Calculus BC Summer Packet

Date_____ Period____

For each problem, find all points of absolute minima and maxima on the given interval.

1) $y = -\dfrac{4x}{x^2 + 4}$; $(-3, \infty)$

2) $y = \dfrac{9x}{x^2 + 9}$; $(-\infty, -2)$

3) $y = -x^3 + 10x^2 - 28x + 18$; $(-\infty, \infty)$

4) $y = -\dfrac{x^2}{2} + x + \dfrac{3}{2}$; $(-\infty, -2]$

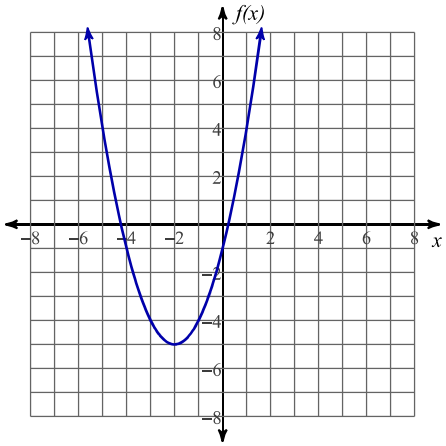
For each problem, find the average rate of change of the function over the given interval.

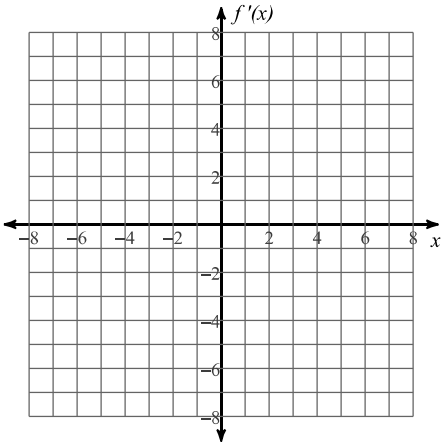
5) $f(x) = 2x^2 + 2$; $[0, 1]$

6) $f(x) = x^2 + 2$; $[0, 1]$

Given the graph of $f(x)$, sketch an approximate graph of $f'(x)$.

7)





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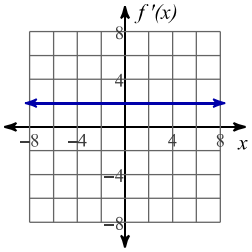

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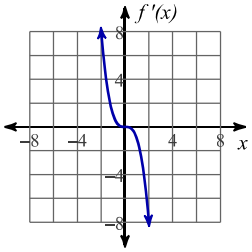

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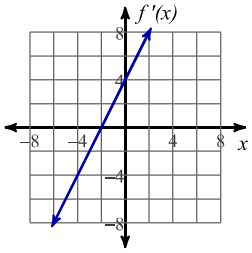

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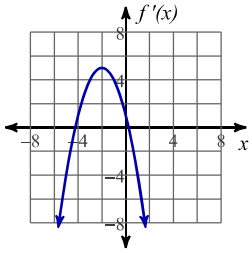

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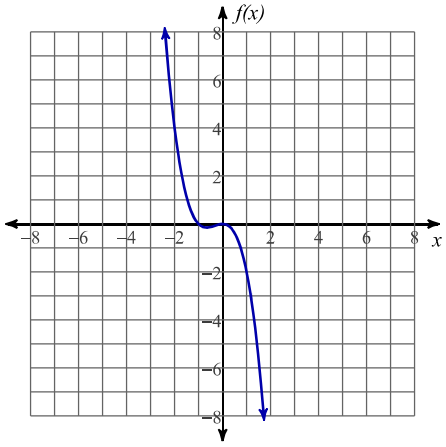

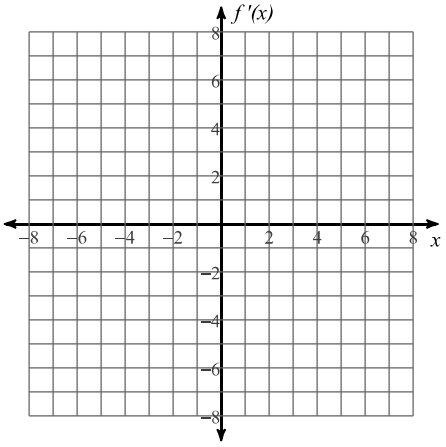

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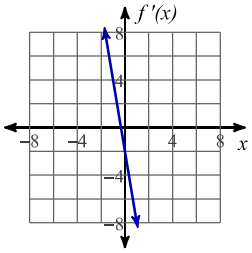

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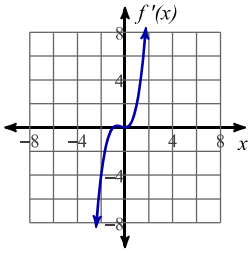

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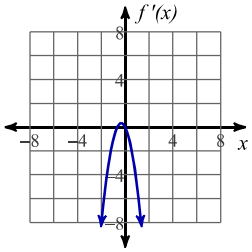

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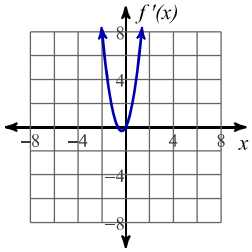
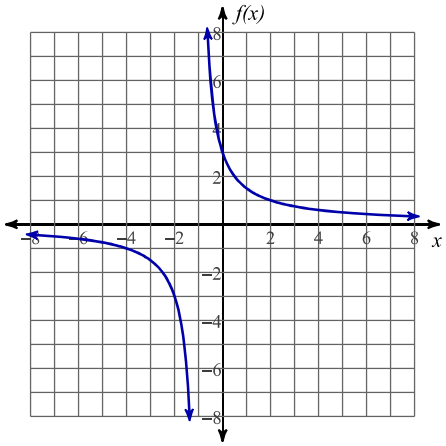
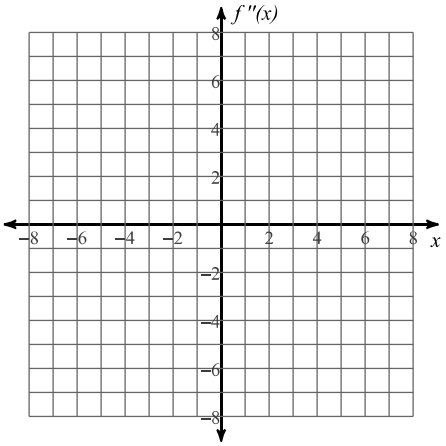


Given the graph of $f(x)$, sketch an approximate graph of $f''(x)$.

9)





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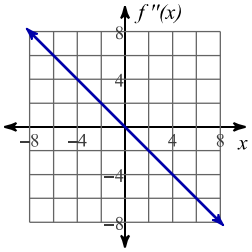

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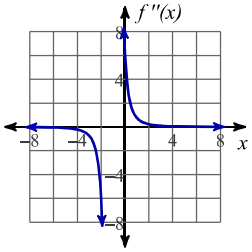

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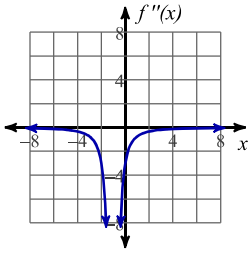

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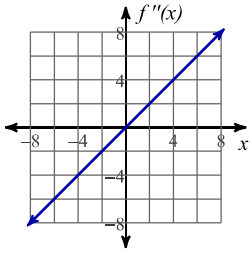

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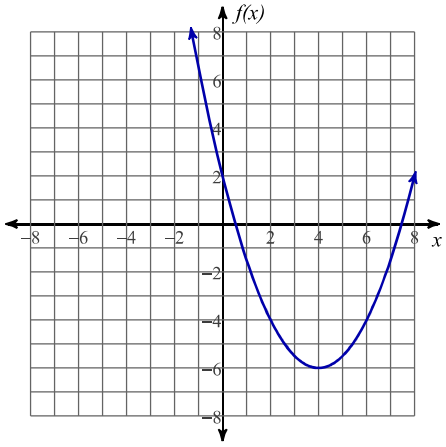

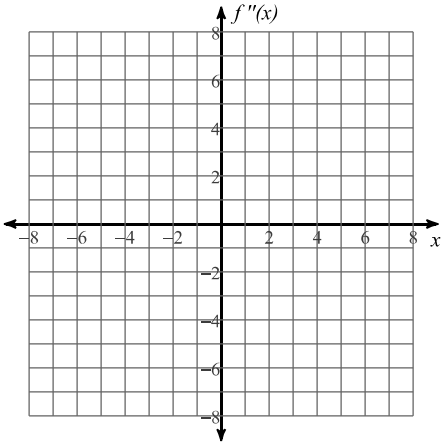

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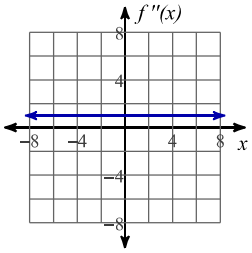

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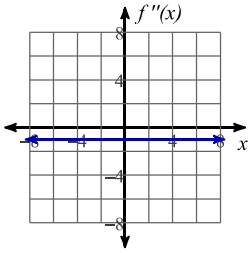

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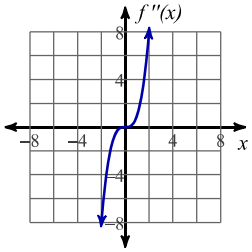

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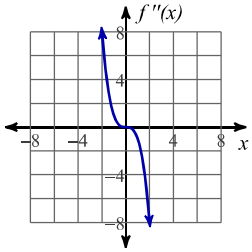
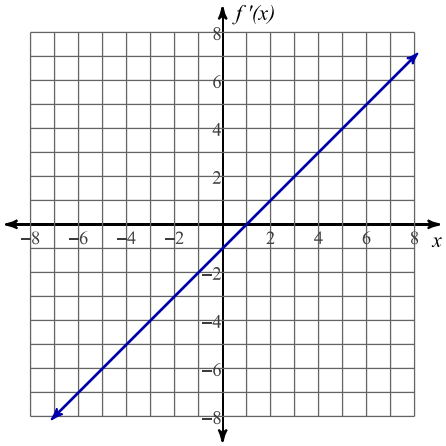
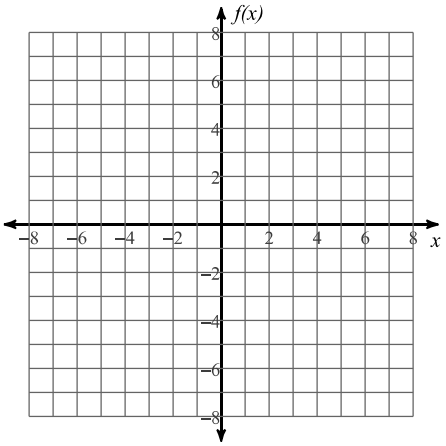


Given the graph of $f'(x)$, sketch a possible graph of $f(x)$.

11)





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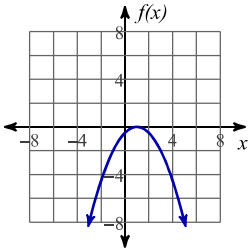

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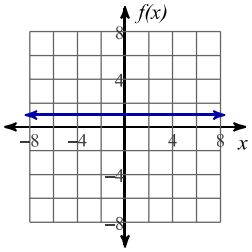

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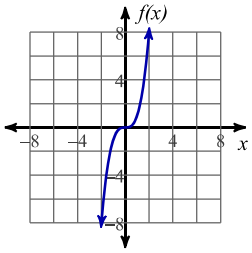

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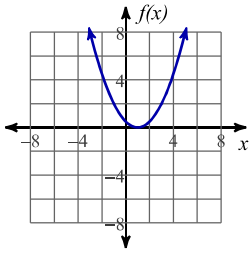

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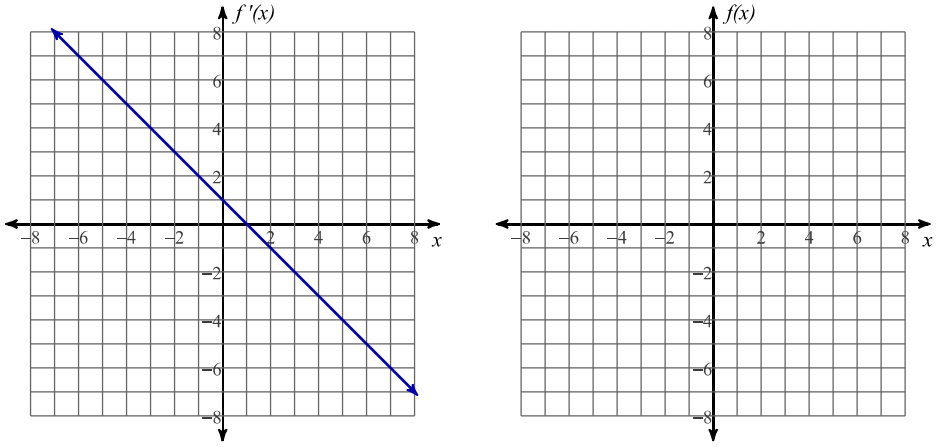

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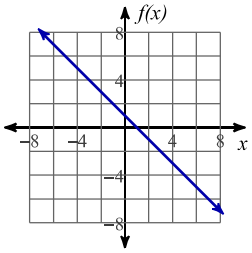

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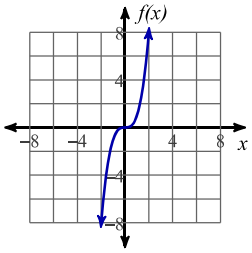

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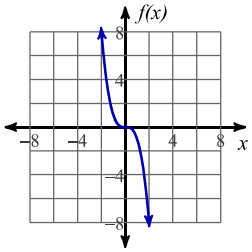

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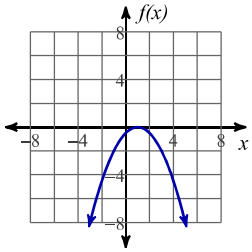


For each problem, find the open intervals where the function is concave up and concave down.

13) $y = -2x^2 + 8x - 4$

14) $y = -x^3 + 3x^2 - 6$

For each problem, find the x-coordinates of all critical points and find the open intervals where the function is increasing and decreasing.

15) $y = -x^3 + x^2$

16) $y = \frac{x^2}{2} - x - \frac{7}{2}$

Evaluate each limit using L'Hôpital's Rule.

17) $\lim_{x \to 0} \frac{5x}{\ln(x+1)}$

18) $\lim_{x \to 0} \frac{e^{3x} - 1}{\sin(3x)}$

19) $\lim_{x \to 0} (2x + 1)^{\frac{1}{x}}$

20) $\lim_{x \to 1} \frac{\ln x}{x - 1}$

For each problem, find the values of *c* that satisfy the Mean Value Theorem.

21) $y = \frac{x^2}{2} - 2x; \quad [-1, 2]$

22) $y = -2x^2 - 4x - 4; \quad [-2, 0]$

A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the velocity at the given value for *t*.

23) $s(t) = -t^4 + 10t^3$; at $t = 8$

24) $s(t) = t^4 - 12t^3$; at $t = 2$

25) $s(t) = -t^3 + 18t^2 - 81t$; at $t = 5$

26) $s(t) = t^3 - 15t^2$; at $t = 8$

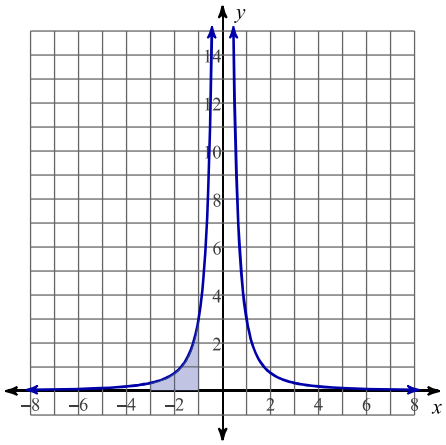
A particle moves along a horizontal line. Its position function is $s(t)$ for $t \geq 0$. For each problem, find the intervals of time when the particle is slowing down and speeding up.

27) $s(t) = -t^3 + 8t^2$

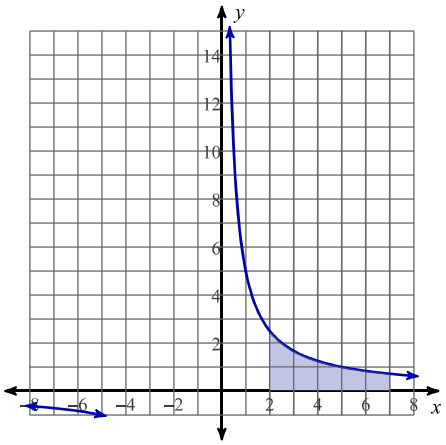
28) $s(t) = -t^3 + 24t^2 - 144t$

For each problem, find the area under the curve over the given interval.

29) $y = \frac{3}{x^2}$; $[-3, -1]$



30) $y = \frac{5}{x}$; $[2, 7]$



For each problem, find the area of the region enclosed by the curves.

31) $y = \frac{x^3}{2} - x^2 - \frac{7x}{2}$, $y = \frac{x}{2}$

32) $y = -\frac{x^3}{2} - \frac{x^2}{2} + 2x$, $y = -x$

For each problem, approximate the area under the curve over the given interval using 4 left endpoint rectangles.

33) $y = x + 5;\ [0, 4]$

34) $y = -x + 5;\ [-2, 2]$

Differentiate each function with respect to *x*.

35) $y = \left((x^5 - 3)^2 - 1\right)^5$

36) $y = \left(4x^2 + 3\right)^2$

Evaluate each definite integral.

37) $\int_{-4}^{1} \left(\frac{x^2}{2} + 2x + 2\right) dx$

38) $\int_{-1}^{1} \left(x^4 - x^3 - x^2 + 1\right) dx$

39) $\int_{-3}^{1} \left(x + 1\right) dx$

40) $\int_{-\frac{3\pi}{4}}^{\pi} \cos x \, dx$

For each problem, use a left-hand Riemann sum to approximate the integral based off of the values in the table.

41) $\int_0^8 f(x)\,dx$

x	0	4	5	6	8
$f(x)$	7	5	3	4	5

42) $\int_0^9 f(x)\,dx$

x	0	1	3	8	9
$f(x)$	8	9	7	9	8

43) $\int_0^8 f(x)\,dx$

x	0	1	3	4	8
$f(x)$	4	5	6	4	5

For each problem, find *F′*(*x*).

44) $F(x) = \int_{-3}^{x} (-t^2 - 2t - 1)\,dt$

45) $F(x) = \int_{-3}^{x} (2t + 2)\,dt$

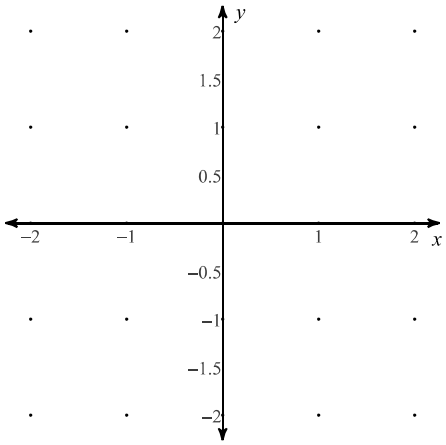
Use *u* substitution to express each definite integral in terms of *u*. Do not evaluate the integral.

46) $\int_{-1}^{0} 12x^2(2x^3+1)^2\,dx$

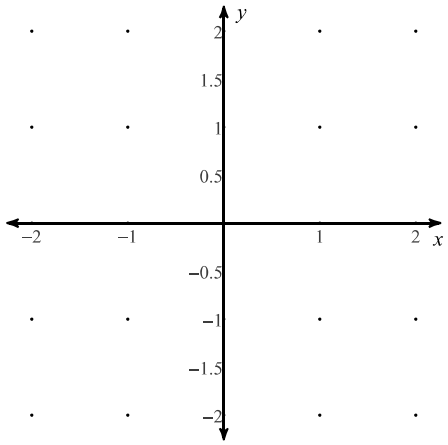
47) $\int_{0}^{2} \frac{4x}{(x^2+4)^2}\,dx$

Sketch the slope field for each differential equation.

48) $\frac{dy}{dx} = xy$



49) $\frac{dy}{dx} = \frac{y}{x}$



For each problem, find the indicated derivative with respect to *x*.

50) $f(x) = -3x^3$ Find f'''

51) $f(x) = 4x^4$ Find $f^{(4)}$

52) $f(x) = 4x^4 + 3x$ Find $f^{(4)}$

53) $f(x) = x^4 + 2x^3$ Find f'''

For each problem, use implicit differentiation to find $\frac{dy}{dx}$ in terms of *x* and *y*.

54) $1 = 4x^2 - 5y^2$

55) $1 = 5x^3 + 3y^2$

56) $2y^2 + 3 = 2x$

57) $5x + y^2 = 4y$

For each problem, find the instantaneous rate of change of the function at the given value.

58) $f(x) = -x^2 + 2;\ \ 2$

59) $f(x) = x^2 + 2x + 2;\ \ -3$

Differentiate each function with respect to *x*.

60) $y = (-5x^3 + 5)(3x^5 - 5)$

61) $y = x^2(3x^5 - 4)$

62) $y = \dfrac{3}{3x^3 + 5}$

63) $y = \dfrac{3x^5 - 2x^4 - 2x^2}{3x^2 + 4}$

64) $y = \cot(\sin 3x^3)$

65) $y = \sin(\cos 5x^3)$

For each problem, find all points of relative minima and maxima.

66) $f(x) = -x^4 + 4x^2 - 3$

67) $y = x^2 - 4x + 3$

68) $y = x^4 - 3x^2 + 2$

69) $y = \dfrac{4x}{x^2 + 4}$

Evaluate each indefinite integral.

70) $\int -3 \, dx$

71) $\int 20x^4 \, dx$

72) $\int (5x^5 + 3)^3 \cdot 25x^4 \, dx$

73) $\int (4x^4 - 3)^3 \cdot 16x^3 \, dx$

74) $\int -4\cos x \, dx$

75) $\int 5\cos x \, dx$

Evaluate each limit.

76) $\lim_{x \to \infty} -\frac{x^4}{2x^2 + 4}$

77) $\lim_{x \to \infty} -\frac{x^2}{x^2 - 9}$

78) $\lim_{x \to 2} -\frac{x^2 - 2x}{x - 2}$

79) $\lim_{x \to -2} \frac{x}{\frac{1}{2 + x} - \frac{1}{2}}$

80) $\lim_{x \to 2} \frac{x - 2}{x^2 - 3x + 2}$

81) $\lim_{x \to 2} \frac{x^2 + 2x - 8}{x - 2}$

82) $\lim_{x \to -3} (2x + 4)$

83) $\lim_{x \to \frac{2\pi}{3}} \sec(x)$

84) $\lim_{x \to -2} (x + 5)$

85) $\lim_{x \to 1} \frac{x - 4}{x^2 - 6x + 9}$

A particle moves along a coordinate line. Its acceleration function is $a(t)$ for $t \geq 0$. For each problem, find the position function $s(t)$ and the velocity function $v(t)$.

86) $a(t) = -6t + 60;\ s(0) = 0;\ v(0) = -225$

87) $a(t) = -6t + 44;\ s(0) = 0;\ v(0) = -121$

88) $a(t) = 6t - 20;\ s(0) = 0;\ v(0) = 0$

89) $a(t) = 6t - 2;\ s(0) = 0;\ v(0) = -56$