

| Marking Period: 2   | Unit Title: Polynomial, Power, and Rational Functions | Recommended Instruction Days: 15 - 20 |
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| <b>Standard-New Jersey Student Learning Standards: N-CN, A-SSE, A-APR, F-IF, S-ID Polynomial, Power, and Rational Functions (Chapter 2)</b>   |   |                                       |
| <p><b>Strand:</b><br/><b>N-CN: The Complex Number System</b><br/><b>Perform arithmetic operations with complex numbers</b></p> <ol style="list-style-type: none"><li>1. Know there is a complex number <math>i</math> such that <math>i^2 = -1</math>, and every complex number has the form <math>a + bi</math> with <math>a</math> and <math>b</math>.</li><li>2. Use the relation <math>i^2 = -1</math> and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.</li></ol> <p><b>Use Complex Numbers in Polynomial Identities and Equations</b></p> <ol style="list-style-type: none"><li>7. Solve quadratic equations with real coefficients that have complex solutions.</li><li>8. Extend polynomial identities to the complex numbers.</li><li>9. Know the Fundamental Theorem of Algebra; show that is true for quadratic polynomials.</li></ol> <p><b>A-SSE: Seeing Structure in Expressions</b><br/><b>Write expressions in equivalent form to solve problems</b></p> <ol style="list-style-type: none"><li>3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.<ol style="list-style-type: none"><li>a. Factor a quadratic expression to reveal the zeros of the function it defines.</li><li>b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.</li></ol></li></ol> <p><b>A-APR: Arithmetic with Polynomials and Rational Expressions</b><br/><b>Perform Arithmetic Operations on Polynomials</b></p> <ol style="list-style-type: none"><li>1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.</li></ol> <p><b>Understand the Relationship between Zeros and Factors of Polynomials</b></p> <ol style="list-style-type: none"><li>2. Know and apply the Remainder Theorem: For a polynomial <math>p(x)</math> and a number <math>a</math>, the remainder on division by <math>x - a</math> is <math>p(a)</math>, so <math>p(a) = 0</math> if and only if <math>(x - a)</math> is a factor of <math>p(x)</math>.</li><li>3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</li></ol> <p><b>Rewrite Rational Expressions</b></p> |   |                                       |

6. Rewrite simple rational expressions in different forms; write  $a(x)/b(x)$  in the form  $q(x) + r(x)/b(x)$ , where  $a(x)$ ,  $b(x)$ ,  $q(x)$ , and  $r(x)$  are polynomials with the degree of  $r(x)$  less than the degree of  $b(x)$ , using inspection, long division, or, for the more complicated examples, a computer algebra system.
7. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

**F-IF: Interpreting Functions**

**Analyze functions using different representations**

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
  - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
  - d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

**S-ID: Interpreting Categorical and Quantitative Data**

**Summarize, Represent, and Interpret Data on Two Categorical and Quantitative Variables**

6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
  - a. Fit a function to the data (including with the use of technology); use functions fitted to data to solve problems in the context of the data.
  - b. Informally assess the fit of a function by plotting and analyzing residuals, including with the use of technology.
  - c. Fit a linear function for a scatter plot that suggests a linear association.

**Interpret Linear Models**

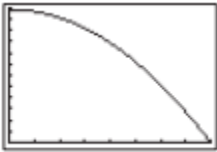
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*

Neil Devine - American Stellar and planetary astrophysicist whose work centered on the understanding of star formation.

The mission is to ensure that every student is able to see themselves in our rich and diverse history.

| <b>Social and Emotional Learning:<br/><i>Competencies</i></b>   |   | <b>Social and Emotional Learning:<br/><i>Sub-Competencies</i></b>  |
|---|---|--|
| <p>Self-Awareness</p> <p>Social Awareness</p> <p>Self-Management</p> <p>Relationship Skills</p> <p>Responsible Decision-Making</p>  |   | <ul style="list-style-type: none"> <li>● Recognizing the importance of self-confidence in handling daily tasks and challenges.</li> <li>● Demonstrate an awareness of the expectations for social interactions in a variety of ways.</li> <li>● Demonstrate an understanding of the need for mutual respect when viewpoints differ.</li> <li>● Recognize the skills needed to establish and achieve personal and educational goals.</li> <li>● Utilize positive communication and social skills to interact effectively with others.</li> <li>● Develop, implement, and model effective problem solving and critical thinking skills.</li> </ul> |
| <p><b>Recommended Activities, Investigations,<br/>Interdisciplinary Connections, and/or Student<br/>Experiences to Explore NJSLM-M within Unit</b></p>  |   |  |
| <b>Essential Questions</b>  | <b>Progress Indicators</b>  | <b>Activity Description</b>  |
| <ul style="list-style-type: none"> <li>● What determines the graphs for polynomial and quadratic functions?</li> <li>● What algebraic methods are used to both determine both real-and complex-number solutions of</li> </ul> | <ul style="list-style-type: none"> <li>● Tests</li> <li>● Quizzes</li> <li>● Practice problems for homework</li> <li>● Worksheets</li> <li>● Leveled assessments</li> <li>● Projects</li> </ul> | <ul style="list-style-type: none"> <li>❖ Linear and Quadratic Functions and Modeling</li> <li>❖ Power Functions with Modeling</li> <li>❖ Polynomial Functions of Higher Degree with Modeling</li> <li>❖ Real Zeros of Polynomial Functions</li> <li>❖ Complex Zeros and the Fundamental Theorem of Algebra</li> <li>❖ Graphs of Rational Functions</li> <li>❖ Example Tasks Below</li> </ul> <p style="text-align: center;"><b>Interdisciplinary Connections:</b><br/><b>Science Domain:From Molecules to Organisms: Structures and Processes</b></p>  |

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| <p>polynomial equations?</p> <ul style="list-style-type: none"><li>• What is the Fundamental Theorem of Algebra and how is it used in determining solutions of a polynomial function?</li></ul> |  | <p>Research conducted at a national health research project shows that the speed at which a blood cell travels in an artery depends on its distance from the center of the artery. The function <math>v = 1.19 - 1.87r^2</math> models the velocity (in cm per second) of a cell that is <math>r</math> cm from the center of an artery.</p> <ol style="list-style-type: none"><li>Find a graph of <math>v</math> that reflects values of <math>v</math> appropriate for this problem. Record the viewing-window dimensions.</li><li>If a blood cell is traveling at 0.975 cm/sec, estimate the distance the blood cell is from the center of the artery.</li></ol> <p><b>Answer:</b></p> <p>a) <br/>[0, 0.8] by [0, 1.20]</p> <p>b) 0.3391 cm</p> <p><b>Task</b></p> |
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Fill in each blank:

- a.  $f(x) = 4x^3 - 7x^2 + 3x - 11$  is a polynomial of degree \_\_\_\_\_.
- b.  $f(x) = 4x - 6$  is a \_\_\_\_\_ function.
- c. The rate of change of  $f(x) = -13x + 9$  is \_\_\_\_\_.
- d. The y-intercept of the graph of  $y = 2/3x + 4$  is the point \_\_\_\_\_.
- e. The equation of a parabola is a \_\_\_\_\_ of degree 2.
- f. If  $f(x) = mx + b$  is decreasing,  $m$  is \_\_\_\_\_ than zero.
- g. The parabola  $f(x) = ax^2 + bx + c$  open upward if \_\_\_\_\_.
- h. The vertex of the parabola  $f(x) = a(x - h)^2 + k$  is located at the point \_\_\_\_\_.
- i. The vertex of the parabola  $f(x) = ax^2 + bx + c$  is located at the point \_\_\_\_\_.
- j. The line of symmetry of the parabola  $f(x) = ax^2 + bx + c$  is \_\_\_\_\_.

**Answer**

- a. 3
- b. Linear
- c. -13
- d. (0,4)
- e. Polynomial
- f. Less
- g.  $a > 0$
- h. (h,k)
- i.  $(-b/2a, c - b^2/4a)$
- j.  $X = -b/2a$

**Task**

Solve the following problems:

- a. The numbers 5, 7,  $2 + 3i$ ,  $3 + 5i$ , and  $3 - 5i$  are zeros of the polynomial function  $h(x)$ , which has real coefficients. What is the minimum possible degree of  $h(x)$ ?
- b. Explain how to use the discriminant  $b^2 - 4ac$  to determine whether or not a quadratic factor  $ax^2 + bx + c$  is irreducible over the reals.
- c. Solve the equation  $(2x/x+2) = (5/x^2 - x - 6) - (1/x-3)$  algebraically. Check for extraneous solutions.

**Answer**

a. 6: If you multiply  $(x - 5)(x - 7)(x - (2 + 3i))(x - (2 - 3i))(x - (3 + 5i))$  and  $(x - (3 - 5i))$  you will have  $x$  to the 6<sup>th</sup> power because you are multiplying 6 terms together.

b.  $ax^2 + bx + c$  is irreducible over the reals if  $b^2 - 4ac < 0$ .

c.  $(x + 2)(x - 3)[(2x/x+2) = (5/x^2 - x - 6) - (1/x-3)]$

$$(x - 3)2x = 5 - 1(x + 2)$$

$$2x^2 - 6x = 5 - 1x - 2$$

$$2x^2 - 6x = -1x + 3$$

$$2x^2 - 5x - 3 = 0$$

$$(2x + 1)(x - 3) = 0$$

$$x = -1/2 \text{ or } 3$$

Plug in  $-1/2$  :  $(2(-1/2)/(-1/2)+2) = (5/(-1/2)^2 - (-1/2) - 6) - (1/(-1/2)-3)$ .

$$x = -1/2$$

Plug in  $x = 3$ .

3 is extraneous because  $1/3-3$  is undefined

**Spot Light On:** *Use random response strategies.*

**Mathematical Practices**

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others.**
4. **Model with mathematics.**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

**Assessments (Formative)**

*To show evidence of meeting the standard/s, students will successfully engage within:*

**Formative Assessment:**

- Entry and Exit Slips
- Quizzes
- Self Assessments
- Focus Packets

**Assessments (Summative)**

*To show evidence of meeting the standard/s, students will successfully complete:*

**Benchmarks:**

- Chapter Tests
- Projects

**Summative Assessments:**

- District assessments

**Differentiated Student Access to Content:  
Teaching and Learning *Resources/Materials***

**Core  
Resources**

**Alternate  
Core Resources  
*IEP/504/At-Risk/ESL***

**ELL  
Core Resources**

**Gifted & Talented  
Core Resources**

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| <p>online albert resource<br/>online achievethecore resource<br/>online learnzillion resource<br/>online khanacademy resource<br/>online desmos resource<br/>online edulastic resource</p>        | <p>Reteaching worksheets<br/>Skill building workbook<br/>Math manipulatives<br/>Leveled practice worksheets</p>  | <p>Dictionary for native language<br/>Video tutorial in native language<br/>Success for English Learners<br/>worksheets<br/>Leveled Strategies for English<br/>Learners<br/>Linguistic Support</p>  | <p>Enrichment worksheets<br/>Art of Problem Solving<br/>Leveled assessments</p>   |
| <p><b>Supplemental Resources</b></p>  |  |   |   |
| <ul style="list-style-type: none"> <li>● Technology: Chromebooks, Graphing Calculators, Smartboards,</li> <li>● Other: Zoom and Google Meets, Schoology, Google Classroom</li> </ul>              |  |   |   |
| <p><b>Differentiated Student Access to Content:<br/>Recommended <i>Strategies &amp; Techniques</i></b></p>  |  |   |   |
| <p><b>Core<br/>Resources</b></p>  | <p><b>Alternate<br/>Core Resources<br/><i>IEP/504/At-Risk/ESL</i></b></p>  | <p><b>ELL<br/>Core Resources</b></p>  | <p><b>Gifted &amp; Talented<br/>Core Resources</b></p>  |
| <p>Deliver instruction utilizing varied learning styles including audio, visual, and tactile/kinesthetic, provide individual instruction as needed, modify assessments and/or rubrics, repeat</p> | <p>Utilize a multi-sensory (VAKT) approach during instruction, provide alternate presentations of skills by varying the method (repetition, simple explanations, additional examples, modeling, etc.), modify test content and/or format, allow students to retake test for additional credit, provide additional times and preferential</p> | <p>Extend time requirements, preferred seating, positive reinforcement, check often for understanding/review, oral/visual directions/prompts when necessary, supplemental materials including use of an online bilingual dictionary, and modified assessment and/or rubric.</p> | <p>Create an enhanced set of introductory activities, integrate active teaching/learning opportunities, incorporate authentic components, propose interest-based extension activities, and connect student to related</p> |

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|  | seating as needed, review, restate and repeat directions, provide study guides, and/or break assignments into segments of shorter tasks. |  |  |
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| New Jersey Legislative Statutes and Administrative Code<br>(place an "X" before each law/statute if/when present within the curriculum map) |   |   |          |   |          |   |   |
|---|---|---|----------|---|----------|---|---|
|   | Amistad Law:<br><i>N.J.S.A. 18A 52:16A-88</i> | Holocaust Law:<br><i>N.J.S.A. 18A:35-28</i> | <b>x</b> | LGBT and Disabilities Law:<br><i>N.J.S.A. 18A:35-4.35</i> | <b>x</b> | Diversity & Inclusion: <i>N.J.S.A. 18A:35-4.36a</i> | Standards in Action:<br><i>Climate Change</i> |