Amity Regional High School

Precalculus Essential Skills

Welcome to your Pre Calculus Success Problem-set! This packet is designed to help you review key concepts and strengthen your math skills before the next school year. It covers essential topics such as equations, functions, and quadratics to reinforce what you've learned and preparing you for future lessons.

Complete each section carefully. Show all work and use the provided formulas when necessary. If you get stuck, you can look up the **bolded words** on the internet or ask for help. By working through this packet, you'll build confidence and stay sharp in algebra.

Let's get started—happy learning!

1. Writing Equations of Linear and Quadratic Functions

- **Linear Functions**: Review slope-intercept form $(y = mx + b)$, point-slope form, and standard form.

- **Quadratic Functions**: Focus on vertex form $(y = a(x - h)^2 + k)$, standard form $(y = ax^2 + bx + c)$, and factoring to find roots.

Practice Problems:

1. Write the equation of the line that passes through $(2, -3)$ with a slope of $-\frac{1}{2}$.

2. Write the equation of a quadratic function with a vertex at $(3, -2)$ that passes through $(1, 6)$.

3. Convert the quadratic function $y = x^2 + 6x + 8$ to vertex form.

4. Write the equation of a line parallel to $y = 3x - 7$ passing through $(-1, 4)$.

5. Write the equation of a quadratic function with roots $x = -2$ and $x = 5$, passing through $(0, -10)$.

*Students should know how to complete the square to convert a quadratic equation from standard form to vertex form.

6. Word Problems

- Apply linear, quadratic, and piecewise functions to solve real-world problems.

Practice Problems:

1. A ball is thrown, and its height is modeled by $h(t) = -16t^2 + 48t + 5$. When does the ball reach its maximum height?

2. A company's revenue is modeled by $R(x) = -2x^2 + 40x$, where x is the number of units sold. Find the number of units that maximizes revenue.

3. Write a piecewise function for the cost of parking: $2 per hour for the first 3 hours and $1.50 per hour after that.

4. A farmer wants to build a rectangular enclosure with 200 ft of fencing. Write an equation for the area of the enclosure in terms of one dimension.

5. The path of a diver is modeled by $h(t) = -4t^2 + 8t + 6$. Find the time at which the diver hits the water.

3. Parent Functions and Function Transformations

- Parent functions: linear ($y = x$), quadratic ($y = x^2$), absolute value ($y = |x|$), square root, cube root, and cubic.

- Transformation rules: translations, reflections, stretches, and compressions.

Practice Problems:

1. Describe the transformation of $y = -2(x - 3)^2 + 5$.

2. Sketch the graph of $y = \sqrt{x + 4} - 2$.

3. Determine the transformation from $f(x) = |x|$ to $g(x) = -|x - 1| + 3$.

4. Graph $y = 3\sqrt{x - 2} + 1$.

5. Compare the graph of $f(x) = x^3$ to $g(x) = -\frac{1}{2}(x + 2)^3 + 4$.

4. Domain and Range

- Domain: all possible x-values.

- Range: all possible y-values.

Practice Problems:

1. Find the domain and range of $f(x) = \frac{1}{x-3}$.

2. Determine the domain and range of $g(x) = \sqrt{x + 5}$.

3. Identify the domain and range of $h(x) = |x - 2| + 1$.

4. What are the domain and range of $y = x^2 - 4$ for $x \geq 0$?

5. Determine the domain of $f(x) = \frac{\sqrt{x+2}}{x-1}$.

5. Analyzing Graphs of Functions

- Focus on intercepts, turning points, asymptotes, and end behavior.

Practice Problems:

1. Identify the vertex, axis of symmetry, and intercepts of $f(x) = -x^2 + 4x - 3$.

2. Sketch the graph of $y = \frac{1}{x}$, noting key features.

3. Find the x- and y-intercepts of $f(x) = \frac{x^2-9}{x+3}$.

4. Describe the end behavior of $y = 2x^3 - 5x^2 + 4x - 1$.

5. Identify all turning points and asymptotes for $y = \frac{1}{x^2+4}$.

6. Word Problems

- Apply linear, quadratic, and piecewise functions to solve real-world problems.

Practice Problems:

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7. Polynomials

- **Multiplying**: Distribute and combine like terms.

- **Factoring**: GCF, trinomials, grouping, difference of squares.

- **Dividing**: Use synthetic or long division.

- **Factor/Rational Root Theorem**: Identify potential roots.

- **End Behavior**: Determine based on the degree and leading coefficient.

Practice Problems:

1. Multiply $(x + 3)(x^2 - 2x + 4)$.

2. Factor $2x^3 - 3x^2 - 2x + 3$.

3. Divide $x^3 + 2x^2 - 5x - 6$ by $x + 3$.

4. Use the Rational Root Theorem to find one root of $x^3 - 3x^2 - x + 3$.

5. Sketch the graph of $f(x) = -x^3 + 2x^2 - x$, identifying end behavior.

8. Exponential and Logarithmic Functions

- Review properties of exponents and logarithms.

- Understand graphing and transformations.

- Solve equations involving exponents and logarithms.

Practice Problems:

1. Simplify $\log(100) + 2\log(5) - \log(2)$.

2. Solve $3^{2x} = 81$.

3. Graph $f(x) = e^x - 2$.

4. Solve $2^{x+1} = 16$.

5. Expand $\ln(x^3 \sqrt{y})$.

9. Radical Functions

- Graph and transform radical functions.

- Perform operations and solve radical equations.

Practice Problems:

1. Simplify $\sqrt{50} + \sqrt{18}$.

2. Solve $\sqrt{x + 3} = 5$.

3. Graph $f(x) = \sqrt{x - 2} + 1$.

4. Simplify $(2\sqrt{5} + \sqrt{2})^2$.

5. Solve $\sqrt{3x + 7} - 2 = 1$.

10. Rational Functions

- Simplify and perform operations on rational expressions.

- Analyze asymptotes, holes, and graphing.

Practice Problems:

1. Simplify $\frac{x^2 - 9}{x^2 + 3x + 2}$.

2. Solve $\frac{1}{x - 2} + \frac{1}{x + 2} = \frac{5}{x^2 - 4}$.

3. Find the vertical and horizontal asymptotes of $f(x) = \frac{2x^2 - 3x}{x^2 - 4}$.

4. Multiply and simplify $\frac{x}{x + 1} \cdot \frac{x + 2}{x^2 - 1}$.

5. Determine the holes and asymptotes of $f(x) = \frac{x^2 - 4}{x^2 + 2x}$.

Answers

1. Writing Equations of Linear and Quadratic Functions

1. $y = -\frac{1}{2}x - 2$
2. $y = -2(x-3)^2 - 2$
3. $y = (x+3)^2 - 1$
4. $y = 3x + 7$
5. $y = -\frac{1}{2}(x+2)(x-5)$

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2. Function Notation

1. $f(-2) = 25$
2. $g(5) = 17$
3. $h(3) = 32$
4. $k(a+1) = a^2 - a + 2$
5. $p(-x) = 4 + 2x$

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3. Parent Functions and Function Transformations

1. Reflection over x-axis, vertical stretch by 2, right 3 units, up 5 units.

2. Square root graph shifted left 4 and down 2.

3. Reflection over x-axis, right 1 unit, up 3 units.

4. Cube root graph shifted right 2, vertically stretched by 3, and shifted up 1.

5. Reflection over x-axis, horizontal compress ↓ by 2, left 2 units, up 4 units.

4. Domain and Range

1. Domain: $x \neq 3$, Range: $y \neq 0$.

2. Domain: $x \geq -5$, Range: $y \geq 0$.

3. Domain: All real numbers, Range: $y \geq 1$.

4. Domain: $x \geq 0$, Range: $y \geq -4$.

5. Domain: $x > -2$, $x \neq 1$.

5. Analyzing Graphs of Functions

1. Vertex: $(2, 1)$, Axis: $x = 2$, Intercepts: $(0, -3)$ and $(3, 0)$.

2. Asymptotes: $x = 0$, $y = 0$; Intercepts: $(1, 1)$, $(-1, -1)$.

3. Intercepts: $x = 3$, $y = 0$; Hole at $(-3, 6)$.

4. End behavior: As $x \to \pm\infty$, $y \to \pm\infty$.

5. Horizontal asymptote: $y = 0$; Turning points: none.

6. Word Problems

1. $t = 1.5$ seconds.

2. $x = 10$ units.

3. $C(x) = \begin{cases} 2x & \text{if } x \leq 3 \\ 6 + 1.5(x - 3) & \text{if } x > 3 \end{cases}$

4. $A = x(100 - x)$.

5. $t = 2$ seconds.

7. Polynomials

1. $x^3 - 2x^2 + 4x + 12$.
2. $(x^2 - 1)(2x - 3)$.
3. $x^2 - x - 2$.
4. Root: $x = 1$.
5. End behavior: $y \to -\infty$ as $x \to \pm\infty$.

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8. Exponential and Logarithmic Functions

1. 3.
2. $x = 2$.
3. Asymptote: $y = -2$, passes through $(0, -1)$.
4. $x = 3$.
5. $\ln(x^3\sqrt{y}) = 3\ln(x) + \frac{1}{2}\ln(y)$.

9. Radical Functions

1. $3\sqrt{2} + 3\sqrt{2} = 6\sqrt{2}$.
2. $x = 22$.
3. Square root graph shifted right 2 and up 1.
4. $22 + 4\sqrt{10}$.
5. $x = 4$.

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10. Rational Functions

1. $\frac{x-3}{x+2}$.
2. $x = 0, \pm 4$.
3. Vertical asymptotes: $x = \pm 2$; Horizontal asymptote: $y = 2$.
4. $\frac{x(x+2)}{(x+1)(x-1)}$.
5. Hole at $x = -2$; Vertical asymptote: $x = 0$.