Name: KEY
ALG 3 & TRIG SUMMER ASSIGNMENT

Exponents and Radicals

Integer and Rational Exponents

In general, if b is a real number and r is a positive integer, then $b^r = \underbrace{b \cdot b \cdot b \cdots b}_{r\ factors}$, where r is the exponent and b is the base.

Complete the following properties of exponents.

$(ab)^m = a^m b^m$ $\quad a^{m+n} = a^m \cdot a^n$ $\quad a^{mn} = (a^m)^n$ $\quad a^0 = 1$

$\frac{1}{a^n} = a^{-n}$ $\quad \frac{a^m}{a^n} = a^{m-n}$ $\quad \left(\frac{a}{b}\right)^m = \frac{a^m}{b^m}$ $\quad a^{1/n} = \sqrt[n]{a^1}$

If a is a real number and n is a positive integer such that the principle nth root of a exists, then $a^{1/n} = \sqrt[n]{a}$, where $1/n$ is the exponent. The numerator of a rational exponent denotes the exponent to which the base is raised, and the denominator denotes the index or the root to be taken.

Example 1: Write the radical expression $\sqrt[4]{w^9}$ in exponential form.

$\sqrt[4]{w^9} = w^{\frac{9}{4}}$

Example 2: Simplify the expression $\frac{x^{3/4}}{x^{2/3}}$.

$\frac{x^{\frac{3}{4}}}{x^{\frac{2}{3}}} = x^{\frac{3}{4}-\frac{2}{3}} = x^{\frac{9}{12}-\frac{8}{12}} = x^{\frac{1}{12}}$

Radicals and Their Properties

Let a and b be real numbers. If $a = b^2$, then b is the square root of a. If $a = b^3$, then b is the cube root of a. In $\sqrt[n]{a}$, the positive integer n is the index of the radical, and the number a is the radicand. The radical expression $\sqrt{-36}$ is not a real number because no real # squared is negative

Simplifying Radicals

An expression involving radicals is in simplest form when the following conditions are satisfied:

1) No Perfect Products in Radicand.
2) All FRACTIONS have radical-free denominators.
3) The index of radical is fully reduced.

Radical expressions are like radicals when they have same indexes and same radicands. To add or subtract like radicals combine their coefficents.

Example 1: Simplify each radical expression.

(a) $-\sqrt{\frac{81}{16}}$ (b) $\sqrt[3]{3} - .5\sqrt[3]{3}$ (c) $\sqrt{8}$ (d) $\sqrt{136}$ (E) $\sqrt[3]{m} + \sqrt[3]{9}$

(a) $-\frac{\sqrt{81}}{\sqrt{16}} = -\frac{9}{4}$

(b) $(1-5)\sqrt[3]{3} = -4\sqrt[3]{3}$

(c) $\sqrt{4}\cdot\sqrt{2} = 2\sqrt{2}$

(d) $\sqrt{4}\cdot\sqrt{34} = 2\sqrt{34}$

(E) $= \sqrt[3]{m} + \sqrt[3]{9}$

Rationalizing Denominators and Numerators

The conjugate of the radical expression $a + b\sqrt{m}$ is $a - b\sqrt{m}$. To rationalize a denominator of the form $a - b\sqrt{m}$ or $a + b\sqrt{m}$, multiply $(a - b\sqrt{m})(a + b\sqrt{m}) = a^2 - b^2 m$.

Example 1: Simplify the radical expression by rationalizing the denominator of the expression

(a) $\frac{4}{\sqrt{8}} \cdot \frac{\sqrt{8}}{\sqrt{8}} = \frac{4\sqrt{8}}{\sqrt{64}}$

$= \frac{4\sqrt{8}}{8} = \frac{\sqrt{8}}{2} = \frac{2\sqrt{2}}{2} = \sqrt{2}$

(b) $-\frac{4}{(5-\sqrt{8})} \cdot \frac{(5+\sqrt{8})}{(5+\sqrt{8})}$ DIST FOIL

$\frac{20+4\sqrt{8}}{25+\sqrt{8}-\sqrt{8}-\sqrt{64}} = \frac{20+8\sqrt{2}}{25-8}$

$= \frac{20+8\sqrt{2}}{17}$

Polynomials

In standard form, a polynomial in x is written with exponents in descending order.

Polynomials with one term are called monomials.

Polynomials with two terms are called binomials.

Polynomials with three terms are called trinomials.

For polynomials in one variable, the degree of the polynomial is highest/largest exponent.

Example 1: Write the polynomial $1 - 6y - 5y^3 + 4y^2$ in standard form and find the degree of the polynomial.

STANDARD FORM: $-5y^3 + 4y^2 - 6y + 1$ Degree: 3

Operations with Polynomials

Like terms are terms that have same base raised to same exponent.

To add or subtract polynomials, combine coeffcients of like terms.

To find the product of two polynomials, distribute or FOIL.

Example 1: (a) Subtract: $(5x^3 - 9x + 4) - (2x^3 + 5x^2 - 12x + 4)$

$5x^3 \boxed{-9x} + 4 - 2x^3 - 5x^2 \boxed{+12x} - 4$

$3x^3 - 5x^2 + 3x$

(b) Multiply: $(3x - 2)($[illegible]$- 2x - 1)$

$3x^3 + 6x^2 \boxed{-3x} - 2x^2 \boxed{-4x} + 2$

$3x^3 + 4x^2 - 7x + 2$

Example 2: (a) Multiply: $(3x-4)^2$ FOIL

$(3x-4)(3x-4)$

$9x^2-12x-12x+16$

$9x^2-24x+16$

(b) Multiply: $(x+2)^2$ FOIL

$(x+2)(x+2)$

$x^2+2x+2x+4$

x^2+4x+4

Factoring Polynomials

Factoring is rewriting a product as it's prime factors. If a polynomial does not factor using integer coefficients, then it is prime or unfactorable.

The simplest type of factoring involves a polynomial that can be written as the product of a monomial and another polynomial.

The technique used here is the Distributive Property in reverse: $ab + ac = a(b+c)$

Example 1: Factor: (a) $3w^3 - 12w^2 + 15w$

$3w(w^2-4w+5)$

Factor: (b) $9x^2 - 18$

$9(x^2-2)$

Trinomials with Binomial Factors

To factor a trinomial of the form $ax^2 + bx + c = ([]x+[])([]x+[])$, the goal is to find factors of $a \cdot c$ that add up to b.

Example 1: Factor: (a) $x^2 + 3x - 18$

$(x^2+6x)(-3x-18)$

$x(x+6)-3(x+6)$

$(x-3)(x+6)$

$(x+6)(x-3)$

Factor: (b) $9x^2 + 12x + 4$

$(3x+2)(3x+2)$

$(3x+2)^2$

$(9x^2+6x)(+6x+4)$

$3x(3x+2)+2(3x+2)$

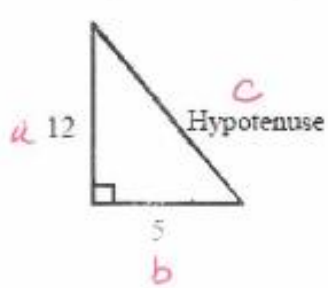
$(3x+2)(3x+2)$

$(3x+2)^2$

Mathematical Models

The **Pythagorean Theorem** states that for a right triangle with hypotenuse of length c and sides of lengths a and b, the mathematical relationship between a, b, and c is $a^2+b^2=c^2$.

Example 1: In the right triangle below, find the length of the hypotenuse, by using the **Pythagorean Theorem**



$a^2+b^2=c^2$

$(12)^2+(5)^2=c^2$

$144+25=c^2$

$169=c^2$

$\sqrt{169}=c$

$c=13$

The Distance Formula states that $d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$.

Example 1: Find the distance between the points $\overset{x_1,\,y_1}{(4, 2)}$ and $\overset{x_2,\,y_2}{(5, -1)}$.

$$d = \sqrt{(5-4)^2 + (-1-2)^2} = \sqrt{(1)^2 + (-3)^2} = \sqrt{1+9} = \sqrt{10} \approx 3.2$$

The Midpoint Formula gives the midpoint of the segment joining the points (x_1, y_1) and (x_2, y_2) as $(x_m, y_m) = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$.

Example 1: Find the midpoint of the line segment with endpoints at $\overset{x_1\,y_1}{(-8, 2)}$ and $\overset{x_2\,y_2}{(6, -10)}$. Then find the coordinates of the midpoint.

$$(x_m, y_m) = \left(\frac{-8+6}{2}, \frac{2+(-10)}{2}\right) = \left(\frac{-2}{2}, \frac{-8}{2}\right) = (-1, -4)$$

Linear Equations in One Variable

A linear equation in one variable x is an equation that can be written in the standard form $ax + b = 0$, where a and b are real numbers with $a \neq$ 0. A linear equation has exactly one solution(s).

Example 1: Solve $5(x + 3) = 35$.

$5x + 15 = 35$
$5x = 20$
$x = 4$

OR

$\frac{5(x+3)}{5} = \frac{35}{5}$
$x + 3 = 7$
$x = 4$

Rational Equations That Lead to Linear Equations

A rational equation is an equation with variable fractions. To solve a rational equation, multiply by the Least Common Denominator. A rational equation with a single fraction on each side can be cleared of denominators by cross multiplying that is, multiplying the left numerator by the right denominator and the right numerator by the left numerator. Then, solve the linear equation

LCD: 14

CROSS multiply

Example 1: Solve: (a) $\frac{5x}{7} = \frac{9}{14}$

$14\left(\frac{5x}{\cancel{7}}\right) = \left(\frac{9}{\cancel{14}}\right)14$
$2(5x) = 9$
$10x = 9$
$x = \frac{9}{10}$

$70x = 63$
$x = \frac{63 \div 7}{70 \div 7}$
$x = \frac{9}{10}$

LCD: $(x+1)(x-1)$

Solve: (b) $\frac{1}{x+1} + \frac{5}{x-1} = \frac{4}{x-1}$

$$\cancel{(x+1)}(x-1)\left(\frac{1}{\cancel{x+1}}\right) + (x+1)\cancel{(x-1)}\left(\frac{5}{\cancel{x-1}}\right) = (x+1)\cancel{(x-1)}\left(\frac{4}{\cancel{x-1}}\right)$$

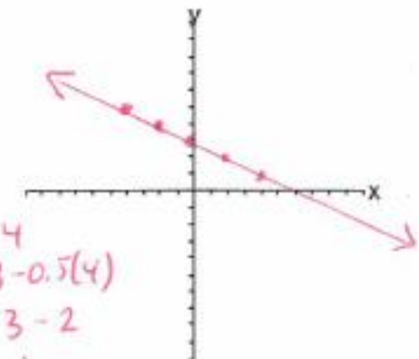
$x - 1 + 5(x+1) = 4(x+1)$
$x - 1 + 5x + 5 = 4x + 4$
$6x + 4 = 4x + 4$
$2x = 0$
$x = 0$

The Graph of a Linear Equation in Two Variables

The linear equation in two variables $y = mx + b$ is called *linear* because it's graph is a line.
A line whose slope is positive rises from left to right. A line whose slope is negative falls from from left to right. The **slope-intercept form** of the equation of a line is $y = mx + b$, where m is the slope and the y-intercept is (0 , b). A vertical line has an equation of the form $x = x\text{-intercept}$.
The equation of a vertical line cannot be written in the form $y = mx + b$ because it's slope is undefined.
To sketch the graph of a linear equation in two variables use the point-plotting method.

Example 1: Complete the table. Then use the resulting solution points to sketch the graph of the equation $y = 3 - 0.5x$.

x	-4	-2	0	2	4
y	5	4	3	2	1



$x = -4$
$y = 3 - 0.5(-4)$
$y = 3 + 2$
$y = 5$

$x = -2$
$y = 3 - 0.5(-2)$
$y = 3 + 1$
$y = 4$

$x = 0$
$y = 3 - 0.5(0)$
$y = 3$

$x = 2$
$y = 3 - 0.5(2)$
$y = 3 - 1$
$y = 2$

$x = 4$
$y = 3 - 0.5(4)$
$y = 3 - 2$
$y = 1$

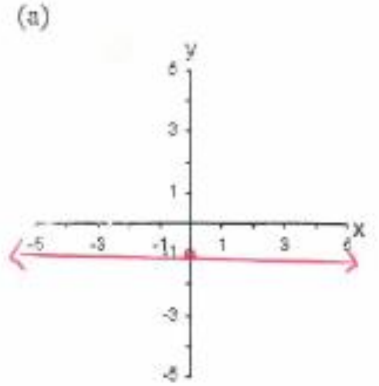
The slope and the y-intercept of a linear equation in two variables can be used to sketch the graph of a linear equation in two variables

Complete the following steps to sketch the graph of a linear equation in two variables using slope and the y-intercept

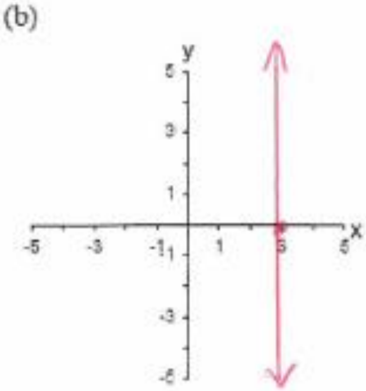
1) Plot y-intercept (0, b)
2) Use slope to plot other points (ie. $\frac{rise}{run}$)
3) Connect points with straight line

Example 1: Sketch the graph of (a) $y = -1$ (b) $x = 3$ and (c) $y = -2/3x - 4$.

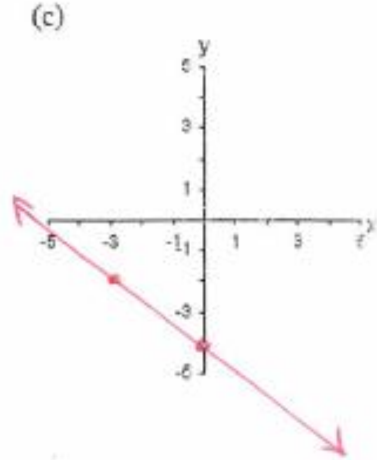
(a)



(b)



(c)



Finding the Slope of a Line

The slope m of the nonvertical line through (x_1, y_1) and (x_2, y_2) is $m = \dfrac{y_2 - y_1}{x_2 - x_1}$

Example 1: Find the slope of the line through the points $(\overset{x_1}{-2}, \overset{y_1}{5})$ and $(\overset{x_2}{4}, \overset{y_2}{-3})$.

$$m = \frac{-3-5}{4-(-2)} = \frac{-8 \div 2}{6 \div 2} = -\frac{4}{3}$$

If a line falls from left to right, it has negative slope. If a line is horizontal, it has zero slope. If a line is vertical, it has undefined slope. If a line rises from left to right, it has positive slope.

Parallel and Perpendicular Lines

The relationship between the slopes of two lines that are parallel is that they are equal

The relationship between the slopes of two lines that are perpendicular is that they are opposite reciprocals

Example 1: (a) A line that is parallel to a line whose slope is 2 has slope 2.

(b) A line that is perpendicular to a line whose slope is 2 has slope $-\frac{1}{2}$.

Writing Linear Equations in Two Variables

Complete the following steps to write linear equations in two variables

1) Find the slope m and y-int b.
2) Plug m and b into y=mx+b.

Functions and Function Notation

The symbol $f(x)$ is **function notation** read as *the value of f at x* or simply *f of x*. Keep in mind that f is the name of the function, whereas $f(x)$ is the value of the function at x.

Example 1: If $f(w) = 4w^3 - 5w^2 - 7w + 13$, find $f(-2)$.

$$f(-2) = 4(-2)^3 - 5(-2)^2 - 7(-2) + 13$$
$$f(-2) = 4(-8) - 5(4) + 14 + 13$$
$$f(-2) = -32 - 20 + 14 + 13$$
$$f(-2) = -52 + 14 + 13$$
$$f(-2) = -38 + 13$$
$$f(-2) = -25$$

Example 2: If $f(x) = \sqrt{5x} + 13$, find $f(5)$

$$f(5) = \sqrt{5(5)} + 13$$
$$f(5) = \sqrt{25} + 13$$
$$f(5) = 5 + 13$$
$$f(5) = 18$$