Probability and Statistics - Unit 6 Normal Probability Distribution	20 - 25 Days Established 14-15 Revised 20-21 Revised Nov 2021

Standard – NJSLS: S.ID and S.IC
Normal Probability Distribution

Strand
S-ID: Interpreting Categorical and Quantitative Data
Summarize, represent, and interpret data on a single count or measurement variable
1. *Represent data with plots on the real number line (dot plots, histograms, and box plots).*

S-IC: Making Inferences and Justifying Conclusions
Understand and evaluate random processes underlying statistical experiments.
1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
6. Evaluate reports based on data

Curriculum aligned with: 2009 New Jersey Core Curriculum Content Standards for 21st Century Skills (9.1 A-F)

21st Century Theme: *Global Awareness ☐, Financial, economic, business and entrepreneurial literacy , , Civic literacy ☐, Health literacy ☐ Environmental Literacy ☐,*

21st Century Skills: *Critical Thinking & Problem Solving , Creativity and Innovation ☐, Collaboration, Teamwork and Leadership ☐, Cross-Cultural Understanding and Interpersonal Communications ☐, Communication and Media Fluency ☐, Accountability, Productivity and Ethics ☐*

Interdisciplinary Connection: Math=MA, English=ELA, Science=SCI, Social Studies=SS, Physical Education=PE, Art=ART, Music=MU, Technology=TECH, World Language=WL, Business = BU

Essential Questions	Enduring Understandings	Activities, Investigation, and Student Experiences
1. What are real life examples of normal distributions?	*Students will understand…* ● A continuous random variable that has a	***Task 1:*** A scale is designed so that when items are weighed, the errors in the indicated weights are normally distributed with a mean of 0 g and a standard deviation of 1 g. (If the scale reading is too low, the error is negative. If the scale reading is too high, the error is positive.)

2. How is the Central Limit Theorem used in statistical procedures?	distribution with a graph that is symmetric and bell-shaped is a normal distribution. ● The standard normal distribution has a mean equal to 0 and has a standard deviation equal to 1. ● A continuous random variable has a uniform distribution if its values are spread evenly over the range of possibilities. ● A density curve is a graph of a continuous probability distribution that has a total area under the curve	● If an item is randomly selected and weighed, what is the probability that it has an error between -0.5 g and 0.5 g? ● If 16 items are randomly selected and weighed, what is the probability that the mean of the errors is between -0.5 g and 0.5 g? ● What is the 90th percentile for the errors? ***Answer:*** ● P(z < 0.5) = 0.6915 and P(z < -0.5) = 0.3085 so 0.6915-0.3085 = 0.3830, which is the P(-0.5 < z < 0.5). $z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{0.5 - 0}{\frac{1}{\sqrt{16}}} = 2.00$ ● , P(z < 2.0) = 0.9772 and $z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{-0.5 - 0}{\frac{1}{\sqrt{16}}} = -2.00$ P(z < -2.00) = 0.0228 so 0.9772-0.0228 = .9544, which is the P(-0.5 < z < 0.5) when 16 items are selected. Also according to the Empirical Rule, approximately 95% of the data lies within two standard deviations of the mean which would apply here. ● Since the cumulative area under the standard normal curve for a z score of 1.28 is almost equal to 0.9000, this value corresponds to the 90th percentile. ***Task 2:*** Scores for men on the verbal portion of the SAT-I test are normally distributed with a mean of 509 and a standard deviation of 112 (based on data from the College Board). Randomly selected men are given the Columbia Review Course before taking the SAT test. Assume the course has no effect. ● If 1of the men is randomly selected, find the probability that his score is at least 590.

	that must equal 1. • There is a correspondence between area and probability. • Values can be converted to standard scores, *z* scores, so that the procedures working with all normal distributions are the same as those for the standard normal distribution. • The sampling distribution of a statistic is the distribution of all values of that statistic when all possible samples of the same size are taken from the same population.	• If 16 of the men are randomly selected, find the probability that their mean score is at least 590. • In finding the probability for part (b), why can the central limit theorem be used even though the sample size does not exceed 30? • If the random sample of 16 men does result in a mean score of 590, is there strong evidence to support the claim that the course is effective? Why or why not? ***Answer:*** • $z = \frac{x - \mu}{\sigma} = \frac{590 - 509}{112} = 0.72$ P(z < 0.72) = 0.7642 so the P(z >0.72) = 0.2358, which is 1-0.7642. • $z = \frac{\bar{x} - \mu}{\sigma_{\bar{x}}} = \frac{590 - 509}{\frac{112}{\sqrt{16}}} = 2.89$ P(z < 2.89) = 0.9981 so the P(z>2.89) = 0.0019, which is 1-0.9981. • Because the original population is normally distributed, the sampling distribution of sample means will be normally distributed for any sample size. • Yes, because the probability of 0.0019 shows that it is highly unlikely that by chance, a randomly selected group would get a mean as high as 590. ***Task 3** **Interdisciplinary BU** The amount of regular unleaded gasoline purchased every week at a gas station near UCLA follows the normal distribution with mean 50000 gallons and standard deviation 10000 gallons. The starting supply of gasoline is 74000 gallons, and there is a scheduled weekly delivery of 47000 gallons. Find the probability that, after 11 weeks, the supply of gasoline will be below 20000 gallons. **Answer:**

	• A sample mean is an unbiased estimator of the population mean. • A sample variance is an unbiased estimator of the population variance. • A sample proportion is an unbiased estimator of a population proportion. • The Central Limit Theorem says that when n is large, the sampling distribution of the sample mean is approximately normal. • The standard deviation of the sample means is denoted by σ_x.	The supply will be below 20000 gallons if the total purchased in these 11 weeks is more than 591000 – 20000 = 571000 gallons. Therefore we need to find P(T> 571000) = $P\left(z > \frac{571000-11(50000)0)}{\sqrt{111000000}}\right)$ = P(z > 0.63) = 1 – 0.7357 = 0.2643. **Modifications and/or Accommodations:** • **Special Education:** Utilize a multi-sensory (VAKT) approach during instruction, provide alternate presentations of skills by varying the method (repetition, simple explanations, additional examples, modeling, etc.), modify test content and/or format, allow students to retake test for additional credit, provide additional times and preferential seating as needed, review, restate and repeat directions, provide study guides, and/or break assignments into segments of shorter tasks. • **English Language Learners:** Extend time requirements, preferred seating, positive reinforcement, check often for understanding/review, oral/visual directions/prompts when necessary, supplemental materials including use of online bilingual dictionary, and modified assessment and/or rubric. • **Students at Risk of School Failure:** Deliver instruction utilizing varied learning styles including audio, visual, and tactile/kinesthetic, provide individual instruction as needed, modify assessments and/or rubrics, repeat instructions as needed. • **Gifted Students:** Create an enhanced set of introductory activities, integrate active teaching/learning opportunities, incorporate authentic components, propose interest-based extension activities, and connect student to related talent development opportunities. **Teacher Resources** online achievethecore resource online learnzillion resource online khanacademy resource

	• The mean of the sample means is denoted by μ_x and equals the mean of the population. • A binomial probability distribution can sometimes be approximated by a normal distribution. • For a normal approximation to be used as an approximation for a binomial distribution, the conditions that $np \geq 5$ and $nq \geq 5$ must be met. Also a correction for continuity may be used for more accurate results.	online desmos resource online ixl resource

	• A normal quantile plot can be used to determine if the requirement of a sample comes from a normally distributed population. • If a histogram departs dramatically from a bell shape then the sample was not taken from a normally distributed population. Normality can be rejected if there is more than one outlier present in the data set.	
Content Statements	**Cumulative Progress Indicators**	

Probability and Statistics - Unit 6 Normal Probability Distribution	20 - 25 Days Established 14-15 Revised 20-21 Revised Nov 2021

Students will know… ● How to use the area under a density curve to calculate probability ● How to use the standard normal distribution to calculate the proportion of values in a specified interval ● How to use the standard normal distribution to determine a *z*-score from a percentile ● How to distinguish between a population distribution, sampling distribution and the distribution of sample data ● How to use the Central Limit Theorem to help find probabilities involving a sample mean ● How to calculate the probabilities involving a sample when the population distribution is Normal ● How the shape of the sampling distribution is	● Tests ● Quizzes ● Practice problems for homework ● Workbook pages ● Worksheets	

related to the shape of the population distribution ● How to use normal distributions to approximate a binomial distribution ● How to determine if a sample comes from a normally distributed population		
Desired Results		
● **Area under the curve** ● **Standard deviation** ● **Population and sampling distribution** ● **Central Limit Theorem** **1. Make sense of problems and persevere in solving them.** **2. Reason abstractly and quantitatively.** **3. Construct viable arguments and critique the reasoning of others** **4. Model with mathematics** **5. Use appropriate tools strategically.**		

6. Attend to precision.

7. Look for and make use of structure.

Standards for Mathematical Practices

8. Look for and express regularity in repeated reasoning.

LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*

Stephen Hawking - Despite living with amyotrophic lateral sclerosis, Stephen Hawking is a world-renowned physicist who is credited with groundbreaking discoveries involving quantum theory and general relativity, among others.

The mission is to ensure that every student is able to see themselves in our rich and diverse history.

Social and Emotional Learning: *Competencies*	**Social and Emotional Learning: *Sub-Competencies***
Self-Awareness Social Awareness Self-Management Relationship Skills	• Recognizing the importance of self-confidence in handling daily tasks and challenges.

Responsible Decision-Making	• Demonstrate an awareness of the expectations for social interactions in a variety of ways. • Demonstrate an understanding of the need for mutual respect when viewpoints differ. • Recognize the skills needed to establish and achieve personal and educational goals. • Utilize positive communicatio n and social skills to interact

	effectively with others. ● Develop, implement, and model effective problem solving and critical thinking skills.