Calculus - SUMMER PACKET

NAME:__________________________

NO CALCULATOR!!!

Given $f(x) = x^2 - 2x + 5$, find the following.

1. $f(-2) =$

2. $f(x+2) =$

3. $f(x+h) =$

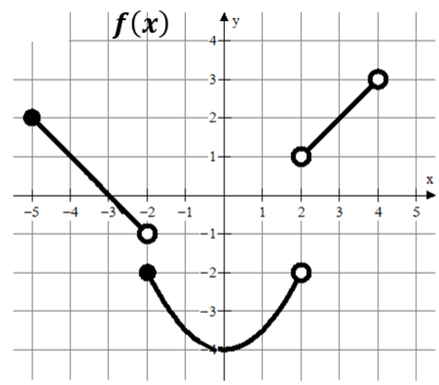
Use the graph $f(x)$ to answer the following.

4. $f(0) =$ $\quad$ $f(4) =$

$f(-1) =$ $\quad$ $f(-2) =$

$f(2) =$ $\quad$ $f(3) =$

$f(x) = 2$ when $x = ?$ $\quad$ $f(x) = -3$ when $x = ?$



Write the equation of the line meets the following conditions. Use point-slope form.
$\boldsymbol{y - y_1 = m(x - x_1)}$ & then convert to slope-intercept (y=mx+b)

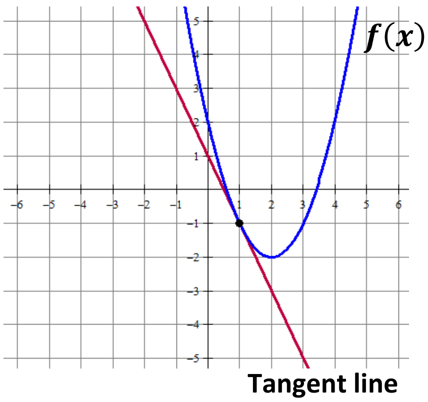
5. slope $= 3$ and $(4, -2)$

6. $m = -\frac{3}{2}$ and $f(-5) = 7$

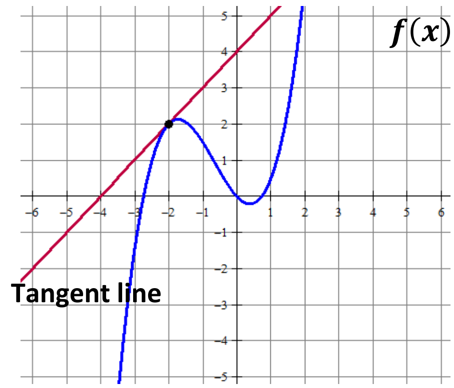
7. $f(4) = -8$ and $f(-3) = 12$

Write the equation of the tangent line in point slope form. $y - y_1 = m(x - x_1)$

8. The line tangent to $f(x)$ at $x = 1$

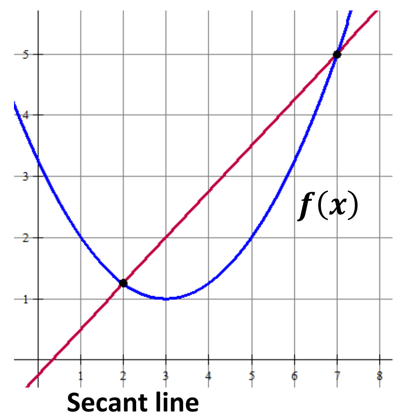


9. The line tangent to $f(x)$ at $x = -2$



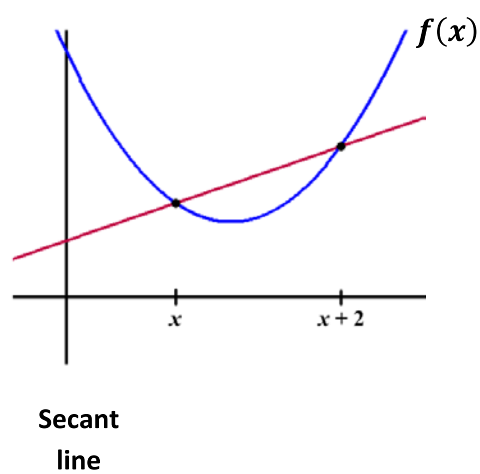
MULTIPLE CHOICE! Remember slope $= \frac{y_2 - y_1}{x_2 - x_1}$

10. Which choice represents the slope of the secant line shown?



A) $\frac{7-2}{f(7)-f(2)}$ B) $\frac{f(7)-2}{7-f(2)}$ C) $\frac{7-f(2)}{f(7)-2}$ D) $\frac{f(7)-f(2)}{7-2}$

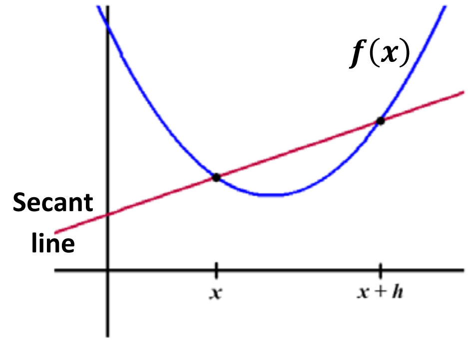
11. Which choice represents the slope of the secant line shown?



A) $\frac{f(x)-f(x+2)}{x+2-x}$ B) $\frac{f(x+2)-f(x)}{x+2-x}$ C) $\frac{f(x+2)-f(x)}{x-(x+2)}$

D) $\frac{x+2-x}{f(x)-f(x+2)}$

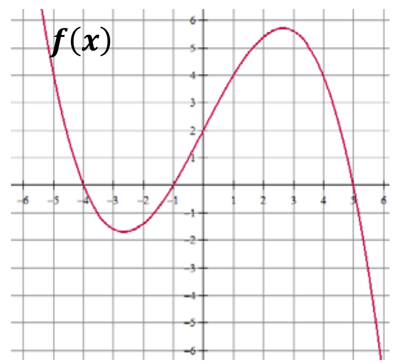
12. Which choice represents the slope of the secant line shown?



A) $\frac{f(x+h)-f(x)}{x-(x+h)}$ B) $\frac{x-(x+h)}{f(x+h)-f(x)}$ C) $\frac{f(x+h)-f(x)}{x+h-x}$

D) $\frac{f(x)-f(x+h)}{x+h-x}$

13. Which of the following statements about the function $f(x)$ is true?

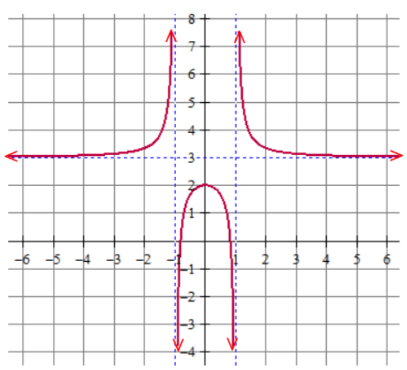


I. $f(2) = 0$
II. $(x + 4)$ is a factor of $f(x)$
III. $f(5) = f(-1)$

(A) I only
(B) II only
(C) III only
(D) I and III only
(E) II and III only

Find the domain and range (express in interval notation). Find all horizontal and vertical asymptotes.

14.



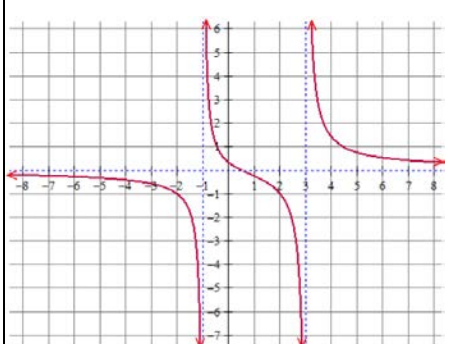
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

15.



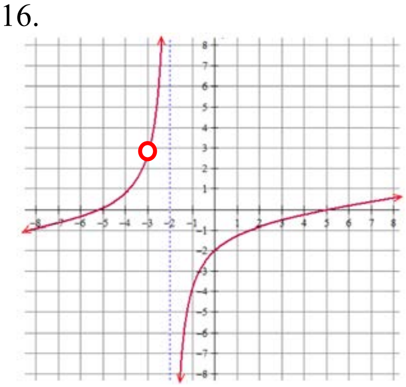
Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

16.



Domain:

Range:

Horizontal Asymptote(s):

Vertical Asymptotes(s):

MULTIPLE CHOICE!

17. Which of the following functions has a vertical asymptote at $x = 4$?

(A) $\frac{x+5}{x^2-4}$

(B) $\frac{x^2-16}{x-4}$

(C) $\frac{4x}{x+1}$

(D) $\frac{x+6}{x^2-7x+12}$

(E) None of the above

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18. Consider the function: $(x) = \frac{x^2-5x+6}{x^2-4}$. Which of the following statements is true?

I. $f(x)$ has a vertical asymptote of $x = 2$
II. $f(x)$ has a vertical asymptote of $x = -2$
III. $f(x)$ has a horizontal asymptote of $y = 1$

(A) I only

(B) II only

(C) I and III only

(D) II and III only

(E) I, II and III

Rewrite the following using rational exponents. Example: $\frac{1}{\sqrt[3]{x^2}} = x^{-\frac{2}{3}}$

19. $\sqrt[5]{x^3} + \sqrt[5]{2x}$	20. $\sqrt{x+1}$	21. $\frac{1}{\sqrt{x+1}}$
22. $\frac{1}{\sqrt{x}} - \frac{2}{x}$	23. $\frac{1}{4x^3} + \frac{1}{2}\sqrt[4]{x^3}$	24. $\frac{1}{4\sqrt{x}} - 2\sqrt{x+1}$

Write each expression in radical form and positive exponents. Example: $x^{-\frac{2}{3}} + x^{-2} = \frac{1}{\sqrt[3]{x^2}} + \frac{1}{x^2}$

25. $x^{-\frac{1}{2}} - x^{\frac{3}{2}}$	26. $\frac{1}{2}x^{-\frac{1}{2}} + x^{-1}$	27. $3x^{-\frac{1}{2}}$
28. $(x+4)^{-\frac{1}{2}}$	29. $x^{-2} + x^{\frac{1}{2}}$	30. $2x^{-2} + \frac{3}{2}x^{-1}$

Need to know basic trig functions in RADIANS! We never use degrees. You can either use the Unit Circle or Special Triangles to find the following.		
31. $\sin\frac{\pi}{6}$	32. $\cos\frac{\pi}{4}$	33. $\sin 2\pi$
34. $\tan\pi$	35. $\sec\frac{\pi}{2}$	36. $\cos\frac{\pi}{6}$
37. $\sin\frac{\pi}{3}$	38. $\sin\frac{3\pi}{2}$	39. $\tan\frac{\pi}{4}$
40. $\csc\frac{\pi}{2}$	41. $\sin\pi$	42. $\cos\frac{\pi}{3}$
43. Find x where $0 \le x \le 2\pi$, $\sin x = \frac{1}{2}$	44. Find x where $0 \le x \le 2\pi$, $\tan x = 0$	45. Find x where $0 \le x \le 2\pi$, $\cos x = -1$

Solve the following equations. Remember $e^0 = 1$ and $\ln 1 = 0$.		
46. $e^x + 1 = 2$	47. $3e^x + 5 = 8$	48. $e^{2x} = 1$
49. $\ln x = 0$	50. $3 - \ln x = 3$	51. $\ln(3x) = 0$
52. $x^2 - 3x = 0$	53. $e^x + xe^x = 0$	54. $e^{2x} - e^x = 0$

Solve the following trig equations where $0 \le x \le 2\pi$.

55. $\sin x = \frac{1}{2}$	56. $\cos x = -1$	57. $\cos x = \frac{\sqrt{3}}{2}$
58. $2\sin x = -1$	59. $\cos x = \frac{\sqrt{2}}{2}$	60. $\cos\left(\frac{x}{2}\right) = \frac{\sqrt{3}}{2}$
61. $\tan x = 0$	62. $\sin(2x) = 1$	63. $\sin\left(\frac{x}{4}\right) = \frac{\sqrt{3}}{2}$

For each function, determine its domain and range.

Function	**Domain**	**Range**
64. $y = \sqrt{x-4}$		
65. $y = (x-3)^2$		
66. $y = \ln x$		
67. $y = e^x$		
68. $y = \sqrt{4-x^2}$		

Simplify.

69. $\frac{\sqrt{x}}{x}$	70. $e^{\ln x}$	71. $e^{1+\ln x}$

72. $\ln 1$

73. $\ln e^7$

74. $\log_3 \frac{1}{3}$

75. $\log_{1/2} 8$

~~76. $\ln\frac{1}{2}$~~

77. $27^{\frac{2}{3}}$

78. $\left(5a^{2/3}\right)\left(4a^{3/2}\right)$

79. $\frac{4xy^{-2}}{12x^{-\frac{1}{3}}y^{-5}}$

80. $\left(4a^{5/3}\right)^{3/2}$

If $\boldsymbol{f(x) = \{(3,5),(2,4),(1,7)\}}$ $\boldsymbol{g(x) = \sqrt{x-3}}$ **, then determine each of the following.**
$\boldsymbol{h(x) = \{(3,2),(4,3),(1,6)\}}$ $\boldsymbol{k(x) = x^2 + 5}$

81. $(f+h)(1)$

82. $(k-g)(5)$

83. $f\big(h(3)\big)$

84. $g\big(k(7)\big)$

85. $h(3)$

86. $g\big(g(9)\big)$

87. $f^{-1}(4)$

88. $k^{-1}(x)$

89. $k\big(g(x)\big)$

90. $g(f(2))$

Find The volume of each solid. Keep answers in terms of pi. (r = radius, d = diameter)

91. Sphere; $r = 5$ *in*

92. Hemisphere; $d = 10$ *cm*

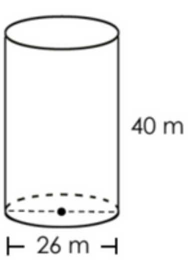
93. Cone; height of *4 cm* and a radius of *4 inches*.

94. Pyramid; the base has a length of *2 in.* and a ~~height~~ width of *3 in*. The pyramid has a height of *4 in*.

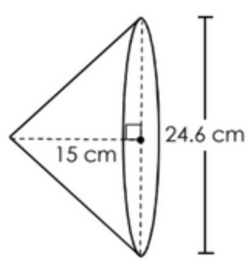
95. Find the radius of a circle whose circumference is 144 pi.

96. Cylinder; height of *6 ft* and a radius of *3 ft*

97. Find the volume:



98. Find the volume:

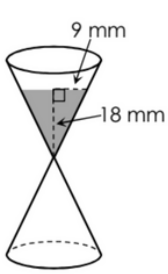


99.

a. Find the radius of a sphere whose volume is 288 pi

b. Find the surface area of the sphere

100. You are playing a game in which you must answer a question before the sand in the timer falls to the bottom. If the sand is falling at a rate of 50 cubic millimeters per second, how long do you have to answer the question?



Things to Know for Calculus

TRIGONOMETRY

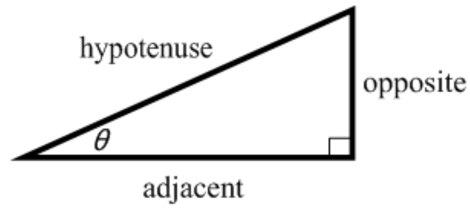
Trig Functions

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

SOH-CAH-TOA



Reciprocal Functions

$\csc\theta = \dfrac{1}{\sin\theta} = \dfrac{\text{hyp}}{\text{opp}}$

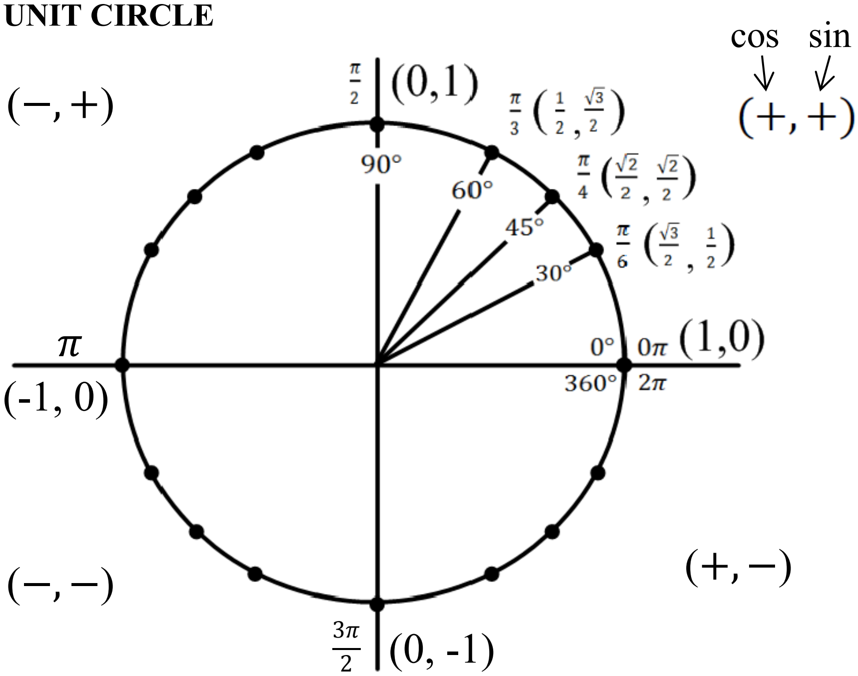
$\sec\theta = \dfrac{1}{\cos\theta} = \dfrac{\text{hyp}}{\text{adj}}$

$\cot\theta = \dfrac{1}{\tan\theta} = \dfrac{\text{adj}}{\text{opp}}$

TEST ONLY USES RADIANS!

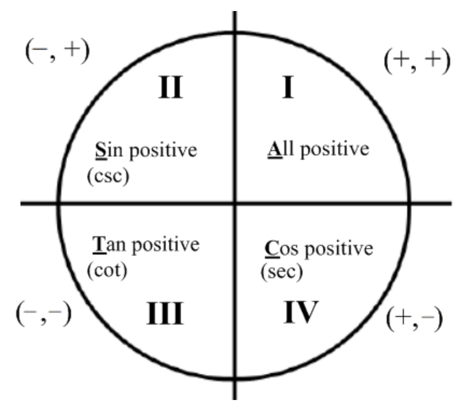
Must know trig values of special angles $0\pi, \frac{\pi}{6}, \frac{\pi}{4}, \frac{\pi}{3}, \frac{\pi}{2}, \pi, \frac{3\pi}{2}, 2\pi$ using Unit Circle or Special Right Triangles.

UNIT CIRCLE



To help remember the signs in each quadrant

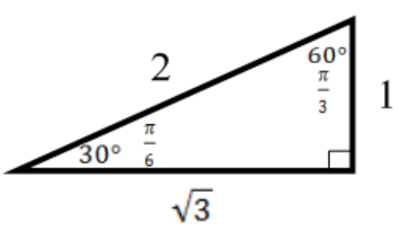
All **S**tudents **T**ake **C**alculus



SPECIAL RIGHT TRIANGLES

$30° - 60° - 90°$ Triangles

Which are $\frac{\pi}{6} - \frac{\pi}{3} - \frac{\pi}{2}$ Triangles

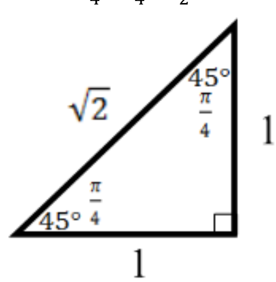


Find $\tan\left(\frac{\pi}{6}\right)$

$\tan\left(\frac{\pi}{6}\right) = \frac{\text{opp}}{\text{adj}} = \frac{1}{\sqrt{3}}$ simplify to $\frac{1}{\sqrt{3}} \frac{\cdot\sqrt{3}}{\cdot\sqrt{3}} = \frac{\sqrt{3}}{3}$

$45° - 45° - 90°$ Triangles

Which are $\frac{\pi}{4} - \frac{\pi}{4} - \frac{\pi}{2}$ Triangles

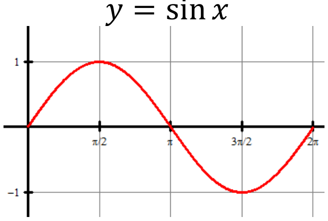


Find $\sin\left(\frac{\pi}{4}\right)$

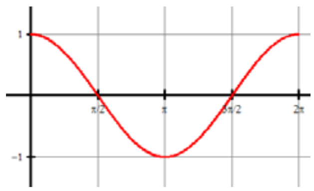
$\sin\left(\frac{\pi}{4}\right) = \frac{opp}{hyp} = \frac{1}{\sqrt{2}}$ simplify to $\frac{1}{\sqrt{2}} \frac{\cdot\sqrt{2}}{\cdot\sqrt{2}} = \frac{\sqrt{2}}{2}$

Graphs of trig functions

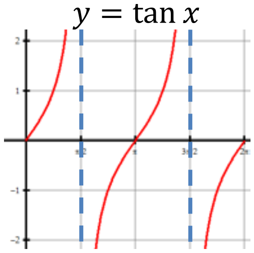
$y = \sin x$



$y = \cos x$



$y = \tan x$



Inverse Trig Function

$\sin^{-1}\theta$ is the same as $\arcsin\theta$

$\sin^{-1}\theta = \left(\frac{\sqrt{3}}{2}\right)$ means what angle has a sine value of $\frac{\sqrt{3}}{2}$

that means $\theta = \frac{\pi}{3} \pm 2\pi n \text{ or } \frac{2\pi}{3} \pm 2\pi n$

Since θ has infinite answers then it isn't a function. Bummer. To make it a function we define inverses like:

sin/csc and tan/cot use quadrant I and IV for inverses
cos/sec use quadrant I and II for inverses

So… $\theta = \frac{\pi}{3}$ because it is in the first quadrant

Trig Identities

There are a bunch, but you really only need to know Pythagorean Identity. $\mathbf{\sin^2 x + \cos^2 x = 1}$

Subtract $\sin^2 x$ to get $\cos^2 x = 1 - \sin^2 x$ or subtract $\cos^2 x$ to get $\sin^2 x = 1 - \cos^2 x$

Divide by $\sin^2 x$ to get $1 + \cot^2 x = \csc^2 x$ or divide by $\cos^2 x$ to get $\tan^2 x + 1 = \sec^2 x$

GEOMETRY

FORMULAS

AREA

Triangle $= \frac{1}{2}bh$

Circle $= \pi r^2$

Trapezoid $= \frac{1}{2}(b_1 + b_2)h$

CIRCUMFERENCE

Circle $= 2\pi r$

SURFACE AREA

Sphere $= 4\pi r^2$

LATERAL AREA

Cylinder $= 2\pi rh$

VOLUME

Sphere $= \frac{4}{3}\pi r^3$

Cylinder $= \pi r^2 h$

Cone $= \frac{1}{3}\pi r^2 h$

Prism $= Bh$

Pyramid $= \frac{1}{3}Bh$

B is the area of the base

DISTANCE FORMULA

The distance between two points (x_1, y_1) and (x_2, y_2) is $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

ALGEBRA

Linear Functions

Slope

$$m = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

y-intercept Form

(slope-intercept Form)

$$y = mx + b$$

Point Slope Form

$$y - y_1 = m(x - x_1)$$

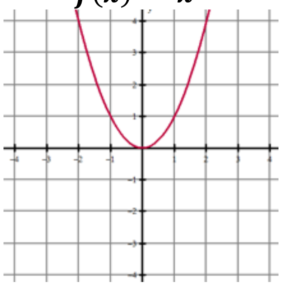
Parallel Lines
Have the same slope

Perpendicular Lines
Have the opposite reciprocal slopes

Functions

Quadratic Function

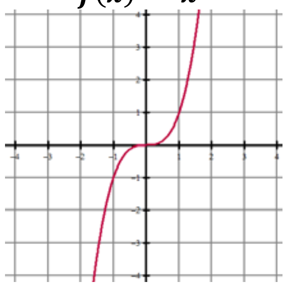
$f(x) = x^2$



$y = a(x-h)^2 + k$

Cubic Function

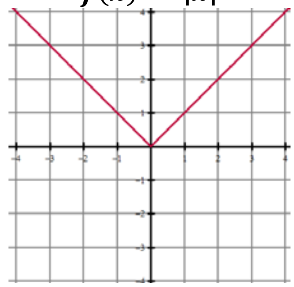
$f(x) = x^3$



$y = a(x-h)^3 + k$

Absolute Value

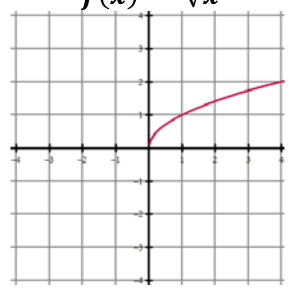
$f(x) = |x|$



$y = a|x-h| + k$

Square Root Function

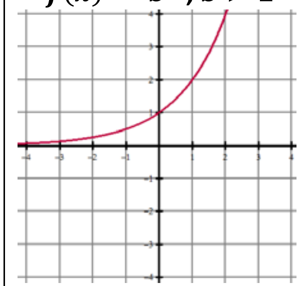
$f(x) = \sqrt{x}$



$y = a\sqrt{x-h} + k$

Exponential Function

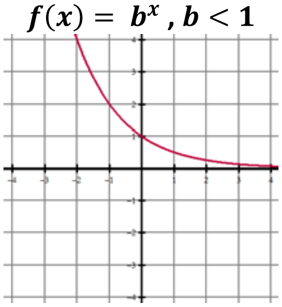
$f(x) = b^x, b > 1$



$y = a \cdot b^{(x-h)} + k$

Exponential Function

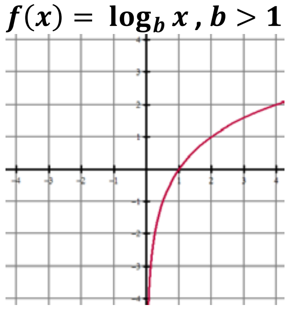
$f(x) = b^x, b < 1$



$y = a \cdot b^{(x-h)} + k$

Logarithmic Function

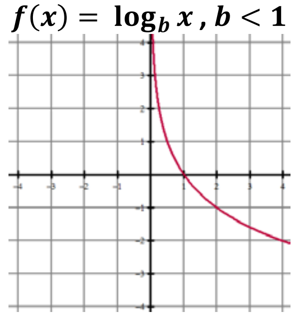
$f(x) = \log_b x, b > 1$



$y = a\log_b(x-h) + k$

Logarithmic Function

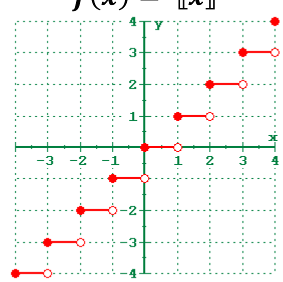
$f(x) = \log_b x, b < 1$



$y = a\log_b(x-h) + k$

Greatest Integer

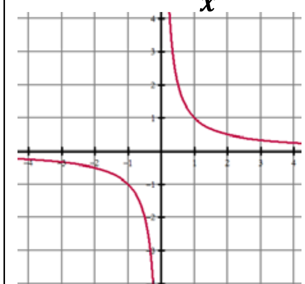
$f(x) = [\![x]\!]$



$y = a[\![x-h]\!] + k$

Rational Function

$f(x) = \dfrac{1}{x}$



$y = \dfrac{a}{x-h} + k$

Translations

All functions move the same way!

Given the parent function $y = x^2$

Move up 4	Move down 3	Move left 2	Move right 1	Move left 2 and down 3
$y = x^2 + 4$	$y = x^2 - 3$	$y = (x+2)^2$	$y = (x-1)^2$	$y = (x+2)^2 - 3$

To flip (reflect) the function vertically $y = -x^2$
To flip (reflect) the function horizontally $y = (-x)^2$

So $f(x) = -\sqrt{x-3} + 1$ is a square root function reflected vertically, shifted right 3 and up 1

Notation

Notice open parenthesis ( ) versus closed [ ]

Inequality		Interval
$-3 < x \le 5$	$\longleftrightarrow$	$(-3,5]$
$-3 \le x \le 5$	$\longleftrightarrow$	$[-3,5]$
$-3 < x < 5$	$\longleftrightarrow$	$(-3,5)$
$-3 \le x < 5$	$\longleftrightarrow$	$[-3,5)$

Infinity is always open parenthesis

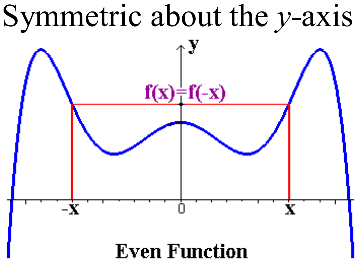
Inequality		Interval
$x < 3$	$\longleftrightarrow$	$(-\infty, 3)$
$x \le 3$ or $x > 5$	$\longleftrightarrow$	$(-\infty, 3](5, \infty)$
$x \ne 3$	$\longleftrightarrow$	$(-\infty, 3)(3, \infty)$
all Real numbers	$\longleftrightarrow$	$(-\infty, \infty)$

Even and Odd Functions

EVEN

$f(-x) = f(x)$

Symmetric about the y-axis

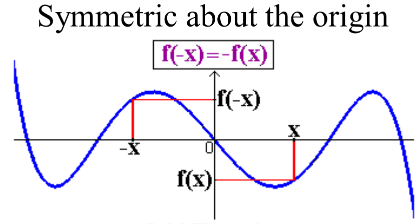


Even Function

ODD

$f(-x) = -f(x)$

Symmetric about the origin



Odd Function

Domain and Range

Domain = all possible x values

Range = all possible y values

Algebraically

You can't divide by zero

You can't square root a negative

$$y = \sqrt{2x+5}$$

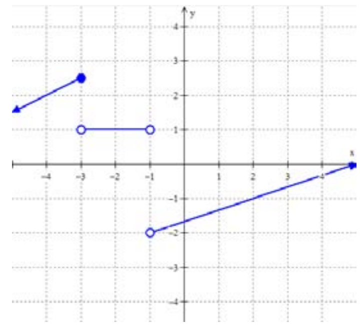
D: $[-\frac{5}{2}, \infty)$

$$y = \frac{x^2 - 1}{x^2 + 7x + 12}$$

D: $(-\infty, -4)(-4, -3)(-3, \infty)$

Graphically

Just look at it



D: $(-\infty, -1)(-1, 5]$

R: $(-\infty, 2.5]$

Finding zeros

Must be able to factor and use the quadratic formula: $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Special products

Sum of cubes: $a^3 + b^3 = (a+b)(a^2 - ab + b^2)$

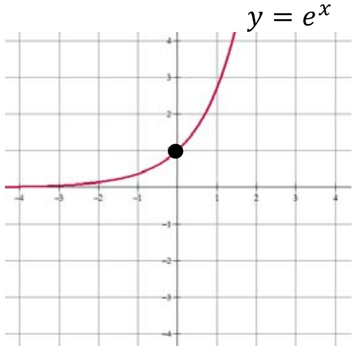
Difference of cubes: $a^3 - b^3 = (a-b)(a^2 + ab + b^2)$

Exponential and Logarithmic Properties

The exponential function b^x of base b is one-to-one which means it has an inverse which is called the logaritmic function of base b or logarithm of base b which is denoted $\log_b x$ which reads "the logarithm of base b of x" or "log base b of x". So…

BASE e

$y = e^x$

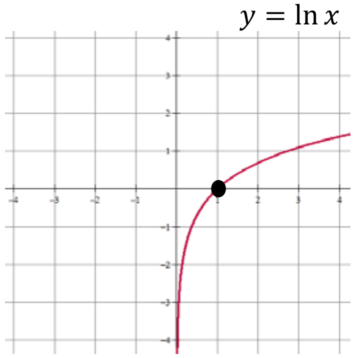


KNOW!

$e^0 = 1$

Horizontal asymptote at $y = 0$

$y = \ln x$



KNOW!

$\ln 1 = 0$

Vertical asymptote at $x = 0$

$$y = \log_b x \longleftrightarrow x = b^y$$

Exponential		Logarithmic
$b^x b^y = b^{x+y}$	Product Rule	$\log_b xy = \log_b x + \log_b y$
$\frac{b^x}{b^y} = b^{x-y}$	Quotient Rule	$\log_b \left(\frac{x}{y}\right) = \log_b x - \log_b y$
$(b^x)^y = b^{xy}$	Power Rule	$\log_b x^y = y \log_b x$
$b^{-x} = \frac{1}{b^x}$		$\log_b \left(\frac{1}{x}\right) = -\log_b x$
$b^0 = 1$		$\log_b 1 = 0$
$b^1 = b$		$\log_b b = 1$
	Change of Base	$\log_b x = \frac{\log_c x}{\log_c b}$
	Natural Log	$\log_e x = \ln e$
	Common Log	$\log_{10} x = \log x$