Statistical Reasoning

MATHEMATICS KEY COMPETENCIES & COURSE STANDARDS

WITH LEARNING OBJECTIVES IN PROGRESSION ORDER

GEORGIA'S K-12 MATHEMATICS STANDARDS 2021

Governor Kemp and Superintendent Woods are committed to the best set of academic standards for Georgia's students – laying a strong foundation of the fundamentals, ensuring age- and developmentally appropriate concepts and content, providing instructional supports to set our teachers up for success, protecting and affirming local control and flexibility regarding the use of mathematical strategies and methods, and preparing students for life. These Georgia-owned and Georgia-grown standards leverage the insight, expertise, experience, and efforts of thousands of Georgians to deliver the very best educational experience for Georgia's 1.7 million students.

In August 2019, Governor Brian Kemp and State School Superintendent Richard Woods announced the review and revision of Georgia's K-12 mathematics standards. Georgians have been engaged throughout the standards review and revision process through public surveys and working groups. In addition to educator working groups, surveys, and the Academic Review Committee, Governor Kemp announced a new way for Georgians to provide input on the standards: the Citizens Review Committee, a group composed of students, parents, business and community leaders, and concerned citizens from across the state. Together, these efforts were undertaken to ensure Georgians will have buy-in and faith in the process and product.

The Citizens Review Committee provided a charge and recommendations to the working groups of educators who came together to craft the standards, ensuring the result would be usable and friendly for parents and students in addition to educators. More than 14,000 Georgians participated in the state's public survey from July through September 2019, providing additional feedback for educators to review. The process of writing the standards involved more than 200 mathematics educators -- from beginning to veteran teachers, representing rural, suburban, and metro areas of our state.

Grade-level teams of mathematics teachers engaged in deep discussions; analyzed stakeholder feedback; reviewed every single standard, concept, and skill; and provided draft recommendations. To support fellow mathematics teachers, they also developed learning progressions to show when key concepts were introduced and how they progressed across grade levels, provided examples, and defined age/developmentally appropriate expectations.

These teachers reinforced that strategies and methods for solving mathematical problems are classroom decisions -- not state decisions -- and should be made with the best interest of the individual child in mind. These recommended revisions have been shared with the Academic Review Committee, which is composed of postsecondary partners, age/development experts, and business leaders, as well as the Citizens Review Committee, for final input and feedback.

Based on the recommendation of Superintendent Woods, the State Board of Education will vote to post the draft K-12 mathematics standards for public comment. Following public comment, the standards will be recommended for adoption, followed by a year of teacher training and professional learning prior to implementation.

Statistical Reasoning

Overview

This document contains a draft of Georgia's 2021 K-12 Mathematics Standards for the High School Statistical Reasoning Course, which is a fourth mathematics course option in the high school course sequence.

The standards are organized into big ideas, course competencies/standards, and learning objectives/expectations. The grade level key competencies represent the standard expectation of learning for students in each grade level. The competencies/standards are each followed by more detailed learning objectives that further explain the expectations for learning in the specific grade levels.

New instructional supports are included, such as clarification of language and expectations, as well as detailed examples. These have been provided for teaching professionals and stakeholders through the Evidence of Student Learning Column that accompanies each learning objective.

Course Description:

The Statistical Reasoning course offers students opportunities to strengthen their understanding of the statistical method of inquiry and statistical simulations. Students will formulate statistical investigative questions to be answered using data, will design and implement a plan to collect the appropriate data, will select appropriate graphical and numerical methods for data analysis, and will interpret their results to make connections with the initial question. The Standards for Mathematical Practices and Mathematical Modeling, through a statistical lens, will provide the foundation for instruction and assessment. Topics should be introduced and assessed using simulations and appropriate supporting technology.

Prerequisite:

This course is designed for students who have completed *Advanced Algebra / Algebra II.*

Georgia's K-12 Mathematics Standards - 2021
Mathematics Big Ideas and Learning Progressions, High School

Mathematics Big Ideas, HS

HIGH SCHOOL
MATHEMATICAL PRACTICES (MP)
MATHEMATICAL MODELING (MM)
NUMERICAL (QUANTITATIVE) REASONING (NR)
PATTERNING & ALGEBRAIC REASONING (PAR)
FUNCTIONAL & GRAPHICAL REASONING (FGR)
GEOMETRIC & SPATIAL REASONING (GSR)
DATA & STATISTICAL REASONING (DSR)
PROBABILISTIC REASONING (PR)

The 8 Mathematical Practices and the Mathematical Modeling Framework are essential to the implementation of the content standards presented in this course. More details related to these concepts can be found in the links below and in the first two standards presented in this course:

Mathematical Practices

Mathematical Modeling Framework

Statistical Reasoning

The six course standards listed below are the key content competencies students will be expected to master in this course. Additional clarity and details are provided through the classroom-level learning objectives and evidence of student learning details for each course standard found on subsequent pages of this document.

COURSE STANDARDS
SR.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.
SR.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.
SR.DSR.2: Formulate statistical investigative questions of interest to students that can be answered with data.
SR.DSR.3: Collect data by designing and implementing a plan to address the formulated statistical investigative question.
SR.DSR.4: Analyze data by selecting and using appropriate graphical and numerical methods.
SR.DSR.5: Interpret the results of the analysis, making connections to the formulated statistical investigative question.

Statistical Reasoning

MATHEMATICAL MODELING		
SR.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.		
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**
SR.MM.1.1	Explain contextual, mathematical problems using a mathematical model.	***Fundamentals*** • Students should be provided with opportunities to learn mathematics in the context of real-life problems. • Contextual, mathematical problems are mathematical problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (model with mathematics).
SR.MM.1.2	Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or the humanities.	***Fundamentals*** • Students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.
SR.MM.1.3	Using abstract and quantitative reasoning, make decisions about information and data from a real-life situation.	
SR.MM.1.4	Use various mathematical representations and structures with this information to represent and solve real-life problems.	

DATA & STATISTICAL REASONING – *Formulate Statistical Investigative Questions*			
SR.DSR.2: Formulate statistical investigative questions of interest to students that can be answered with data.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**	
SR.DSR.2.1	Formulate statistical investigative questions about a population using samples taken from the population.	***Terminology*** • A sample is a subset of a population. • Samples are taken when examining the entire population is not possible or feasible. • Statistical questions are set in a context where one wants to know something; are based in variability or uncertainty; are always data based; and are approximations or estimates that emerge from data analysis. • Deterministic questions are based upon exact calculations or theoretical deductions elicited from prior certain knowledge.	***Fundamentals*** • Students can distinguish between situations where a small group (e.g., a classroom) is the entire population (census) and when it is a sample from a larger population (e.g., the classroom is used to answer a question

			about an entire grade level in a school).
SR.DSR.2.2	Formulate comparative and associative statistical investigative questions for surveys, observational studies, and experiments to compare two or more groups or to investigate the association of two or more variables.	***Fundamentals*** • Students will draft statistical questions for which appropriate data can be collected and analyzed to answer the statistical investigative question.	
SR.DSR.2.3	Formulate multivariable statistical investigative questions.	***Fundamentals*** • As students engage in multivariable thinking, the types of statistical investigative questions should expand to include questions concerning association and prediction.	***Example*** • "Given a list of the arm spans of 9th grade students, what can be predicted about the heights of those students?"
SR.DSR.2.4	Formulate inferential statistical investigative questions regarding association and prediction.	***Fundamentals*** • Students will pose statistical investigative questions for a particular sample to determine any association of the variables of interest for that sample.	

DATA & STATISTICAL REASONING *– Collect Data*				
SR.DSR.3: Collect data by designing and implementing a plan to address the formulated statistical investigative question.				
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**		
SR.DSR.3.1	Apply an appropriate data-collection plan when collecting primary or secondary data for the statistical investigative question of interest.	***Fundamentals*** • Students will use appropriate sampling techniques, critique a poorly constructed survey, and make suggestions for good questions.	***Terminology*** • Primary data is collected through first-hand sources such as surveys, experiments, and other studies. • Secondary data is obtained from previously conducted studies or research.	***Examples*** • Students should consider features such as whether the population is well-defined, whether the sampling procedure is random or non-random, and whether the objectivity or bias of questions will result in valid/invalid answers.
SR.DSR.3.2	Distinguish between surveys, observational studies, and experiments.	***Terminology*** • Surveys involve the collection of data from a pre-defined group to gain insight and information about the statistical investigative question. • Observational studies measure a sample as it is without attempting to influence the results. • Experiments involve the use of a treatment to explore the effects of the treatment on a sample.		***Fundamentals*** • Students should understand the advantages and disadvantages of each data collection method for specific statistical questions.

SR.DSR.3.3	Design sample surveys, experiments, and observational studies using accepted practices.	***Fundamentals*** • Students are able to identify, discuss, and explain the aspects of best statistical practice for designing an experimental study, including: the clear identification of the statistical question to be investigated; the variables under investigation; and the random selection of experimental units and/or the random assignment of treatments to the experimental units.	
SR.DSR.3.4	Distinguish between random selection and random assignment and identify their impact on conclusions.	***Fundamentals*** • Students should be able to design and conduct comparative experiments using random assignment and demonstrate correct methods for planning data collection for comparison of treatments. • Students should be able to randomly assign treatments to experimental units.	
SR.DSR.3.5	Describe potential sources and effects of bias and confounding variables	***Fundamentals*** • Students should be able to design and conduct surveys from both non-random and randomly selected participants. • Students should be able to explain why random samples can provide more unbiased information about a population than other types of samples, such as convenience samples or self-selected samples.	***Examples*** • Types of bias include wording bias, under coverage, non-response bias, selection bias, and experimenter bias.
SR.DSR.3.6	Describe and adhere to the ethical use of data (e.g., sensitive information, privacy, and living subjects).	***Relevance and Application*** • Practices for handling data that enhance reproducibility and ensure ethical use include providing descriptions of alterations to collected data, proper treatment of sensitive information, maintaining the confidentiality of data and experimental units, and using Institutional Review Boards to review study designs.	***Fundamentals*** • Students should be able to describe the ethical consequences of their experiments and analyses.
SR.DSR.3.7	Identify when data can be generalized to a target population.	***Relevance and Application*** • Samples must be randomly selected from the appropriate population to allow for generalizations that extend beyond the sample from which the data were collected. • Sampling procedures that are not random do not allow for generalizations to the sampled population because they may be biased.	

DATA & STATISTICAL REASONING *– Analyze Data*			
SR.DSR.4: Analyze data by selecting and using appropriate graphical and numerical methods.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**	
SR.DSR.4.1	Summarize quantitative or categorical data using tables, graphical displays, and numerical summary statistics.	***Fundamentals*** • Students should identify types of displays that are appropriate for categorical data versus quantitative (numerical) data.	***Examples*** • Graphical displays and tables include histograms, modified boxplots, and relative frequency tables.
SR.DSR.4.2	Summarize and describe relationships among multiple variables.	***Fundamentals*** • Students should be able to identify situations where a change in one attribute may be related to a change in another attribute. • Students should be able to construct scatterplots, and describe positive, negative or no relationship.	***Relevance and Application*** • Strength of association is demonstrated by degree of spread about the line of best fit in a scatterplot.
SR.DSR.4.3	Use sampling distributions developed through simulation to describe the sample-to-sample variability of sample statistics.	***Fundamentals*** • Students should be able to recognize that sample statistics vary with repeated sampling. • Students should be able to interpret the sampling variability in a summary statistic. • Students should be able to interpret the sampling variability from simulation studies of statistics. • Students should be able to recognize how sampling variability is influenced by sample size.	***Examples*** • Sampling variability from simulation studies of statistics include sample mean, sample proportion, median, IQR, and standard deviation.
SR.DSR.4.4	Use sampling distributions to compute simulated p-values.	***Fundamentals*** • Students should be able to approximate p--values using simulation or simulation results, especially for the difference in two means or two proportions. • Students should be able to convert a p-value into a statement about their confidence that the observed data was produced by the treatment rather than by random chance.	
SR.DSR.4.5	Describe the relationship between two quantitative variables by interpreting correlation (r) and a least-square regression line (using technology).	***Fundamentals*** • Students should be able to provide a reasonable estimate of the Pearson's correlation coefficient (*r*) for a scatterplot; identify linear and non-linear relationships in scatterplots; correctly interpret the strength of a linear relationship based on *r*. • Students should be able to understand the magnitude of a correlation coefficient represents the strength of association; understand and able to calculate a residual; understand that any straight line other than the best fit line (by least squares) will have a larger sum of squared residuals than the best fit line.	***Example*** • Correlation coefficients of *r* =- -.65 and *r* = .65 indicate the same strength.
SR.DSR.4.6	Use simulations to investigate associations between two categorical variables and to compare groups.	***Fundamentals*** • Students should be able to interpret measures of association to determine if there is a relationship between variables. • should be able to recognize that “association” does not imply “cause and effect.”	

DATA & STATISTICAL REASONING *– Interpret Results*			
SR.DSR.5: Interpret the results of the analysis, making connections to the formulated statistical investigative question.			
Expectations		**Evidence of Student Learning** **(not all inclusive; see Course Overview for more details)**	
SR.DSR.5.1	Use statistical evidence from analyses to answer the formulated statistical investigative questions.	***Fundamentals*** • Students should be able to decide whether an observed difference is something that would be likely to be observed by chance and whether this difference has any practical meaning. • Students recognize that significance is demonstrated by a result that is unlikely to occur by chance • Students recognize that statistical, but not practical, significance is influenced by sample size.	
SR.DSR.5.2	Interpret the impact of outliers, missing values, or erroneous values on the results.	***Fundamentals*** • Students should be able to describe how population estimates may be overstated or understated due to the presence of outliers. • Students should be able to describe how missing or erroneous values can lead to biased or inaccurate estimations.	
SR.DSR.5.3	Use and interpret the p-value to determine whether the estimate for a population characteristic is plausible.	***Fundamentals*** • Students should be able to interpret a p-value to make an inference in the context of a study.	
SR.DSR.5.4	Interpret a given margin of error associated with an estimate of a population characteristic.	***Fundamentals*** • Students should be able to interpret the confidence interval(s) in relation to the situation being examined.	***Relevance and Application*** • Sampling variability is associated with summary statistics and uses the margin of error to form an interval (confidence interval) to estimate the characteristic.
SR.DSR.5.5	Explain the impact of multiple variables on one another.	***Fundamentals*** • Students provide or select appropriate interpretations of graphical displays and numerical summaries to compare two or more groups in the context of a study.	

ESSENTIAL INSTRUCTIONAL SUPPORTS

MATHEMATICAL PRACTICES

The Mathematical Practices describe the reasoning behaviors students should develop as they build an understanding of mathematics – the "habits of mind" that help students become mathematical thinkers. There are eight standards, which apply to all grade levels and conceptual categories.

These mathematical practices describe how students should engage with the mathematics content for their grade level. Developing these habits of mind builds students' capacity to become mathematical thinkers. These practices can be applied individually or together in mathematics lessons, and no particular order is required. In well-designed lessons, there are often two or more Standards for Mathematical Practice present.

Mathematical Practices	
SR.MP: Display perseverance and patience in problem-solving. Demonstrate skills and strategies needed to succeed in mathematics, including critical thinking, reasoning, and effective collaboration and expression. Seek help and apply feedback. Set and monitor goals.	
Code	**Expectation**
SR.MP.1	Make sense of problems and persevere in solving them.
SR.MP.2	Reason abstractly and quantitatively.
SR.MP.3	Construct viable arguments and critique the reasoning of others.
SR.MP.4	Model with mathematics.
SR.MP.5	Use appropriate tools strategically.
SR.MP.6	Attend to precision.
SR.MP.7	Look for and make use of structure.
SR.MP.8	Look for and express regularity in repeated reasoning.

MATHEMATICAL MODELING

Teaching students to model with mathematics is engaging, builds confidence and competence, and gives students the opportunity to collaborate and make sense of the world around them, the main reason for doing mathematics. For these reasons, mathematical modeling should be incorporated at every level of a student's education. This is important not only to develop a deep understanding of mathematics itself, but more importantly to give students the tools they need to make sense of the world around them. Students who engage in mathematical modeling will not only be prepared for their chosen career but will also learn to make informed daily life decisions based on data and the models they create.

The diagram below is a mathematical modeling framework depicting a cycle of how students can engage in mathematical modeling when solving a real-life problem or task.

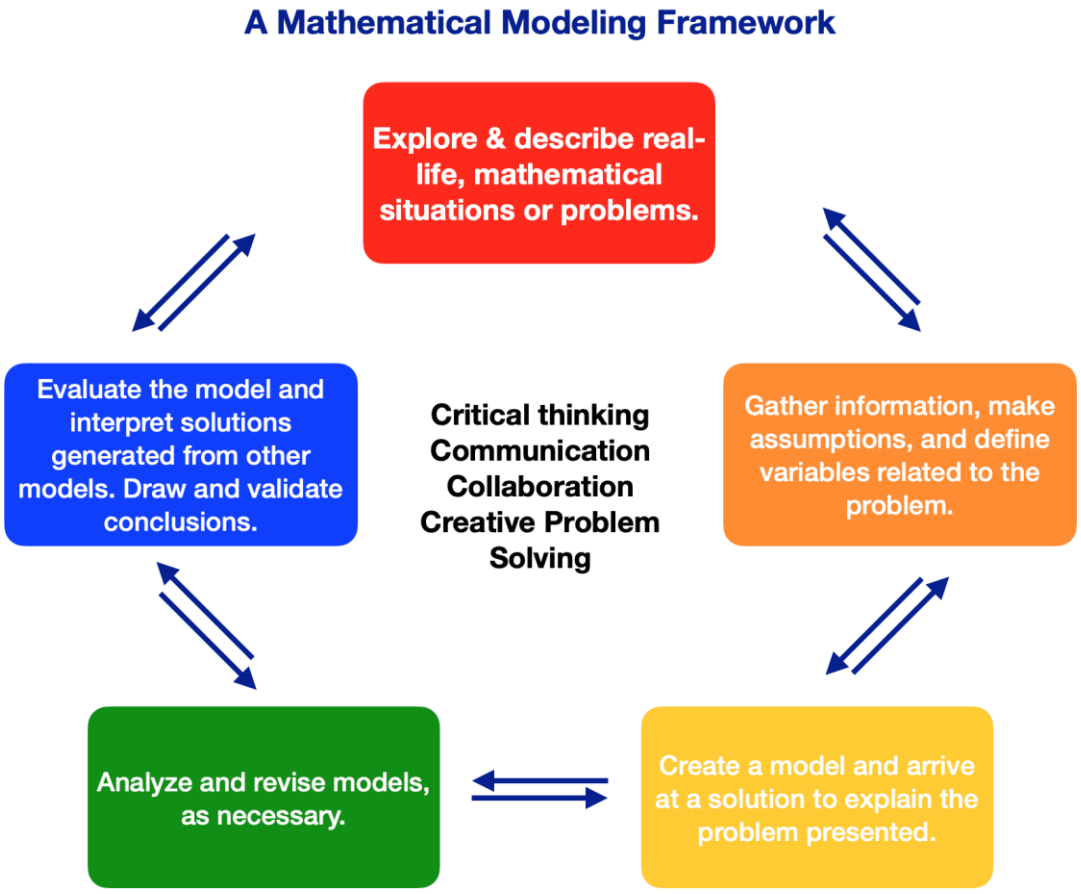

Image adapted from: Suh, Matson, Seshaiyer, 2017

FRAMEWORK FOR STATISTICAL REASONING

Statistical reasoning is important for learners to engage as citizens and professionals in a world that continues to change and evolve. Humans are naturally curious beings and statistics is a language that can be used to better answer questions about personal choices and/or make sense of naturally occurring phenomena. Statistics is a way to ask questions, explore, and make sense of the world around us.

The Framework for Statistical Reasoning should be used in all grade levels and courses to guide learners through the sense-making process, ultimately leading to the goal of statistical literacy in all grade levels and courses. Reasoning with statistics provides a context that necessitates the learning and application of a variety of mathematical concepts.

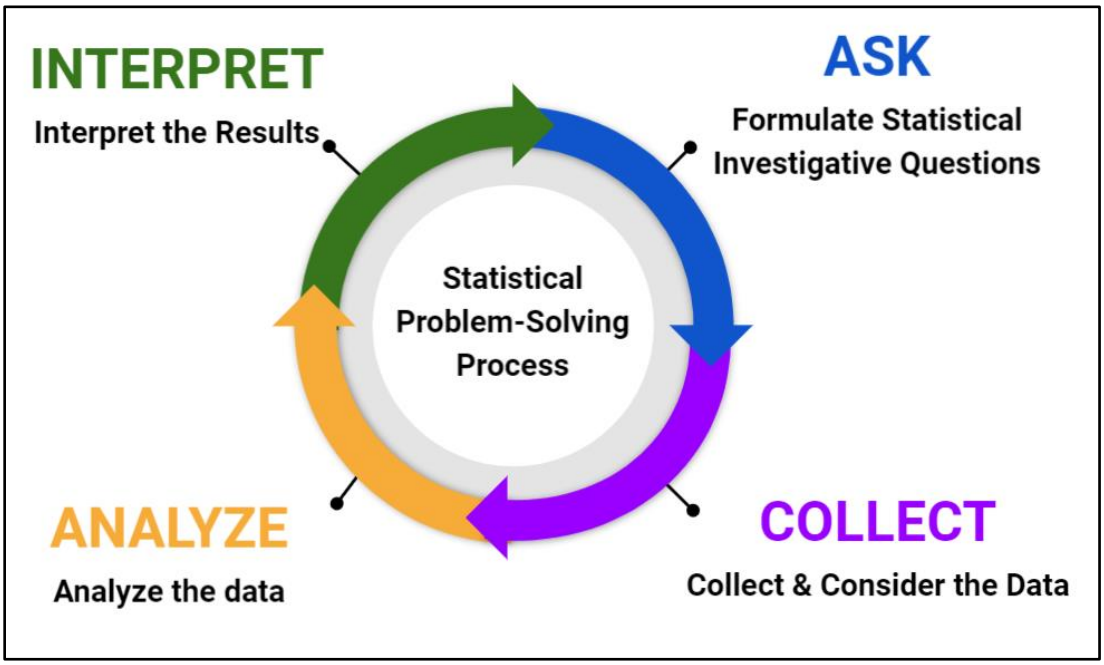

Figure 1: Georgia Framework for Statistical Reasoning

The following four-step statistical problem-solving process can be used throughout each grade level and course to help learners develop a solid foundation in statistical reasoning and literacy:

I. Formulate Statistical Investigative Questions
Ask questions that anticipate variability.

II. Collect & Consider the Data
Ensure that data collection designs acknowledge variability.

III. Analyze the Data
Make sense of data and communicate what the data mean using pictures (graphs) and words. Give an accounting of variability, as appropriate.

IV. Interpret the Results
Answer statistical investigative questions based on the collected data.