Standard – NJSLS: S.IC, S.CP, and S.MD.
Probability

Strand:

S-IC: Making Inferences and Justifying Conclusions

Understand and evaluate random process underlying statistical experiments.

2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

S-CP: Conditional Probability and Rules of Probability

Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of the other events(“or,” “and,” “not”).
2. Understand that two events *A* and *B* are independent if the probability of *A* and *B* occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
3. Understand the conditional probability of *A* given *B* as *P(A* and *B*)/*P(B)*, and interpret independence of *A* and *B* as saying that the conditional probability of *A* given *B* is the same as the probability of *A*, and the conditional probability of *B* given *A* is the same as the probability of *B*.
4. Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. *For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.*
5. Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. *For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.*

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

6. Find the conditional probability of *A* given *B* as the fraction of *B’s* outcomes that also belong to *A*, and interpret the answer in terms of the model.

7. Apply the Addition Rule, *P(A or B) = P(A) + P(B) – P(A and B)*, and interpret the answer in terms of the model.
8. (+) Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
9. (+) Use permutations and combinations to compute probabilities of compound events and solve problems.

S-MD: Using Probability to make decisions

Use probability to evaluate outcomes of decisions.

6. (+) Use probabilities to make fair decisions (e.g., drawing by lots, using a random number generator).
7. (+) Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of the game).

Curriculum aligned with: 2009 New Jersey Core Curriculum Content Standards for 21st Century Skills (9.1 A-F)

21st Century Theme: *Global Awareness ☐, Financial, economic, business and entrepreneurial literacy , ▯, Civic literacy ☐, Health literacy☐ Environmental Literacy☐,*

21st Century Skills: *Critical Thinking & Problem Solving▯, Creativity and Innovation ☐, Collaboration, Teamwork and Leadership ☐, Cross-Cultural Understanding and Interpersonal Communications ☐, Communication and Media Fluency ☐, Accountability, Productivity and Ethics ☐*

Interdisciplinary Connection: Math=MA, English=ELA, Science=SCI, Social Studies=SS, Physical Education=PE, Art=ART, Music=MU, Technology=TECH, World Language=WL, Business = BU

Essential Questions	Enduring Understandings	Activities, Investigation, and Student Experiences
1. What are examples of when probability values are used in every day life? 2. How can we base decisions on chance? 3. How does probability relate to the likelihood of an event happening?	*Students will understand….* ● If the probability of an event is small or less than 0.05, then that event or assumption is unusual. ● Probability values exist	****Task 1:*** **Interdisciplinary SS** Use the following data from the 100 Senators from the 108th Congress of the United States. <table><tr><td></td><td>Republican</td><td>Democrat</td><td>Independent</td></tr><tr><td>Male</td><td>46</td><td>39</td><td>1</td></tr><tr><td>Female</td><td>5</td><td>9</td><td>0</td></tr></table>● If we randomly select one Senator, what is the probability of getting a Republican, given that a male was selected?

	between 0 and 1 inclusive. The probability of an impossible event is 0 and the probability of an event certain to occur is 1. ● The complement of an event is the probability of the event not occurring. ● The law of large numbers says that as trials are repeated, the probability of the event approaches the actual probability. ● A compound event is any event that combines 2 or more simple events. P(A or B) is the probability that both events occur at the same time.	● If we randomly select one Senator, what is the probability of getting a male, given that a Republican was selected? Is this the same as the result found in the question above? ● If we randomly select one Senator, what is the probability of getting a female, given that an Independent was selected? ● If we randomly select one Senator, what is the probability of getting a Democrat or Independent, given that a male was selected? ***Answer:*** ● $P(B \mid A) =$ *P(getting a Republican, given a male)* = $\frac{P(\text{Republican} \,\&\, male)}{P(male)} = \frac{46/100}{86/100} = \frac{46}{86} = 0.535$ ● $P(B \mid A) =$ *P(getting a male, given a Republican)* = $\frac{P(\text{Republican} \,\&\, male)}{P(\text{Republican})} = \frac{46/100}{51/100} = \frac{46}{51} = 0.902$ *;* This result is not the same as 0.535 because it is incorrect to believe that the probability of getting a Republican, given a male was selected is the same as the probability of getting a male, given a Republican was selected. This is known as confusion of the inverse with conditional probability. ****Task 2:*** **Interdisciplinary ELA** Many newspapers carry “Jumble,” a puzzle in which the reader must unscramble letters to form words. For example, the letters TAISER were included in newspapers on the day this exercises was written. ● How many ways can the letters of TAISER are arranged? ● Identify the correct unscrambling, then determine the probability of getting that result by randomly selecting an arrangement of the given letters. ***Answer:***

	• In one trial, to calculate the probability of event A or B use the addition rule but be careful not to double count any outcomes. • If finding the probability that event A occurs on one trial and event B occurs on a second trial, use the multiplication rule but be sure to take into account the fact that even A has already occurred. • The probability of an event and its complement is 1. P(A and B) is the probability of event A occurring in the first trial and event B	• 6! = 720 different ways • Satire; The probability of getting that result would be 1/720. ***Task 3:*** With one method of acceptance sampling, a sample of items is randomly selected without replacement, and the entire batch is rejected if there is at least one defect. The Medtyme Pharmaceutical Company has just manufactured 2500 aspirin tablets, and 2% are defective because they contain too much or too little aspirin. • If 4 of the tablets are selected and tested, what is the probability that the entire batch will be rejected? ***Answer:*** • P(at least one) = 1 – P(none) P(at least one) = 1 – (0.98)^4 = 0.0777 The probability that the entire batch will be rejected is 0.0777. **Modifications and/or Accommodations:** • **Special Education:** Utilize a multi-sensory (VAKT) approach during instruction, provide alternate presentations of skills by varying the method (repetition, simple explanations, additional examples, modeling, etc.), modify test content and/or format, allow students to retake test for additional credit, provide additional times and preferential seating as needed, review, restate and repeat directions, provide study guides, and/or break assignments into segments of shorter tasks. • **English Language Learners:** Extend time requirements, preferred seating, positive reinforcement, check often for understanding/review, oral/visual directions/prompts when necessary, supplemental materials including use of online bilingual dictionary, and modified assessment and/or rubric. • **Students at Risk of School Failure:** Deliver instruction utilizing varied learning styles including audio, visual, and tactile/kinesthetic, provide individual instruction as needed, modify assessments and/or rubrics, repeat instructions as needed. • **Gifted Students:** Create an enhanced set of introductory activities, integrate active teaching/learning opportunities, incorporate authentic components, propose interest-based extension activities, and connect student to related talent development opportunities. **Spot Light On:** *Acknowledge every student's comment or response, even if it's incorrect.*

	occurring in the second trial. ● Conditional probability represents the probability of event B occurring after it is assumed that event A has already occurred. ● The confusion of the inverse is when it is believed P(B|A) = P(A|B). ● The probability of "at least one" is equivalent to one or more. The complement of getting at least one item of a particular type is that you get no items of that type. ● Simulation techniques are often helpful in determining	**Teacher Resources** online achievethecore resource online learnzillion resource online khanacademy resource online desmos resource online ixl resource

	probability values. ● In various probability problems, the total number of outcomes can be found by using the fundamental counting rule, the permutations rule or the combinations rule. ● For a sequence of two events in which the first event can occur *m* ways and the second event can occur *n* ways, the events together can occur a total of $m*n$ ways. ● A collection of *n* different items can be arranged in order $n!$ different ways. ● The number of permutations of *r* items selected	

	from n different available items is $_nP_r = \frac{n!}{(n-r)!}$. ● The number of combinations of r items selected from n different items is $_nC_r = \frac{n!}{(n-r)!r!}$. ● When different orderings of the same items are counted separately, we have a permutation problem, but when different orderings of the same items are not counted separately, we have a combination problem.	

Content Statements	**Cumulative Progress Indicators**	
Students will know… ● How to use the rare event rule for inferential statistics ● What the probability limits are for probability distributions ● How to use the law of large numbers ● How to calculate the actual odds against and for an event ● How to use the payoff odds for a given event ● How and when to use conditional probability ● How to distinguish between mutually exclusive and independent events ● How to work with the multiplication and additional rules for probability ● How to use probabilities with complementary events	● Tests ● Quizzes ● Practice problems for homework ● Workbook pages ● Worksheets	

- How to use permutations and combinations to find the number of outcomes

Desired Results

- **Probability Limits**
- **Law of Large Numbers**
- **Conditional Probability**
- **Mutually exclusive and independent events**
- **Complimentary events**
- **Permutations and Combinations**

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others**
4. **Model with mathematics**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**

Standards for Mathematical Practices

8. **Look for and express regularity in repeated reasoning.**

Probability and Statistics - Unit 4 Probability	Revised August 2023 Revised Nov 2021 20 - 25 Days

LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35* Kate Hutton - "Earthquake Kate." Staff seismologist at the California Institute of Technology. The mission is to ensure that every student is able to see themselves in our rich and diverse history.	
Social and Emotional Learning: *Competencies*	**Social and Emotional Learning: *Sub-Competencies***
Self-Awareness Social Awareness Self-Management Relationship Skills Responsible Decision-Making	• Recognizing the importance of self-confidence in handling daily tasks and challenges. • Demonstrate an awareness of the expectations for social interactions in a variety of ways. • Demonstrate an understanding of the need for mutual respect when viewpoints differ. • Recognize the skills needed to establish and achieve personal and educational goals. • Utilize positive communication and social skills to interact effectively with others. • Develop, implement, and model effective problem solving and critical thinking skills.

New Jersey Legislative Statutes and Administrative Code (place an "X" before each law/statute if/when present within the curriculum map)									
	Amistad Law: *N.J.S.A. 18A 52:16A-88*		Holocaust Law: *N.J.S.A. 18A:35-28*	**x**	LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*	**x**	Diversity & Inclusion: *N.J.S.A. 18A:35-4.36a*		Standards in Action: *Climate Change*