken down into sums to make the multiplication manageable and the area is calculated for each of the component parts. The same is true for the representation of binomial multiplication. The product of the binomials is the area of a rectangle with those binomial side lengths. While the processes and reasoning are the same for these two ideas, the teaching of binomial multiplication is often treated as new for students and unconnected to prior learning.

		$35 \times 24 = 840$		$(3x+5)(2x+4) = 6x^2 + 22x + 20$	
		30	5		
20		$30 \times 20 = 600$	$5 \times 20 = 100$	2x	
					$3x \times 2x = 6x^2$
4		$30 \times 4 = 120$	$5 \times 4 = 20$	4	
					$3x \times 4 = 12x$
					$5 \times 4 = 20$

Rapid changes in technology and artificial intelligence mean that the adults of this upcoming generation will need skills of sensemaking, critical interpretation, and mathematical reasoning and judgment, and that it will be far less common to perform precise calculations or conversions. For example, adults will not often need to carry out long division with large numbers, but they will need to know whether a result is reasonable. Work on conventional algorithms such as long division could center the high-leverage content of place value, yet too often it is focused on the answer more than on understanding the incredible mathematical structure that underlies the procedure. Naming content that is high-leverage can support students and teachers to focus on developing crucial mathematical competence. Understanding what content is high-leverage and therefore of highest priority can help teachers manage the pressure of content “coverage” and pacing guides, ensuring that crucial content is not missed or too lightly or quickly treated. Supporting teachers to be able to make these determinations can be developed through professional learning.

What are some examples of “high-leverage mathematical content?”

Base-ten system of place value and numeration

One clear example of high-leverage mathematical content is the base-ten system of place value and numeration. How does it work? How is it possible to use only 10 digits and yet be able to write any number? What does this compressed system enable in calculating, estimating, and using number sense? Recognizing the power of this system would mean that more attention would be paid to students’ understanding of the structure and less to identifying what place a particular digit is in. For example, why do we say we “add a zero” when we multiply by ten but we say we “move the decimal point over” when we divide by ten? This example highlights the common fragmentation of connected ideas as well as the lack of focus on what makes place value high leverage.



Spotlight on Practice

What does it mean to “add a zero” to multiply by ten? Take the number 54, for example. $54 \times 10 = 540$. We are not “adding” anything, much less a zero. The real explanation is that 54 is composed of 5 tens and 4 ones. When we multiply by 10, we now have ten sets of 5 tens and 4 ones. That means we now have 5 groups of 10 tens (or hundreds) and 4 groups of 10 ones (or tens), so we move the 5 from the tens place to the hundreds place and the 4 from the ones place to the tens place. When we do that, the value of the 5 is now 5 hundreds and the value of the 4 is now 4 tens. The structure of the place value system makes it easy to transform the value of a number by where it is located—its place. As we move to the left, we are multiplying by a factor of ten each time. The zero in 540 is because 54 is 5 tens, 4 ones, and 0 tenths. The zero is not holding a place. Like the 5 and the 4, it has simply been moved one place to the left. This is at the heart of a place value system.

Based on this explanation, how would we explain dividing by 10? It is structurally the same. We are not moving the decimal point; instead, we are again moving the numbers, but in this case to the right.

Number line

Another example of high-leverage mathematical content is the number line and its properties as a fundamental mathematical object. Students see number lines from the earliest grades, but they only encounter whole numbers—rarely negative numbers or fractions—until much later. Moreover, the infinities of the line—in each direction and between any two points—is left unexposed. The number line holds all the rational numbers and is crucial for many ideas and kinds of reasoning. This also highlights that fractions when first introduced do not seem to be numbers at all, but merely parts of wholes. Once again, the lack of attention to these ideas as high-leverage leaves them disconnected and less powerful.

Fractions

Fractions are another key example of high-leverage content. Understanding fractions as rational numbers, connected to decimals which are place-value based representations of rational numbers, is foundational to being able to operate and reason with numbers. Fractions as division are the building blocks of rational functions, which are crucial for competence with algebraic skill and reasoning. Being able to represent quantities in equivalent forms for particular purposes is an example of the leverage that fractions afford.

Solving equations

The basic principles of solving equations are also high-leverage content. Understanding solving an equation as isolating the variable in order to determine the value and understanding that isolating the variable involves performing operations to the two equivalent expressions that maintain equivalence are fundamental to advanced work with equations and functions. Students are exposed to equivalence in very early grades as they learn different ways to represent numbers, but the meaning of equivalence (and the equal sign) are often left implicit, resulting in many students viewing the equal sign as an operator that tells you to find the answer rather than a symbol noting equivalence between two mathematical objects.

Likewise, work to solve equations by isolating the variable begins in elementary school. Students solve “missing value” problems where they use reasoning and their understanding of inverse operations to determine the missing value. As students progress, the solving of equations often gets reduced to a disconnected set of cases where students follow a prescribed series of steps without meaning. Teachers can create opportunities to support students in recognizing that there are many possible ways of solving the equation as long as the operation you perform on the expressions maintains the equivalence. Raising this idea up would mean moving away from phrases like “move the number to the other side,” which hides the reasoning behind why this move is appropriate and helps you to isolate the variable.

Core practices from the New York State Next Generation Mathematics Learning Standards

High-leverage mathematical content is more than concepts and procedures. The [New York State Next Generation Mathematics Learning Standards](#) specify a set of eight core practices that are fundamental to mathematical competence. Making sense of problems and persevering in solving them is broadly important and crucial, for example, for solving word problems. Being able to explain and justify rules and procedures is highlighted for the power gained from understanding why something works. Reasoning, explaining, and proving are high-leverage because they enable learners critical agency in making sense and solving problems. All eight of the Standards for Mathematical Practice (New York State Department of Education, 2019) are high-leverage because they describe both what students should be doing as they learn and what capacities are essential to being a person who can use mathematics and who identifies as a capable doer of mathematics.

Standards for Mathematical Practice

Make sense of problems and persevere in solving them

Reason abstractly and quantitatively

Construct viable arguments and critique the reasoning of others

Model with mathematics

Use appropriate tools strategically

Attend to precision

Look for and make use of structure

Look for and express regularity in repeated reasoning

What Do Teachers Need to Support High-Leverage Mathematical Content?

Teachers need opportunities to develop deep mathematical knowledge because teaching requires specialized mathematical understanding that is fundamental to the work of helping others learn math. Teachers with higher levels of this kind of mathematical knowledge for teaching are more effective in helping students grow mathematically.

Spotlight on Research

Teaching requires understanding the math that students are to learn. But teaching also requires different understanding, different from just being able to do the math. Teaching involves being able to explain why a procedure works and to define mathematical terms in ways that are correct but also accessible.

"The mathematical demands of teaching require specialized mathematical knowledge not needed in other settings. . . . Teachers need to know mathematics in ways useful for, among other things, making mathematical sense of student work, and choosing powerful ways of representing the subject so that it is understandable to students."

(Ball et al., 2008, pp. 401-404)

Understanding the content that students are learning is necessary but insufficient for teachers because the work of teaching is not merely doing the math oneself. It is important for teachers to understand the content from students' perspectives, be aware of what might make a particular procedure or skill confusing, and be able to represent it in ways that make sense to students. This is important for teaching all students but is particularly important for advancing equity in mathematics learning.

Equitable teaching involves knowing different representations, what each is especially good for, and the mathematical differences among them. Sensitivity to language is another dimension of the high-leverage content that teachers need. When students talk in informal ways, teachers need to be able to consider how their language corresponds to formal mathematical terms and definitions. Teachers often have to adapt examples in their materials; doing so while preserving the mathematical focus requires a special kind of mathematical insight and reasoning. Posing mathematically strategic tasks and questions, interpreting student work, and communicating with students' families all require specialized ways of knowing and using math that others do not need. Being able to hear the thinking underlying students' mathematical ideas and to acknowledge their competence is vital to disrupting patterns of inequity that position students as "struggling," a pattern that disproportionately affects students of historically marginalized identities (Gresalfi et al., 2009). Supporting teachers to help students thrive mathematically requires that they have both curriculum-based professional development and opportunities for professional learning that provide practice with these specialized ways of knowing mathematics for teaching.



Key Take-Aways

1. The U.S. school mathematics curriculum is broad, and comprises a wide range of topics, applications, and skills. This breadth can take time away from a deliberate focus on high-leverage mathematical content.
2. Focusing students' learning on high-leverage mathematical content and practices is essential for their ongoing development, including their access to pathways that depend on mathematical knowledge and skill, their disposition and capability to use mathematics in everyday situations, and a positive sense of themselves as "math doers."
3. Teachers need specialized understanding of mathematics to help students thrive mathematically. Being able to discern and emphasize high-leverage mathematical content and practices depends on teachers' own mathematical knowledge for teaching.
4. Teachers need professional learning opportunities to continue to develop their mathematical knowledge for teaching.



Reflect & Analyze:

1. How do you react to situations involving mathematical reasoning—for instance, about how large something is or how to fit something into some space or to figure out how likely something is? How does your experience with mathematics in and out of school play out in situations like these? Did you experience math as bits and pieces of skills or did you develop a broader view that highlighted high-leverage concepts, skills, and practices?
2. What are particular concepts, skills, or practices of mathematics or mathematical reasoning you use in your life, whether formally or not? When do you feel competent in math and what contributes to that?
3. What are moments in teaching where you find yourself trying to explain something you know how to do but feel stuck about how to unpack it? What resources do you have to address those moments? What are moments when you see that you have the special mathematical understanding needed to help a student who is confused?

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The resources included in this brief are designed to provide helpful information. Resources are provided for instructional use purposes only and do not constitute NYSED endorsement of any vendor, author, or other sources. To the best of our knowledge, the resources provided are true and complete.

BRIEF #3: High-Leverage Mathematical Content

DISCUSSION PROMPTS AND QUESTIONS

DIRECTIONS

Step 1 – “Think”

Read the questions and jot down your initial thoughts. Take a moment to reflect on each.

Step 2 – “Pair”

Discuss your responses with the people at your table. Share your insights and listen to theirs.

Step 3 – “Share”

Consider what key points you'd like to share with the larger group, if time allows.

General Discussion Questions ***(for use with any of the numeracy briefs)***

- Identify the strengths in your district, school, or classroom that align with the brief(s).
- Identify existing challenges or barriers in your district, school, or classroom that may hinder implementation of the ideas presented in the brief(s).
- Offer suggestions for regional collaboration to support the ideas presented in the brief(s).

BREIF #3 QUESTIONS	REFLECTION / NOTES
How do you react to situations involving mathematical reasoning – for instance, about how large something is or how to fit something into some space or to figure out how likely something is? How does your experience with mathematics in and out of school play out in situations like these? Did you experience math as bits and pieces of skills or did you develop a broader view that highlighted high-leverage concepts, skills, and practices?	
What are particular concepts, skills, or practices of mathematics or mathematical reasoning you use in your life, whether formally or not? When do you feel competent in math and what contributes to that?	
What are moments in teaching where you find yourself trying to explain something you know how to do but feel stuck about how to unpack it? What resources do you have to address those moments? Or what are moments when you see that you have the special mathematical understanding needed to help a student who is confused?	



Brief 4:

High-Leverage Instructional Practices

Supporting Evidence-Based P–12 Mathematics Teaching Practice

Produced for the New York State Education Department
by Deborah Loewenberg Ball and TeachingWorks at the University of Michigan



No in-school intervention has a greater impact on student learning than an effective teacher.

(National Council for Accreditation of Teacher Education, 2010, p. 1)

What is meant by “high-leverage instructional practices?”

High-leverage instructional practices (TeachingWorks, 2012) are essential to the work of teaching. When they are used with care and judgment, they are key to supporting students' learning and thriving. Some familiar examples include **eliciting and interpreting individual students' thinking**, **explaining and modeling content**, **leading a group discussion**, **setting up and managing small group work**, and **establishing and maintaining community expectations**. High-leverage instructional practices are grounded in research on teaching and learning mathematics and support students' mathematics learning and development.

These instructional practices help students learn important mathematical content and practices. They are also crucial in promoting equity and inclusion in the classroom, and they enable teachers to support students' social and emotional development.

Eliciting and interpreting individual students' thinking (sometimes referred to as “posing questions”)

Teaching mathematics well depends on understanding students' thinking. This high-leverage practice focuses on surfacing novel points of view, new ideas, and ways of thinking, as well as alternative

conceptions. Through this, teachers draw out student thinking using carefully chosen questions and tasks followed by attending closely to what students do and say. Teachers can use what they learn about students to guide instructional decisions, and to surface ideas for the class to consider. Eliciting and interpreting student thinking positions students as sense-makers and centers their thinking as valuable.

Questions asked during mathematics teaching should move beyond eliciting only answers and processes. Studies show that students whose teachers elicit (and listen to) their thinking more frequently outperform their peers in classrooms where teachers elicited thinking less frequently. Students need to have opportunities to articulate, explain, provide detail about, and justify their strategies and thinking. Questions that elicit this level of thinking are also tied to increased student engagement and learning.



Spotlight on Practice

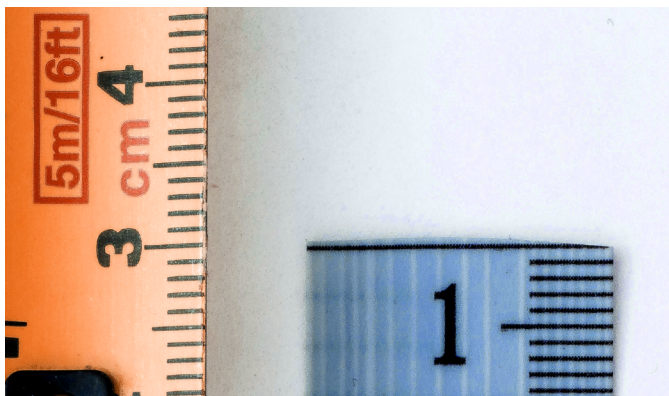
Randy is a four-year-old multilingual learner who is enthusiastic about playing math games with his teacher, Mr. Gonzalez, during center time. While exploring manipulatives, Mr. Gonzalez asks Randy questions like, “how many are there?” and follows up by asking, “how do you know?” Mr. Gonzalez notices and interprets Randy's nonverbal communications and uses them to draw out his thinking, e.g., “I saw you moving your fingers, what were you doing?” He also narrates his own thinking, modeling the kinds of explanations Randy might learn to give, e.g., “I saw two and one and I know that makes a group of three!” Over time Mr. Gonzalez learns that Randy successfully uses his fingers to count and has mastered one-to-one correspondence to 10 (matches one number word to one object while counting). By demonstrating a genuine interest in his mathematical thinking, Mr. Gonzalez communicates to Randy that his ideas are fascinating and important, bolstering Randy's developing mathematical identity.

Explaining and modeling content (sometimes called “cognitive apprenticeship”)

Explaining and modeling are practices for making a wide variety of topics, academic practices, and strategies explicit to students. When teachers explain or model, it can help make visible content and practices that are otherwise tacit, such as which definition of subtraction you are using to solve a problem on the number line. Teachers might use simple explanations when working with straightforward content. They might choose modeling, which includes verbal explanation as well as thinking aloud and demonstrating when sharing the metacognitive process, to provide greater access to students about strategies and practices.

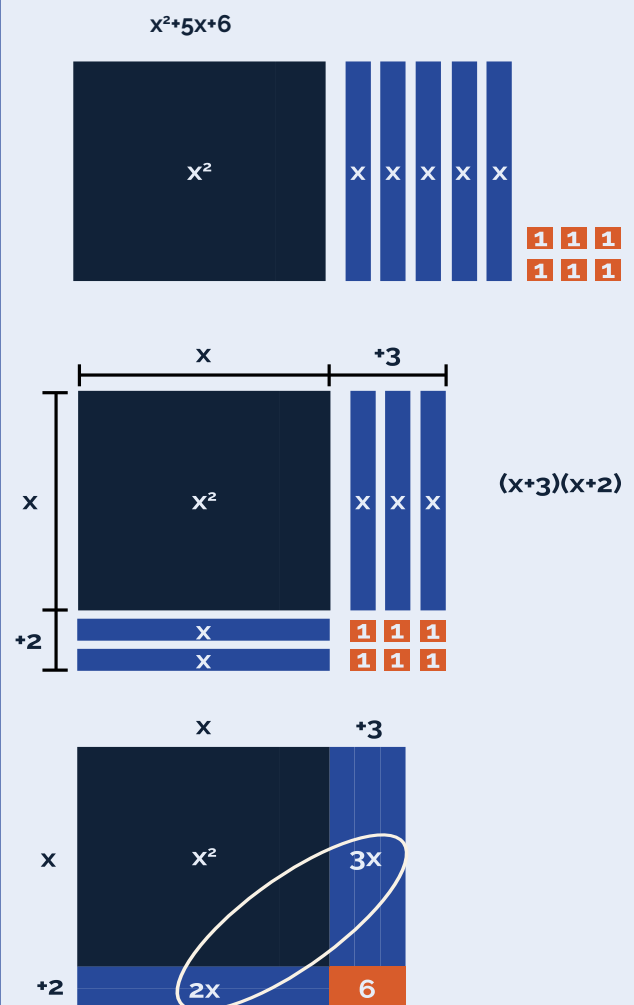
This is different from what people sometimes call “direct instruction” because it builds on the idea that students are sense-makers, not empty vessels to be filled. It also maintains the cognitive demand, ensuring that students are engaging with rich academic content. Explaining and modeling in mathematics often involve the use of manipulatives and representations. These are central resources in mathematics teaching and take special care to support student learning; therefore, the use of manipulatives and representations is outlined in a separate brief (Brief #6).

Explaining and modeling in mathematics is often an instructional practice used to support students in approaching “word problems.” Modeling for students how to identify the mathematical structure underlying the word problem (e.g., combining) can support students in representing and solving word problems. Research has shown this can be more effective than other strategies for supporting students with disabilities, such as identifying keywords. The goal should be for the teacher to be aware of different problem types, ensure students are engaged in a variety of problem types, and support them in naming the key concepts (combining, comparing, grouping, etc.) as a framework for problem solving. Modeling serves as a scaffold for this process—not the teacher’s opening instructional move. For more information on problem structures in mathematics, see the [Common Addition and Subtraction Situations in the New York State Next Generation Mathematics Learning Standards](#).



Spotlight on Practice

Ms. Jackson’s high schoolers have typically learned the procedure for factoring trinomials but seem to lack the foundational conceptual understanding that factoring is rewriting the expression as multiplication. She knows that students learned to represent multiplication using an area model in elementary school and found areas of geometric figures in middle school. She decides to help them connect the process of factoring to multiplication area models. She begins by showing the terms of the trinomial with algebra tiles and rearranging them to create a rectangle (process shown below). She then supports them to name the length of the sides of the rectangle and connects this representation to the factors of the trinomial. Finally, she connects algebra tiles to area models, walking her students through the process of mapping the terms of a trinomial onto the area model and considering how to determine the factors. She describes the process while narrating her thinking at each stage, and carefully documenting her modeling visually on the board. In this way, she is able to help students “see” what may be less visible and to engage with important academic content in ways that are grounded in meaning.



Leading a group discussion

In a group discussion, the teacher guides students as they work together on specific content, using one another's ideas as resources. The purposes of a discussion are to build collective knowledge and capability in relation to specific instructional goals and to allow students to practice listening, speaking, interpreting, questioning, agreeing, and disagreeing. The teacher takes care to use a variety of strategies, including posing questions, showing, providing clarification, and orienting students to one another. The teacher supports a wide range of students to contribute orally, listen actively, and respond to and learn from others' contributions. Teachers work to ensure students are positioned as competent among their peers, that patterns of interaction are respectful, and that the collective work of the group uses students' strengths and benefits each student's learning.

Spotlight on Practice

Mr. Nguyen's second graders have been engaging in number talks. At first, Mr. Nguyen merely implements them as written (he gives his students an equation to solve mentally, collects solutions, then collects strategies). Mr. Nguyen begins to realize that there is an untapped opportunity for his students to experience the value of working collaboratively. He practices a more intentional "launch," where he explains to students that a number talk helps us work on efficiency, flexibility, and accuracy and that a new goal will be to learn a strategy from a friend. He orients students to one another's thinking by asking them to restate each other's strategies, supporting them to ask each other questions, and helping them see connections across strategies as he records and represents the thinking that gets shared on the board. These moves enable him to find authentic opportunities to position each student, especially those from historically marginalized groups, as smart in math. Over time, he sees evidence of many students "trying on" the strategies of others, and they eagerly look forward to engaging in number talks.

Standards for Mathematical Practice

Construct viable arguments and critique the reasoning of others

Mathematically proficient students . . . justify their conclusions, communicate them to others, and respond to the arguments of others . . . Students at all grades can listen or read the arguments of others, decide whether they make sense, and ask useful questions to clarify or improve the arguments (New York State Department of Education, 2019, p. 7).



Teachers and researchers alike know that engaging students in productive discussions can support the building of collective knowledge as well as individual students' understanding of mathematics (O'Connor et al., 2017; Franke et al., 2015). What students see and can collectively make use of in the public space of a classroom can support their engagement and learning in a discussion.

(Garcia et al., 2021, p.927)



Setting up and managing small group work

In order to set up and manage small group work, teachers choose tasks that require and foster collaborative work, provide clear directions that enable groups to work independently, and hold students accountable for collective and individual learning. Teachers use their own time strategically, deliberately choosing which groups to work with, when to work with them, and on what. They work to ensure students are positioned as competent among their peers, that patterns of interaction are respectful, and that the collective work of the group uses the strengths of and benefits each student.

Spotlight on Practice

Mx. Yazzie has given their fifth graders a complex problem, The Train Problem, to work on. While they could have students work on it independently, they see an opportunity to support cooperation and collaboration by assigning them to work in small groups. Mx. Yazzie considers the academic, social emotional, and language needs of each student, taking care to balance the groups in terms of intersecting identities. Before the groups begin working, Mx. Yazzie ensures that the task is clear and that students know what the various roles within the group are—e.g., someone should take notes, someone should be sure that everyone gets to share their idea, etc. As students are working, Mx. Yazzie circulates around the room, spending more time in groups that seem to need support and scaffolding. When they come back together, Mx. Yazzie shares some comments about how groups worked together and makes explicit the value of working collaboratively so that students understand that this is part of what gets worked on in class.

Establishing and maintaining community expectations

Teachers must set and maintain expectations and establish classroom environments that are productive and safe while also ensuring students' dignity. The NYSED Culturally Responsive-Sustaining Education Framework describes such environments as "a space where people can find themselves represented and reflected, and where they understand that all people are treated with respect and dignity. The environment ensures all cultural identities . . . are affirmed, valued, and used as vehicles for teaching and learning" (New York State Education Department, 2018, p. 14). To create this environment, teachers must carefully consider the purpose and impact of management routines. For example, some routines are aimed at supporting students' learning while others merely exercise control. Discerning the difference matters for creating learning environments that welcome learners and develop positive mathematical identities and competence. Classroom norms and agreements can be developed collaboratively with students. Such norms should emphasize the development of students' agency and responsibility for their learning, mutual respect, and collective engagement in the mathematical work. Agreements should also explicitly include how these norms will be maintained within the classroom community. This classroom environment should also support students to engage in challenging mathematics.

Principle #2 from the Four Principles of Culturally Responsive-Sustaining Education

High expectations and rigorous instruction prepare the community for rigor and independent learning. The environment is academically rigorous and intellectually challenging, while also considering the different ways students learn. Instruction includes opportunities to use critical reasoning, take academic risks, and leverage a growth mindset to learn from mistakes. Messages encourage positive self-image and empower others to succeed.

What do high-leverage instructional practices have to do with mathematical identity?

High-leverage instructional practices help teachers support students to broaden their understanding of what it means to do math well. This ensures that all students have opportunities to demonstrate their mathematical brilliance (Martin, 2007; Gholson, 2013).

Historically, not all children have had this opportunity—so high-leverage instructional practices are particularly important for students from "nondominant ethnic, racial, cultural, and language backgrounds, students with varying disabilities, and students from economically challenged areas" (Wilkerson, 2020).

When choosing which high-leverage instructional practices to use, consider the following factors (not exhaustive):

- Content and learning goals
- Grade level
- Common patterns of student thinking
- Language needs of students



Reflect & Analyze:

1. How and when do you decide to use different approaches to teaching mathematics—e.g., eliciting and interpreting individual students' thinking, modeling and explaining content, setting up and managing small group work, and leading a group discussion?
2. What practices do you find yourself using most often? Which ones do your curricular materials recommend the most? Do these align with what we know about student learning outcomes?

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The resources included in this brief are designed to provide helpful information. Resources are provided for instructional use purposes only and do not constitute NYSED endorsement of any vendor, author, or other sources. To the best of our knowledge, the resources provided are true and complete.

BRIEF #4: High-Leverage Instructional Practices

DISCUSSION PROMPTS AND QUESTIONS

DIRECTIONS

Step 1 – “Think”

Read the questions and jot down your initial thoughts. Take a moment to reflect on each.

Step 2 – “Pair”

Discuss your responses with the people at your table. Share your insights and listen to theirs.

Step 3 – “Share”

Consider what key points you'd like to share with the larger group, if time allows.

General Discussion Questions *(for use with any of the numeracy briefs)*

- Identify the strengths in your district, school, or classroom that align with the brief(s).
- Identify existing challenges or barriers in your district, school, or classroom that may hinder implementation of the ideas presented in the brief(s).
- Offer suggestions for regional collaboration to support the ideas presented in the brief(s).

BRIEF #4 QUESTIONS	REFLECTION / NOTES
How and when do you decide to use different approaches to teaching mathematics—e.g., eliciting and interpreting individual students' thinking, modeling and explaining content, setting up and managing small group work, and leading a group discussion?	
What practices do you find yourself using most often? Which ones do your curricular materials recommend the most? Do these align with what we know about student learning outcomes?	



Brief 5:

Mathematics Assessment of and for Student Learning

Supporting Evidence-Based P–12 Mathematics Teaching Practice

Produced for the New York State Education Department
by Deborah Loewenberg Ball and TeachingWorks at the University of Michigan

Assessment is used to understand students' knowledge and skills both at a particular point in time (e.g., with a single test) and across a period of time (e.g., with pre- and post-tests and summative assessments). Yet in practice, what is assessed, how it is assessed, and how the measurement information is used varies. This brief outlines complementary approaches to assessing students' mathematical knowledge, skills, understanding, and mathematical identity, and the uses of different kinds of information with care.

Assessment of mathematics learning can support students' mathematical development if it is systematic and comprehensive, and if the assessment tools are useful for instruction. It can enable teachers and school systems to implement appropriate instructional approaches and interventions to support students with particular needs. Assessments also make it possible for teachers and school systems to ensure that effective Tier I instruction is taking place and make instructional adjustments as needed. Research shows that a majority of students will show grade-level proficiency on assessments when strong Tier I instruction is in place (Swanson et al., 2017; Shapiro, 2014). Information about instructional practices that are core to Tier I instruction can be found in Briefs #1, #2, #4, and #6.

Systems of Assessment

The New York State Education Department brief [Science of Reading: Leading for Literacy](#) outlines the requirements of assessment systems to measure student progress and needs. Such systems include screening assessments administered to all students to serve as early identifiers for intervention, diagnostic assessments to support the identification of appropriate interventions for some learners, and specialized diagnostics to understand needs of specific learners, including those with

mathematics disabilities such as dyscalculia and general mathematics learning disabilities. Systems of assessment also include classroom, formative, and informal assessments of student progress toward learning goals—the primary focus of this brief.

The Role of Assessment in Advancing Equity

Assessing student mathematics learning can support equitable teaching and learning when it is done thoughtfully. But this takes care. Too often, assessment data are interpreted solely in terms of gaps or deficiencies in student understanding and performance instead of an overall appraisal of what students know and can do and what they need to learn. Effective instruction builds on students' strengths, understanding, and resources to support students' continued growth, and seeing and naming these is vital. Although identifying what students do not know or cannot yet do can be important for identifying students in need of greater support or for identifying patterns in performance (e.g., differences in performance across groups), these analyses can also contribute to the reinforcement of deficit orientations associated with marginalized identities (Aguirre et al., 2024). Awareness of this tendency is important, and steps must be taken to look critically at differences in assessment results.





Questions to Consider

As educators, we want the results of assessments to reflect students' knowledge and skills accurately. However, sometimes biases get in the way. When teachers and leaders notice differences in performance associated with race, gender, language, and socio-economic status, they must interrogate the reasons for differences and determine whether their results are accurate. Some considerations and questions to ask include:

- **What biases might be inherent in the assessment? Are the contexts of problems or tasks impeding students' ability to demonstrate their knowledge? Are there accessibility issues?**
- **Was the assessment aligned with the instruction and instructional goals? In other words, was the assessment assessing what was taught?**
- **Were students supported with opportunities to self-assess, reflect on their learning, and build metacognitive skills?**

When identifying differences in performance, teachers and leaders must consider whether biases in the assessments might have influenced the results, and if so, how to change the assessments to more accurately measure student knowledge and skill. This may involve addressing accessibility issues such as adjusting the context or wording of problems or it may involve adding or removing items to better align with instructional goals. In other cases, when the items themselves do not appear to be the source of performance differences, teachers and leaders must consider shifts in instructional practice to better support student learning (see Brief #4) and whether the curriculum materials provide appropriate support (see Brief #7).

Attention to a broad view of students' strengths is also crucial. This involves examining information about how students are reasoning, the resources on which they are drawing, and the strategies they deploy. While accuracy matters for some elements of mathematical skill, other elements of mathematical competence are crucial to consider. These include mathematical reasoning, interpreting problems, using representations, and care with language. Students' verbal or written responses and drawings reveal important information about what students understand about the mathematical ideas, their skill with explaining their thinking and reasoning, how they make use of available mathematical tools, and their ability to represent their mathematical ideas. Identifying strengths in each of these areas as well as the underlying mathematical conceptual understandings is necessary to support students' continuing mathematical development.

Purposes for Assessment

Teachers deliberately use a variety of methods to assess what students are learning during and after lessons. Some assessments are *of* learning while others are used *for* learning. Assessments *of* learning are assessments that take place after learning has occurred and are called summative assessments. Assessments *for* learning are formative and take place regularly throughout the course of instruction. New York State's Educational Assessment Strategy describes the specific purposes and frequency of typical assessment types, and the questions each type can help you to answer.

Methods and tools for assessing students' mathematical development can take a variety of forms and should be aligned with the purposes for assessing. Among the purposes are:

1. **Teacher-led formative assessment:** Checking in up close on how students are making sense of instruction. After a lesson or a few, what understanding and skills are students developing and are there aspects that have been confusing that warrant further experience and instruction?
2. **Self-appraisal:** Supporting students to appraise their own learning. Assessment tasks can provide students opportunities to reflect on what they are doing, what they see themselves developing, and what they want to focus on for their continued development. This information is useful not only for students themselves but also for teachers to gain a view of how students appraise their own levels of understanding, skill, and mathematical confidence.
3. **Assessment of mathematical identity:** Learning how students see themselves as mathematical thinkers and knowers, focused on the development of their mathematical identity. This is an important aspect of their growth, for when students see themselves as capable, this supports engagement and identification with mathematics, both of which are goals of mathematics education. That so many adults do not see themselves as "math people" is an outcome that we should seek to change for this generation.
4. **Summative assessment:** Documenting what students have learned after completing a specific unit of instruction, what they may need further support with, and what they can extend beyond the scope of what was taught. This information provides insight about how students have taken up instruction and can contribute to adjustments in teaching and curriculum for a particular set of mathematical concepts, skills, and practices.

5. Standardized assessment (state, national, and international assessments): Using common rigorously developed assessment items that enable analyses at scale, including comparisons, changes over time, and connections to instructional approaches, curriculum, teachers, and other critical variables that affect learning. State assessments are required by federal law to measure student progress toward state learning standards with the goal of fostering educational equity across the state. National and international assessments provide information about patterns in collective student learning. These assessments allow for comparison across states and countries and the examination of trends in mathematics learning over time.

The remainder of this brief focuses mainly on formative and informal assessments as these are used daily by teachers to understand whether instruction needs adjustment, what supports may need to be put into place, how students are understanding themselves and their learning, and how they are making sense of new topics.



Assessments for Learning

Assessments for learning provide frequent opportunities for teachers and students to monitor and learn about their mathematical learning. These assessments take place during the course of everyday teaching and learning and are an inherent part of several high-leverage instructional practices such as eliciting student thinking, setting up and monitoring small group work, and leading a discussion (see Brief #4). They may be informal verbal techniques, structured opportunities for all students to share their thinking, or opportunities for students to assess their own progress and can include entrance routines and exit tickets. Next, we share examples of different types of assessments for learning.



Ideas and Resources

Looking for ideas and resources that are mathematics specific? Check out the following:

- *The Formative 5: Everyday Assessment Techniques for Every Math Classroom*, by Francis (Skip) Fennell, Beth McCord Kobett, and Jonathan A. Wray. A free webinar related to the book can be found [online here](#).
- *Mathematics Formative Assessment: 75 Practical Strategies for Linking Assessment, Instruction, and Learning*, by Page Keeley and Cheryl Rose Tobey. Free resources related to the book can be found on [Page Keeley's website](#).

Teacher-led formative assessments

These include exit tickets, short quizzes, journal writing, tasks matched to what was done in class, or tasks that slightly extend what was done in class.

Formative assessment data support teachers in determining whether instruction is supporting student learning and how to proceed with the next lesson or series of lessons. If the data gathered from a lesson show that very few students understood the content, the teacher may decide to reteach the content using an alternative activity and/or different instructional strategies. If all students understood the content, the teacher may decide to increase the cognitive demand of the next lesson, anticipating that students are ready for more challenge. If the data show that a small group of students needs support with the content or that there are several differing support needs, a teacher may decide to engage in small group instruction.



Spotlight on Practice

Example of an exit ticket after work in class on properties of the number line



1. Is A less than or more than H? How do you know?

2. Is A less than or more than X? How do you know?

3. Label what numbers go at A, X, M, and H. Then, explain how you know what M is.



Other examples of formative assessments for learning include asking students to:

- Restate something or explain something to the whole class
- Critique or comment on an idea (e.g., a piece of student work or a strategy to solve a problem)
- Contribute one idea or takeaway from the lesson
- Write about something they learned (e.g., in their notebooks)
- Complete a problem that reflects the material covered in class but is new (e.g., a new problem or new text)
- Respond in writing to another's idea or claim
- Indicate in writing something they did well and something that was hard for them



Affordances and Limitations: Teacher-Led Formative Assessments

Affordances: Provide information close to instruction of how students are understanding what was worked on in class; can help to calibrate instruction to what students are taking up, and support students to reflect on their own learning; provide teachers with information about how students are seeing their progress.

Limitations: Questions must be well designed to get useful information, and if the questions are too close to exactly what was done in class or too different from it, the information gained might be misleading. Students might not be used to being asked to reflect on their learning and might need some experience and support to learn to do this.

Self-appraisal

These include exit tickets, questions as part of homework assignments, short surveys.

Self-appraisal helps students to reflect on their own learning and progress. While this can serve to give ownership of learning to students, it can also help teachers to understand what specifically students are finding more or less difficult, whether students need individual support, and which students may need scaffolds to assess their own learning. This information can help teachers to design upcoming lessons and to build additional metacognitive moments into their teaching as needed.



Spotlight on Practice

Example of a self-appraisal

Please look back over your work. Remember to write complete answers to these questions.

What was the hardest question on the homework? Why?

What was the easiest? Why?

Which one or ones was the most interesting to you? What made it interesting?

Assessments of mathematical identity

These include short questionnaires, drawings, journal entry prompts.



Spotlight on Practice

Examples of assessments focused on mathematical identity

DO YOU MOSTLY AGREE OR DISAGREE WITH THESE STATEMENTS?

Explain why or give an example.

1. I think of myself as good at math.
2. When a math problem is hard, I think I won't be able to do it.
3. I am not a math person.
4. I can help other people with math.
5. I enjoy learning math most of the time.
6. I am good at solving math problems in everyday life.
7. I avoid doing math.

Draw a picture of yourself doing math. What does it show about you?



Affordances and Limitations: Self-Appraisal

Affordances: Broaden assessment to include data on how students are seeing themselves as mathematical thinkers and doers; signal that this is an important goal of mathematics instruction.

Limitations: Students might not be used to being asked to reflect on their learning and might need some experience and support to learn to do this; questions must be well designed to get useful information.

Task-based assessments

Mathematics is not simply following procedures to produce numeric solutions, so problems that focus only on procedures and answers limit the information that can be gained about students' mathematical development. Mathematics also requires problem-solving, explanation, justification, representation, and selection of mathematical tools. Many mathematical problems and situations that require mathematical analyses and reasoning have more than one solution. Assessments should include tasks that have different solution spaces:

- unique (single) answers;
- multiple but finite solutions (and justifying that all solutions have been found);
- infinitely many solutions; and
- no solutions (impossible) problems.

An appropriately full portrait of students' mathematical development is difficult to capture if mathematics assessments do not include tasks that require these skills.

Assessing what mathematical competence actually involves means utilizing tasks that require students to demonstrate these skills in order to solve them. Such tasks are difficult to design, but there are resources available to help. For more than 30 years, international researchers at the Mathematics Assessment Resource Service have designed, researched, and made available such tasks through the [Mathematics Assessment Project](#). Resources are also available through [Inside Mathematics](#) which hosts a variety of resources, including performance tasks and non-routine problem solving tasks.



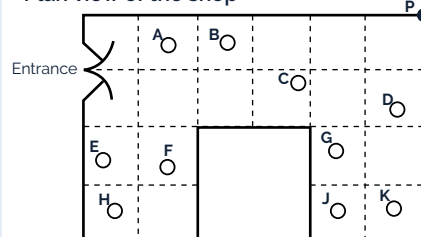
Spotlight on Practice

Example of a task-based assessment

This annotated high school example illustrates how one task can be structured to call upon a number of different mathematical skills:

A shop owner wants to prevent shoplifting. He decides to install a security camera on the ceiling of his shop. The camera can turn right round through 360° . The shop owner places the camera at point P, in the corner of the shop.

Plan view of the shop



1. The plan shows ten people who are standing in the shop. These are labelled A, B, C, D, E, F, G, H, J, K. Which people cannot be seen by the camera at P? Tell how you know.
2. The shopkeeper says that "15% of the shop is hidden from the camera." Show clearly that he is right.
3. Show the best place for the camera, so that it can see as much of the shop as possible. Explain how you know that this is the best place.

Mathematics Assessment Project, 2011



Key Take-Aways

1. Assessment of students' mathematical development is essential for instruction that is aligned with students' strengths, needs, and trajectories.
2. Different kinds and tools for assessment serve different purposes and a strong system for attending to students' development includes multiple types of assessments. These should include deliberate attention to high-leverage content and practices (see Brief #3) and mathematical identities.
3. Regularly assessing students' understanding can help to attune instruction and learning experiences to students' development. Summative assessment can provide information about what students have taken up and mastered.
4. To advance equity and disrupt problematic deficit orientations to students' competence, there must be careful analyses of the tools used and potential biases they may hold.

Key References

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BRIEF #5: Mathematics Assessment of and for Student Learning

DISCUSSION PROMPTS AND QUESTIONS

DIRECTIONS

Step 1 – “Think”

Read the questions and jot down your initial thoughts. Take a moment to reflect on each.

Step 2 – “Pair”

Discuss your responses with the people at your table. Share your insights and listen to theirs.

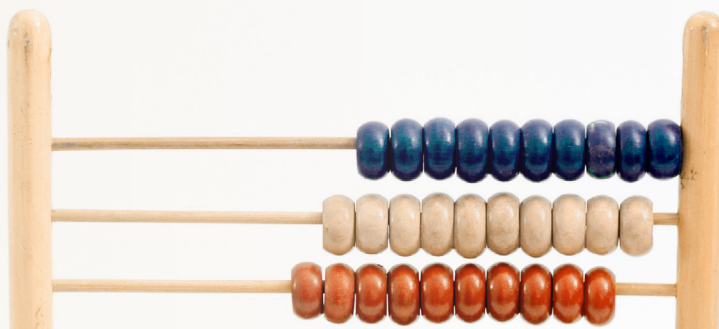
Step 3 – “Share”

Consider what key points you'd like to share with the larger group, if time allows.

General Discussion Questions ***(for use with any of the numeracy briefs)***

- Identify the strengths in your district, school, or classroom that align with the brief(s).
- Identify existing challenges or barriers in your district, school, or classroom that may hinder implementation of the ideas presented in the brief(s).
- Offer suggestions for regional collaboration to support the ideas presented in the brief(s).

BREIF #5 QUESTIONS	REFLECTION / NOTES
What biases might be inherent in the assessment? Are the contexts of problems or tasks impeding students' ability to demonstrate their knowledge? Are there accessibility issues?	
Was the assessment aligned with the instruction and instructional goals? In other words, was the assessment assessing what was taught?	
Were students supported with opportunities to self-assess, reflect on their learning, and build metacognitive skills?	



Brief 6:

The Role and Challenges of Using Representations

Supporting Evidence-Based P–12 Mathematics Teaching Practice

Produced for the New York State Education Department
by Deborah Loewenberg Ball and TeachingWorks at the University of Michigan



Instructional programs from prekindergarten through grade 12 should enable all students to: (a) create and use representations to organize, record, and communicate mathematical ideas; (b) select, apply, and translate among mathematical representations to solve problems; (c) use representations to model and interpret physical, social, and mathematical phenomena.

(National Council of Teachers of Mathematics, 2000, p. 67).

Representations: Central Resources and Practices in Mathematics

In mathematics teaching and learning, representations are central resources in mathematical reasoning, problem-solving, and communicating for both students and teachers. "Representations" refers to several different kinds of models and materials important in mathematics itself and in support of mathematics learning (Lesh et al., 1987):

- **Physical:** e.g., manipulative materials such as Unifix cubes, base ten blocks, algebra tiles
- **Visual:** e.g., diagrams, drawings, graphs
- **Symbolic:** e.g., numbers, expressions, equations
- **Verbal:** e.g., explanations and discussions
- **Contextual:** e.g., real-world situations and problem-based contexts

Representations allow teachers and students to work with abstract mathematical ideas in a range of ways. In teaching, each of these types of representations can highlight different aspects of the mathematical

ideas, helping to support student learning. Physical representations, often referred to as concrete representations or manipulatives, allow students to physically move objects, make trades for equivalent objects, and rearrange objects to help them to solve problems. Some physical objects maintain key mathematical features they are trying to represent. For example, base ten blocks and bean sticks show ten as one object made of ten individual objects connected together. Other physical objects are more abstract. Place value disks are simply colored chips representing each of the place values but do not make visible the differences in quantity or the concept of grouping fundamental to place value.



Visual representations more permanently represent the mathematical ideas, but like physical representations can be more or less abstract. For example, multiplication arrays closely represent the individual objects being multiplied as well as the product, but area models make clear that the answer is an area that is a product of lengths. Although there exists a debate around the order in which these representations should be used in learning, there is agreement that engagement with them is necessary for conceptual understanding and meaningful use of procedural skill.

Choosing and building representations are also core to doing mathematics. As explained by the National Council of Teachers of Mathematics (2014), “When students learn to represent, discuss, and make connections among mathematical ideas in multiple forms, they demonstrate deeper mathematical understanding and enhanced problem-solving skills” (p. 24). Representation is not only an external process involving, for example, diagramming, drawing, and explaining. It is also an internal process, as students develop rich mental models of mathematical concepts over time through making sense of and using multiple representations. Choosing and using appropriate representations is a key part of mathematical fluency and problem solving.

The Work of Using Representations in Teaching Mathematics

Using representations with care requires a repertoire appropriate for particular mathematical content and specialized mathematical understanding of how specific representations can highlight crucial aspects of the math. Equitable and effective mathematics teaching engages students in creating, interpreting, using, and making connections across representations. Students are given opportunities both to exercise agency in using representations that make sense to them and to learn to use new representational forms to deepen their understanding (Taylor & Dyer, 2014). Representations are central in providing access to mathematics for students with diverse background knowledge and approaches. In particular, research has demonstrated that multilingual learners, students with special needs, and those struggling with a concept are better able to engage in mathematical discourse when they are able to gesture and show their thinking with the supports of manipulatives and visual representations (Fuson & Murata, 2007).

Additionally, the classroom environment must be welcoming and supportive of the broad use of representations which can support engagement with and understanding of rigorous mathematics. A welcoming and affirming mathematics classroom creates an environment where students can comfortably use the mathematical tools at their disposal to make sense of mathematical ideas, share their thinking, and justify their reasoning. Without this welcoming environment paired with an emphasis on symbolic reasoning, students are likely to see manipulatives and representations as “less than” symbolic reasoning rather than a powerful tool for sense-making. For more information on creating a welcoming culture and fostering high expectations, see the [Four Principles of Culturally Responsive-Sustaining Education](#).

Spotlight on Research

As students interact with representations in conversation with one another, they authentically and organically create “bridging language”—transitional language that bridges the language of specific problem contexts with official mathematical language. For example, in examining and discussing tables and graphical representations of slope, students generated language like “it increased by X each time.” These various forms of language mutually reinforce students’ understanding of the underlying mathematical concepts while also developing official mathematical vocabulary (Herbal-Eisenmann, 2002).

Using Representations: What Teachers Need to Know and Be Able to Do

The National Council of Teachers of Mathematics (2014) guidance identifies using and connecting mathematical representations as one of eight core mathematics teaching practices. In order to use representations effectively in instruction, teachers need to understand, among other things, the instructional and mathematical affordances and constraints of different representations in the service of learning goals. Opportunities to explore and compare different representations and materials is a useful focus for professional learning. Using them well also depends on deliberate mathematical consideration of specific models, contexts, materials, or other forms. Selecting and adapting tasks that prompt students to interact with representations and to create their own representations to solve problems matters. All of this requires teachers to build a flexible repertoire of practices for supporting students to work with and make sense of representations.

Preparing for instruction using representations

In using curriculum materials to prepare for instruction, teachers attend not only to the mathematical goals of a unit or lesson but also to the set of representational resources provided to support those goals. Teachers must weigh the strengths and limitations of different representations based on the mathematics they make available. Teachers’ decisions are also informed by what they know about their students’ previous experiences and the challenges students are likely to encounter. For example, which models will feel familiar to students and which ones may need to be more formally introduced? How will students likely interact with a set of manipulatives for the first time? Teachers must translate this understanding of their students and the relative affordances of different representations into their planning.

Mathematical tasks vary in the representational demands they place on students. Some tasks are designed for students to use particular manipulatives or models to develop conceptual understanding. Some tasks invite students to be critical consumers of data representations. Other tasks allow students to invent or choose a representation that supports their problem-solving. Teachers consider these task demands as they make decisions about when to adapt them and how to implement them with their students and mathematical purposes in mind.

Using representations during instruction

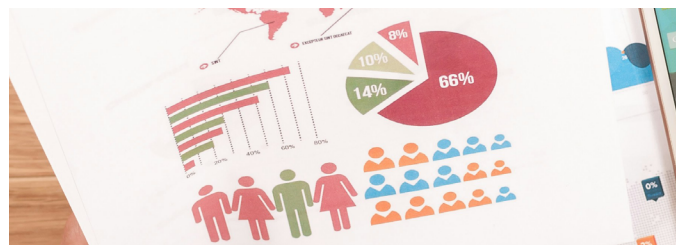
As students engage in mathematical tasks, teachers encourage sense-making by providing time for students to explore and experiment with representations as they solve problems, offering support when needed. This support may sometimes include teaching students how to use a particular manipulative (e.g., algebra tiles) and setting up the language and routines for their productive use. Making connections across representations is crucial, and although teachers may explicitly show how one representation maps onto another, it is also important to support students in developing both the disposition and skill to do so themselves. Teachers may model this skill by asking questions to elicit student thinking while deliberately recording those ideas on the board using a combination of symbolic representations, models, and annotations (Garcia et al., 2021). This representational work supports students in making sense of one another's ideas and models for students how to use representations to communicate about mathematics.

To help students make connections across representations, teachers decide how to select and sequence student work to present a range of representations and strategies (Smith & Stein, 2011). They take care to use board space strategically in the service of collective sense-making (Leatham et al., 2023) and they ask questions that press students to look across representations to make comparisons and notice connections.



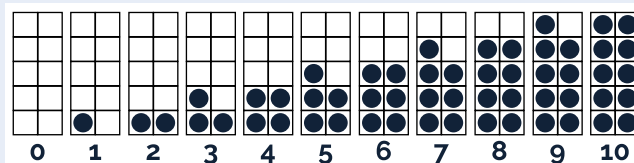
Learn More about Student Representations

For examples of the kinds of questions teachers can ask along with a vignette of a classroom discussion, check out [Mathematical Representations: A Window into Student Thinking](#), a blog post on the website of the Institute for Learning at the University of Pittsburgh by Amaru Pareja, a sixth-grade teacher in Syracuse.



Spotlight on Research

As Christine Losq (2005) clearly demonstrates in her article "Number Concepts and Special Needs Students: The Power of Ten-frame Tiles," not all representations are created equal. There are some crucial affordances of ten-frame tiles that make them a good representational choice for teachers seeking to foster students' counting and computation skills. Ten-frame tiles show a unique image for each number from 0 to 10.



The 5 x 2 array structure of the ten-frame allows each number from 0 to 10 to have a unique picture. This structure supports children to subitize quantities as they come to "internalize the 'shape' of a quantity in instantly recognizable pictures." Unlike cube trains or base ten blocks, the dots on ten-frame tiles are "clearly separate and countable." Additionally, children can see each represented quantity in a range of ways, supporting flexibility and number sense.



Spotlight on Research

Number lines are a powerful representation that support both number sense and operations. In their article "Building a Strong Conception of the Number Line," John Lannin, Delinda van Garderen, and Jessica Kamuru (2020) explain how carefully designed task selection can help students to build and refine their own conceptions of the number line, including the spatial relationship between numbers. They share case studies of children's number line understanding, examining how their understanding changed as they engaged with a series of tasks. They argue that questions such as "what friendly numbers are close to [a particular number]," "what friendly numbers are far away from [a particular number]," and "how are these numbers on the number line related" can keep the focus on big mathematical ideas while refining number line conceptions. When working with number lines, teachers should keep in mind that the number line is not only a representation that can support learning, but is also itself an important mathematical object in later mathematics.

Spotlight on Research

The Class Party Problem

Ms. Simpson formed a committee of students to investigate sites for a class party. The committee does not know how many of the students will attend and has not yet decided whether each class member can invite a guest. However, they have brought back the following details on parties:

- Water World • Swimming, hot dog, chips, and drink. Cost: \$100 to reserve the pool and \$5 per person.
- Pizza Pi and MoviePlex • Pizza, drink, movie. Cost: \$10 per person.
- Skate Til Late • Skating and ice cream (eat before you come). Cost: \$200 to rent the skating rink and \$2 per person for skate rental.

If price is the primary focus of your decision, which party option is the best?

How would you convince other class members of this?

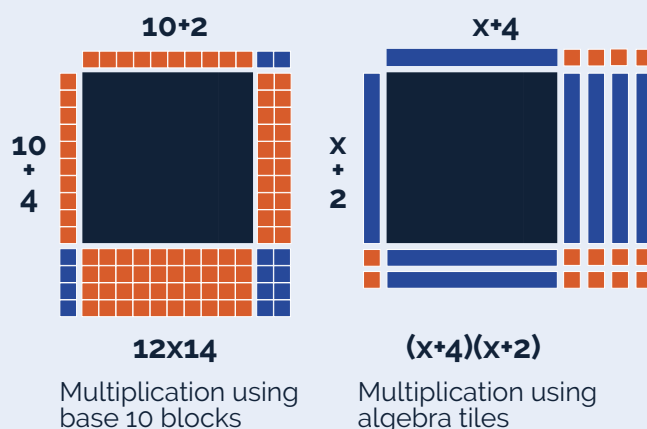
How would you advise other groups planning a party, so that they could make a decision, no matter how many guests they had?

Fig. 1 The Class Party problem

When students are supported to make decisions about the usefulness of various representations, they not only can deepen their mathematical understanding but also can develop a sense of agency. In "Representation as a Vehicle for Solving and Communication," Ron Preston and Amanda Garner (2003) illustrate the power of designing and using tasks like the "Class Party Problem" to engage students in selecting, using, communicating about, and evaluating a range of representations to solve a real-world problem. The task prompted 7th grade pre-algebra students working in small groups to use all of the following representations: word rules, informal and formal language, tables, graphs, equations, patterns, and hand gestures. Through discussion of the various groups' solution strategies, students made connections across representational forms and evaluated the advantages and disadvantages in relation to the problem context. Check out their classroom vignette in the article for more on how students engaged with the task and one another's solutions. They also provide a useful table summarizing some of the typical uses, advantages, and disadvantages of different representations, and share insights on teaching dilemmas that arise when inviting students to choose their own representations.

Spotlight on Research

Representations can support students to experience mathematics as a coherent interconnected system of concepts. In their article "The Area Model: Building Mathematical Connections," Alyson Lischka and Christopher Stephens (2020) illustrate the power of manipulatives and rectangular area models to do just that. Using examples from across grade levels, they demonstrate how manipulatives and area models can support the teaching and learning of a range of concepts involved in multiplicative reasoning with whole numbers, from fractions and binomial expressions in algebra to derivatives in calculus. "Students who learn to visualize the numerical application of the distributive property in this way [using area models]," they report, "build a foundation for considering multidigit, fraction, and binomial multiplication with an area model" (p. 189).



Reflect & Analyze:

1. How do you currently support students to invent, use, compare, and connect representations? Were there aspects of using representations in the mathematics classroom that you hadn't thought about before?
2. Which representational forms would you say your students are most comfortable with? Which are you most comfortable with? Why? How does that impact instruction in your classroom?
3. What would you like to learn more about in order to feel more confident using different representations in your classroom? Who in your context could help provide resources for you with this? How might you work with grade-level or building colleagues to learn more about representations?

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BRIEF #6: The Role and Challenges of Using Representations

DISCUSSION PROMPTS AND QUESTIONS

DIRECTIONS

Step 1 – “Think”

Read the questions and jot down your initial thoughts. Take a moment to reflect on each.

Step 2 – “Pair”

Discuss your responses with the people at your table. Share your insights and listen to theirs.

Step 3 – “Share”

Consider what key points you'd like to share with the larger group, if time allows.

General Discussion Questions ***(for use with any of the numeracy briefs)***

- Identify the strengths in your district, school, or classroom that align with the brief(s).
- Identify existing challenges or barriers in your district, school, or classroom that may hinder implementation of the ideas presented in the brief(s).
- Offer suggestions for regional collaboration to support the ideas presented in the brief(s).

BREIF #6 QUESTIONS	REFLECTION / NOTES
How do you currently support students to invent, use, compare, and connect representations? Were there aspects of using representations in the mathematics classroom that you hadn't thought about before?	
Which representational forms would you say your students are most comfortable with? Which are you most comfortable with? Why? How does that impact instruction in your classroom?	
What would you like to learn more about in order to feel more confident using different representations in your classroom? Who in your context could help provide resources for you with this? How might you work with grade-level or building colleagues to learn more about representations?	

Brief 7:

Understanding, Using, and Modifying Curriculum Materials

Supporting Evidence-Based P–12 Mathematics Teaching Practice

**Produced for the New York State Education Department
by Deborah Loewenberg Ball and TeachingWorks at the University of Michigan**

Background

Curricular materials are key resources for teachers and are necessary, but not sufficient, for ensuring student learning. Skillful evidence-based instruction (see Briefs #1, #2, #4, and #6), appropriate assessment of learning (see Brief #5), and deep understanding of high-leverage mathematics content (see Brief #3) must be present alongside high-quality materials and be supported through professional learning and strong leadership (see Brief #8). Curriculum materials should be selected by a team to ensure alignment with research-based instructional practices and grade-level content standards. However, even when strong, research-based materials are selected, teachers must analyze, adapt, and implement their school's chosen materials in ways that meet the learning needs of students.

Analyzing Curricular Materials with a Critical Eye

When analyzing their school's chosen materials, teachers examine them carefully, considering the content of the curriculum, how mathematics is represented, and the diversity of cultures and people shown in the curriculum. They consider the ways the materials align with learning standards and evidence-based practices, center culturally responsive-sustaining practices and social-emotional learning, and support the needs of all learners.

Learning standards and evidence-based practices

How is mathematics represented in the curriculum materials?

Engaging in mathematics proficiently requires conceptual understanding, procedural fluency, strategic competence, adaptive reasoning, and a productive disposition (National Research Council, 2001) (see Brief #1). Building this proficiency requires

opportunities to engage in mathematics activities that make space to practice these skills. As teachers consider how mathematics is represented in materials, they look at the mathematical activities that students are asked to engage in and the types of skills regularly called for. Do those activities represent, in an age-appropriate way, the fullness of mathematical proficiencies and practices as outlined in the [New York State Next Generation Mathematics Learning Standards](#)? Or do they overemphasize a narrow band of mathematical activities (e.g., calculational procedures) that can lead to incomplete mathematical learning and/or the development of negative mathematical identities (Lockhart, 2009)? Are students only asked to engage with these proficiencies and processes in “application” lessons at the ends of units or are these woven throughout the materials? To build such proficiencies, students need regular and repeated opportunities to utilize these skills.

In addition to analyzing opportunities for students' mathematical engagement, teachers should consider how mathematical coherence is built over the course of the curriculum. Are mathematical topics treated in isolation from each other as a set of independent skills, or are students supported to build connections, see the “big picture” of the mathematics, and develop concepts across topics? Are too many topics “covered,” leading to surface coverage or are fewer focal topics addressed deeply?



How are evidence-based practices represented in the curriculum materials?

Curriculum materials designed to support student learning are structured in ways that allow teachers to engage in high-leverage practices (see Brief #4). Materials should include opportunities for students to engage in rich, cognitively demanding mathematical tasks (Stein et al., 2000), making space for teachers to elicit students' mathematical thinking and press for reasoning and justification. Supports for teachers should include prompts and guidance for mathematics discussions, guidance for the use of representations and manipulatives, and structured opportunities for students to engage in collaborative group work. These supports should also include sufficient professional learning that helps teachers to understand and use the materials as well as guidance on differentiation. Depending on the specific content, explicit instruction and teacher modeling should sometimes come first, but in order to ensure that students develop conceptual understanding, it should often take place after strategically designed exploration activities to support students' sense-making.

Culturally responsive-sustaining practices and social-emotional learning

Who is represented in the curriculum materials and how are they shown?

Learning that mathematics is a human endeavor that has evolved over time and across contexts is vital for students' mathematical development. This involves looking to see that multiple cultures and identities, including the home cultures and identities of one's own students, are authentically represented in instructional materials. Although the use of names and cultural practices (e.g., food and holiday traditions) may be included in the text and tasks, this is insufficient and does not represent the wide range of cultural contributions to and uses of mathematics. Is the development of mathematics across time and place made visible? Does the curriculum provide opportunities to learn about others' experiences and mathematical contributions? Reading curriculum materials with a critical eye also involves examining whether people from a variety of cultures and ability statuses are represented as creators and doers of mathematics and whether harmful stereotypes are reproduced or reinforced (Nguyen & Ryan, 2008; Spencer et al., 1999).

How does the curriculum support a welcoming and affirming environment?

A welcoming and affirming environment is core to culturally responsive-sustaining education. Curriculum materials can contribute to the creation of such an environment when they focus on thinking, reasoning, and sense-making in appropriate balance with fluency and accuracy. They encourage a strong classroom environment by making space for mathematical discourse, discussion, and collaborative work.



Supporting the needs of all learners

What supports are integrated into materials for multilingual learners and students with disabilities?

Strong curriculum materials integrate supports for students into lessons as a natural part of instruction rather than treating these supports as "add-ons." Research-based mathematics supports for multilingual learners such as visual representations of mathematical ideas and the Stanford Graduate School of Education's mathematical language routines (Zwiers et al., 2017) should be a regular part of lessons, integrated into plans rather than sitting as an external resource. The Institute of Education Sciences *What Works Clearinghouse Practice Guides* can be a resource for determining appropriate supports for multilingual learners. Tasks with multiple entry points, strategies, or approaches should be present in a majority of lessons, allowing access for students with disabilities as well as challenge for students.

Adapting Curricular Materials with Care

A high-quality curriculum supports students' development of both conceptual and procedural knowledge and attends consistently to the use of mathematical practices. To contribute to students' learning across all of these domains, instructional materials must be used with skill. In particular, teachers must understand how to adapt materials without changing students' learning opportunities, while also ensuring that materials contain sufficient scaffolds and framing to support cultural relevance and equitable practice. Following are principles to guide and pitfalls to avoid when adapting curricular materials.

Principles	Example
Account for your students' prior knowledge and skills as you prepare to use and adapt particular lessons/ materials.	Administer a brief formative assessment of the prerequisite skills and knowledge that students need for an upcoming unit. This will help you know what supports your students might need to be successful, and what kinds of adaptations you might need to make.
Consider with care whether explicit instruction guidance is mathematically accurate and not just getting students to get right answers, and modify as needed.	Attend to the examples used and the language used to make the ideas clear and conceptually and mathematically accurate. Revise or change explanations that distort or mislead.
Consider where additional explicitness might be needed or where it might constrain students' sensemaking.	Look closely at the learning goal. Ensure that the instruction is aligned with what students are to learn.
Ensure that you keep and/or create additional opportunities for students to engage in mathematical practices.	Expand opportunities for students to explain their mathematical thinking (e.g., turn-and-talks). Use a variety of instructional routines that engage students in mathematical practices (Kelamanik et al., 2016).
Make changes that motivate students to engage in the curriculum.	Use a variety of contexts that students find interesting, including imaginative ones. Pose novel tasks/questions that challenge and intrigue students' attention.
Maintain the instructional/learning goal when making revisions and differentiating.	When differentiating during independent practice, small group, or partner activities, adjust tasks in ways that keep the mathematical focus.
Pitfalls	Example
Taking away opportunities for students to engage in mathematical practices	Removing opportunities for students to share their mathematical explanations
Reducing or omitting opportunities for students to develop conceptual knowledge	Proceduralizing a complex, open-ended task by modeling a solution strategy before students have an opportunity to work on the task
Changing the mathematics to make it easier	Omitting key parts of a task because students may struggle with it rather than providing appropriate learning supports
Using math tasks that contain contexts that reproduce or reinforce gendered and/or racialized stereotypes about students' home cultures and who can do mathematics	Using tasks where girls are using mathematics only for stereotypically female tasks such as baking and playing with dolls
Using math tasks that contain contexts that reproduce/ reinforce reductive views of what it means to do mathematics	Using math tasks that contain "pseudocontexts" (Boaler, 2015) that represent math as about questions no one would ever ask (e.g., a problem about a dog pen that is much too small for a dog) Using contrived scenarios that are not how mathematics is actually done and used in the world—unless the purpose is clearly playful
Eliminating opportunities for student or teacher-led synthesis of ideas	Removing direct instruction during the consolidation/ synthesis portion of the lesson



Key Take-Aways

1. Reading curriculum materials with a critical stance means attending not only to whether a variety of cultures are represented but also to how they are represented. Pay attention to whether curriculum materials reduce cultures to “food and flags” or reinforce harmful stereotypes. Attend to whether materials represent the range of ways that cultures have made and used mathematics, the contexts in which mathematics is done, and the people who do mathematics.
2. Adapting curriculum materials with care means that any adaptations maintain the core mathematical ideas, structures, practices, and cognitive demand while supporting children to develop positive mathematical identities.



Reflect & Analyze:

1. How are both the discipline of mathematics and students' home cultures represented in one unit (or even just one lesson) in your curriculum? Are they represented in a way that will aid in the development of authentic, positive mathematical identities?
2. Have you seen any of the pitfalls in your curriculum? Did you respond to them? If so, how? If not, in what ways could you respond to address them?

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BRIEF #7: Understanding, Using, and Modifying Curriculum Materials

DISCUSSION PROMPTS AND QUESTIONS

DIRECTIONS

Step 1 – “Think”

Read the questions and jot down your initial thoughts. Take a moment to reflect on each.

Step 2 – “Pair”

Discuss your responses with the people at your table. Share your insights and listen to theirs.

Step 3 – “Share”

Consider what key points you'd like to share with the larger group, if time allows.

General Discussion Questions ***(for use with any of the numeracy briefs)***

- Identify the strengths in your district, school, or classroom that align with the brief(s).
- Identify existing challenges or barriers in your district, school, or classroom that may hinder implementation of the ideas presented in the brief(s).
- Offer suggestions for regional collaboration to support the ideas presented in the brief(s).

BREIF #7 QUESTIONS	REFLECTION / NOTES
How are the discipline of mathematics and students' home cultures represented in one unit (or even just one lesson) in your curriculum? Are they represented in a way that will aid in the development of authentic, positive mathematical identities?	
Have you seen any of the pitfalls in your curriculum? Did you respond to them? If so, how? If not, in what ways could you respond to address them?	



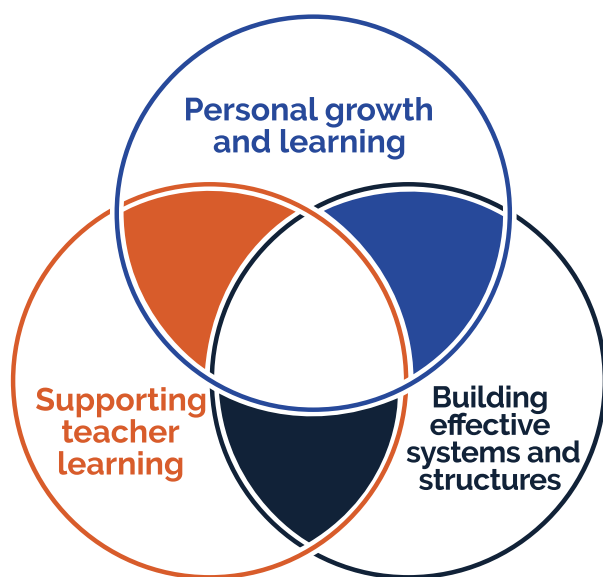
Brief 8:

The Role of Leadership

Supporting Evidence-Based P–12 Mathematics Teaching Practice

Produced for the New York State Education Department
by Deborah Loewenberg Ball and TeachingWorks at the University of Michigan

Leadership Focus: Equitable and Impactful Mathematics Teaching



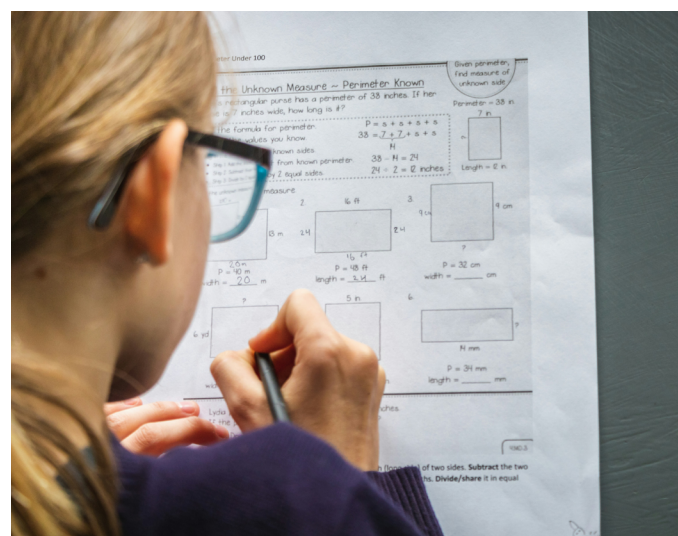
Ensuring that all students have opportunities to thrive mathematically, develop strong identities as mathematics learners, and engage in meaningful collective mathematics learning in tandem with building skill and fluency requires leaders who understand, support, and make space for the continued development of evidence-based mathematics teaching. Teachers must have opportunities for content-specific collaborative learning that provide models of effective mathematics teaching practice paired with coaching and feedback to support their growth (Darling-Hammond et al., 2017; Hill & Papay, 2022). Leaders cannot provide this support alone, but there is work that every leader can do to support equitable and impactful mathematics teaching. The diagram above shows the focal areas for leaders in mathematics teaching and learning.

Leadership Area 1: Personal Growth and Learning

Ensuring equitable and impactful mathematics teaching in our schools requires that leaders at all levels have a vision of evidence-based mathematics instruction and that they understand the tensions inherent in mathematics teaching and learning in schools. Below are three key areas of personal growth and learning for leaders.

Understand effective mathematics teaching and learning

The body of research on equitable and impactful mathematics teaching practice is extensive. Leaders need to be familiar with what is known about such teaching and not just use the common labels. They also need to learn about the differences in effective mathematics teaching across grade levels and develop a clear vision of what effective mathematics teaching looks like at each grade band.



Learn More about the Evidence

Check out other briefs in this series:

- Brief #1: The Research Base for Mathematics Teaching and Learning
- Brief #2: Debunking Myths about Mathematics Teaching and Learning

Look at citations. Read the research that supports particular recommendations. Is it from the field of education or is it from other fields? What claims are made? What groups of students are involved in the studies, and in what sorts of settings? It will often turn out that valid knowledge about what we know about mathematics teaching and learning is much more complex and nuanced. Other resources to investigate include the *Equity in K-12 STEM Education: Framing Decisions for the Future* report by the National Academies of Sciences, Engineering, and Medicine, the Institute of Education Sciences *What Works Clearinghouse Practice Guides*, and the National Mathematics Advisory Panel *Foundations for Success* report.

Understand tensions

Mathematics teachers and leaders must manage several tensions in their work to support student learning. Much of the time, these points of tension between what often feel like competing goals are important but serve different and complementary purposes. Leaders must build their own awareness of: (1) the purposes that each goal serves; (2) why these goals may feel in competition with each other to teachers; and (3) how their messaging to teachers can either heighten or ease the tension. Such tensions include:

- Increasing students' mathematical fluency and automaticity vs. building understanding through problem-based instruction (Hiebert & Grouws, 2007).
- Utilizing explicit instruction vs. centering student thinking and reasoning (National Mathematics Advisory Panel, 2008).
- Increasing test scores vs. developing positive mathematical identities (Yang et al., 2024).
- Ensuring fidelity to the curriculum and pacing guide vs. adjusting pacing and materials to meet the needs of students.

Investigate how communication style and messaging impact teachers' work and support connections

One challenge of leadership is communicating clearly and sensitively, anticipating how the message will be heard and received by teachers. When communicating with teachers about mathematics teaching, leaders must ensure that their message is strong while at the same time acknowledging the nuance and tension involved in teaching. Ensuring that mathematics experts in the building or district are involved in key conversations and messaging can help to ensure that the leader is communicating as intended.

Questions to Consider when Crafting a Message

- Are you communicating about a change in teaching practice in your system? If so, what do you need to do to ensure readiness for the change? If not, how can you communicate that this is not a change, but reinforcing strong practice already taking place?
- In what ways might the message be heard and interpreted by others?
- Is this message part of a leadership-led shift or is it the result of a grassroots teacher effort (different change models require different ways of communicating)?
- Are you acknowledging and addressing the issues at tension with the core of the message and providing assurance that these tensions were considered?
- Are you lifting up expertise in your system, acknowledging the strengths that others in the system bring to the work?

Leadership Area 2: Supporting Teacher Learning

Mathematics teaching that continually provides opportunities for students to thrive mathematically, develop identities as mathematics learners, and engage in meaningful collective mathematics learning in tandem with building skill and fluency requires that teachers have opportunities to learn, reflect, and grow. But schools and systems are not only responsible for mathematics teaching; leaders must balance content-specific teacher learning needs with systems' needs.



Ensure subject-specific learning opportunities for teachers and leaders

Although teaching practices that enable student learning may be the same across disciplines (see Brief #4), enacting those practices in mathematics requires integration of mathematical knowledge for teaching (see Brief #3), an understanding and utilization of the research base for mathematics teaching and learning (see Brief #1), and a shared vision of equitable mathematics teaching. This means that teachers (and leaders) require regular mathematics-specific opportunities to continue to learn about and improve their teaching. Professional learning research (Darling-Hammond et al., 2017; Hill & Papay, 2022) shows that these opportunities should:

- Be content-specific and teaching-practice focused.
- Model and provide examples of effective mathematics teaching practice.
- Allow for teacher collaboration.
- Involve reflection, coaching, and/or feedback.

Ensure curriculum-specific learning opportunities for teachers and leaders

Mathematics teachers and leaders need regular professional learning to support their use and adaptation of curriculum materials (see Brief #7). Too often this professional learning is a brief orientation to the resources available in the curriculum. Although this is necessary, regular professional learning that supports teachers and leaders in examining unit and lesson structures and connections, trajectories of topics and representations across grades, and embedded supports for learners are needed. These may take the form of start-of-unit professional learning sessions, allowing teachers and administrators to focus on the content and materials to be used immediately.

Opportunities to work with colleagues to examine and adjust lessons that focus on high-leverage content (see Brief #3) can help ensure that these crucial mathematical ideas are taught well. Teachers and leaders may meet regularly to try out, unpack, and adjust lessons, ensuring that all teachers understand the structure, purpose, and mathematical goals of essential lessons.

Provide space and structures for practice-focused reflection and feedback

Research on professional learning, coaching, and improvement science all name reflection and focused feedback as key levers for growth. When our goal is to improve mathematics teaching practice, the focus of reflection and feedback should be squarely on teaching practice and its relationship with student learning. However, reflection and feedback do not typically occur spontaneously.



Ways to Create Space for Reflection and Feedback

- Build regular opportunities for reflection and feedback into existing structures like department meetings or staff meetings.
- Encourage mathematics teachers to observe one another's teaching and provide tools and protocols for productive conversations about the observation.
- Provide and use tools that specify agreed-upon work that teachers do when carrying out a specific mathematics teaching practice for both feedback and reflection.
- Develop a shared vision of effective mathematics teaching practice together with mathematics teachers and leaders.
- Consider the balance between facilitative (coach asks questions to prompt teaching thinking), directive (coach provides expertise and does most of the thinking), and dialogical coaching (coach shares expertise as needed and engages in thinking side-by-side with the teacher) (Knight, 2021) and which approach best suits the teachers you support.



Support connections between other frameworks and initiatives

For many teachers, the numerous frameworks and initiatives championed by leaders can feel disjointed, disconnected, and unrelated to their daily work of teaching mathematics. For instance, a system-level initiative focused on developing disciplinary literacy may leave mathematics teachers asking “what does this have to do with me?” if connections are not made between disciplinary literacy, equitable mathematics teaching, and the daily work that mathematics teachers do.

Leaders should:

- Be clear themselves about the connections, overlaps, and distinctions between system frameworks and initiatives. These frameworks include the [Educator Effectiveness Framework](#), the [Culturally Responsive-Sustaining Education Framework](#), the [Social Emotional Learning Benchmarks](#), and the [New York State Next Generation Mathematics Learning Standards](#), all of which are outlined below.
- Provide reflective prompts and processing time for teachers to make sense of the connections, overlaps, and distinctions between frameworks and initiatives and a shared vision of equitable mathematics teaching.

Framework	Description	Connection with equitable mathematics teaching
Educator Effectiveness Framework	To ensure equitable access to effective educators, this framework creates coherent systems of development and support that place instructional practices tied to student learning at their center.	The framework identifies the components of a district system that are reflective of broad root causes that can serve as barriers to equitable access to high-quality mathematics teaching.
Culturally Responsive-Sustaining Education Framework	The framework names principles for teachers in developing a student-centered learning environment that values and utilizes students' many identities as: (1) creating a welcoming and affirming environment; (2) fostering high expectations and rigorous instruction; (3) identifying inclusive curriculum and assessment; and (4) engaging in ongoing professional learning and support.	Equitable mathematics teaching addresses these principles through: <ul style="list-style-type: none"> • Ensuring environments that build positive mathematical identities; • Utilizing high-leverage teaching practices that engage students in rigorous mathematical work; • Modifying curriculum materials to be inclusive of learners; and • Encouraging continued teacher learning specific to mathematics.
NY Social Emotional Learning Benchmarks	These benchmarks outline social emotional learning indicators and grade-band teaching opportunities aligned to these indicators to help support students to become empowered learners.	The benchmarks and supporting activities are directly tied to equitable mathematics teaching by supporting identity development, reinforcing interpersonal skills required for collaboratively engaging in mathematics, and creating safe and supportive learning environments necessary for sharing thinking.
New York State Next Generation Mathematics Learning Standards	These standards represent the mathematical knowledge, skills, and understanding that students need to function in a world dependent upon the application of mathematics. They reflect a balance of conceptual understanding, procedural fluency, and application, and represent a significant level of achievement in mathematics that will enable students to successfully transition to post-secondary education and the workforce.	The NGMLS are directly tied to equitable mathematics teaching as they emphasize the core strands of content that students should learn and the ways in which students engage with mathematics across the grades. These are closely linked with the mathematical knowledge for teaching required for effective teaching, and the high-leverage teaching practices closely tied to student learning.



Leadership Area 3: Building Effective Systems and Structures

Build systems and schedules that support teacher learning

Leaders make space for teacher learning by building systems and structures that routinely allow for learning, feedback, and growth. Leaders do this by:

- Building regular opportunities for reflection and feedback into the school calendar. Regularity translates into routines, making reflection and feedback a part of the way teachers work. This may be weekly collaborative planning time, monthly problem-solving communities, or quarterly professional learning sessions, among other possibilities. It could also take the form of regularly scheduled learning walks, a means to make teaching practice public.
- Schedule opportunities for mathematics teachers to observe one another's teaching and provide tools and protocols for productive discussion and feedback. This may include supporting teacher observations during prep time, providing release time for teachers to observe one another teaching the same lesson, or providing equipment to record their class to share with each other.
- Develop systems and supports for experienced mathematics teachers to take on mathematics leadership roles, such as peer observation with feedback, formal mentoring roles, professional development leaders, etc.
- Advocate for restructuring the professional work of teaching so that built into the schedule are regular opportunities for the work necessary for effective instruction. In some schools and districts, this might look like a block of time (e.g., half a day) set aside each week for teachers to examine and look for patterns in student work, study data on student learning (Horn & Kane, 2015), design instruction to respond to evidence of student learning, communicate with families, search for and develop materials and resources, engage with colleagues about specific aspects of student learning and instruction, and participate in professional learning. In others, it may look like regularly scheduled professional development days to do this work. Regardless of structure, this will require funding and supplying high-quality supplementary programs for students.

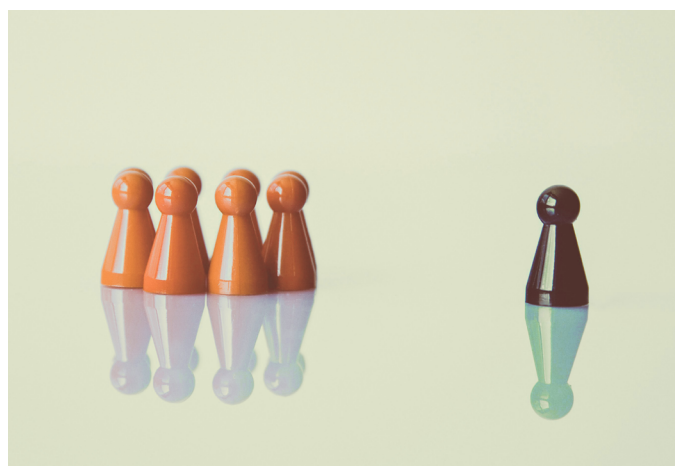
Assessment

Assessment frameworks are abundant (see Brief #5). In fact, the [NYSED's Literacy Initiative](#) outlines [key action steps for leaders](#) around literacy assessment that apply to mathematics assessment, including implementing assessment systems for identifying student progress and needs as well as building systems for effective assessment-instruction links to support educators and serve all learners. Mathematics assessment systems for identifying student progress and needs should make use of screening tools and curriculum-based measures (as called for in response-to-intervention guidance). The National Center on Intensive Intervention includes a review of common screening and progress monitoring tools.

In addition to these action steps that support considering how to use data to impact learning, leaders must understand what mathematics assessments are available (and are being used), know how data from these assessments should impact decisions, and support teachers to engage in collaborative data analysis.

Make use of building resources

Building leaders are not solely responsible for supporting mathematics teaching and learning. Leaders should empower others in their buildings to make use of their expertise to support mathematics teaching. This might involve leveraging experts to provide feedback on teaching, to support the development and implementation of professional learning, or to contribute to collaborative efforts such as curriculum selection and development.





Key Take-Aways

1. Messaging matters! Know what strong mathematics instruction looks like and the research that backs it up and ensure that you are communicating these ideas to teachers with nuance.
2. Mathematics teachers need mathematics-specific teacher learning opportunities. These should be focused on the intersection of mathematics content and teaching practice, allow for modeling of teaching practice, and provide opportunities for collaboration and feedback.
3. Developing and sustaining effective mathematics teaching practice takes time and space. Leaders can use creative scheduling and others with expertise to support this work.



Reflect & Analyze:

1. What messages are you sending your mathematics teachers about instruction? What might teachers be taking away from those messages? How might you improve your messaging and/or understanding of mathematics instruction?
2. Are the professional learning opportunities and supports that you have in place meeting the needs of your teachers? How might you learn more about how teachers are experiencing these supports? What shifts might you make?

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The resources included in this brief are designed to provide helpful information. Resources are provided for instructional use purposes only and do not constitute NYSED endorsement of any vendor, author, or other sources. To the best of our knowledge, the resources provided are true and complete.

BRIEF #8: The Role of Leadership

DISCUSSION PROMPTS AND QUESTIONS

DIRECTIONS

Step 1 – “Think”

Read the questions and jot down your initial thoughts. Take a moment to reflect on each.

Step 2 – “Pair”

Discuss your responses with the people at your table. Share your insights and listen to theirs.

Step 3 – “Share”

Consider what key points you'd like to share with the larger group, if time allows.

General Discussion Questions ***(for use with any of the numeracy briefs)***

- Identify the strengths in your district, school, or classroom that align with the brief(s).

- Identify existing challenges or barriers in your district, school, or classroom that may hinder implementation of the ideas presented in the brief(s).

- Offer suggestions for regional collaboration to support the ideas presented in the brief(s).

BREIF #8 QUESTIONS	REFLECTION / NOTES
What messages are you sending your mathematics teachers about instruction? What might teachers be taking away from those messages? How might you improve your messaging and/or understanding of mathematics instruction?	
Are the professional learning opportunities and supports that you have in place meeting the needs of your teachers? How might you learn more about how teachers are experiencing these supports? What shifts might you make?	

Local Key Takeaways

Site:

Facilitator:

Numeracy Brief	Local Key Takeaways
1: <i>The Research Base for Mathematics Teaching and Learning</i>	
2: <i>Debunking Myths about Mathematics Teaching and Learning</i>	
3: <i>High-Leverage Mathematical Content</i>	
4: <i>High-Leverage Instructional Practices</i>	
5: <i>Mathematics Assessment of and for Students' Learning</i>	
6: <i>The Role and Challenges of Using Representations</i>	
7: <i>Understanding, Using, and Modifying Curriculum Materials</i>	
8. <i>The Role of Leadership</i>	

Suggestions for Regional Collaboration

Site:

Facilitator:

Numeracy Brief	Suggestions for Regional Collaboration
1: <i>The Research Base for Mathematics Teaching and Learning</i>	
2: <i>Debunking Myths about Mathematics Teaching and Learning</i>	
3: <i>High-Leverage Mathematical Content</i>	
4: <i>High-Leverage Instructional Practices</i>	
5: <i>Mathematics Assessment of and for Students' Learning</i>	
6: <i>The Role and Challenges of Using Representations</i>	
7: <i>Understanding, Using, and Modifying Curriculum Materials</i>	
8: <i>The Role of Leadership</i>	

Supporting Culturally and Linguistically Responsive-Sustaining Education

Site:

Facilitator:

Brainstorm ways you can support the implementation of culturally and linguistically responsive-sustaining education and practices for inclusion, belonging, and academic rigor. Check the box(es) to indicate the strategy could be applied to the level(s).

Strategy	PreK	Elementary	Secondary

QR CODE FOR LEADERSHIP MOVES QUESTION

Numeracy Leadership Moves

