Calculus Unit 4 The Definite Integral	25 to 30 days Established 14-15 Revised 20-21 Revised Nov 2021 Revised August 2023

Standard – NJSLS: A.APR, F.IF, F.TF, and G.MG
The Definite Integral (Chapter 5)

Strand

A-APR: Algebra: Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials

1. ***Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.***

F-IF: Functions: Interpreting Functions

Analyze functions using different representations

7. **Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.**
    a. ***Graph linear and quadratic functions and show intercepts, maxima, and minima.***
    b. ***Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.***
    e. **Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.**

F-TF: Functions: Trigonometric Functions

Prove and apply trigonometric identities

8. ***Prove the Pythagorean identity $sin^2(theta) + cos^2(theta) = 1$ and use it to find sin(theta), cos(theta), or tan(theta) given sin(theta), cos(theta), or tan(theta) and the quadrant of the angle.***

G -MG: Geometry: Modeling with Geometry

Apply geometric concepts in modeling situation.

2. ***Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).***

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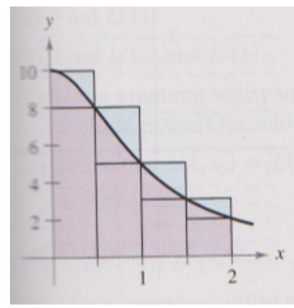
Curriculum aligned with: 2009 New Jersey Core Curriculum Content Standards for 21st Century Skills (9.1 A-F)		
21st Century Theme: *Global Awareness ☐, Financial, economic, business and entrepreneurial literacy ☐, Civic literacy ☐, Health literacy ☐Environmental Literacy ☐*		
21st Century Skills: *Critical Thinking & Problem Solving☐, Creativity and Innovation ☐, Collaboration, Teamwork and Leadership ☐, Cross-Cultural Understanding and Interpersonal Communications ☐, Communication and Media Fluency ☐, Accountability, Productivity and Ethics ☐*		
Interdisciplinary Connection: Math=MA, English=ELA, Science=SCI, Social Studies=SS, Physical Education=PE, Art=ART, Music=MU, Technology=TECH, World Language=WL, Business = BU		
Essential Questions	**Enduring Understandings**	**Activities, Investigation, and Student Experiences**
1. Why is integration more complicated than differentiation? 2. When is each integration technique applicable and how is it applied?	● Just as subtraction is the reverse of addition, division is the reverse of multiplication; integration is harder than differentiation because integrals "undoes" derivatives. Integration is the "inverse" of differentiation. ● A function F is an antiderivative of f on an interval I if F'(x) = f(x) for all x in I. ● Representation of Antiderivatives: If F is an antiderivative of f on an interval I, then G is an antiderivative of f on the interval I if and only if G is of the form G(x) = F(x) + C, for all x in I where C is a constant.	**<u>Task 1:</u>** Find the indefinite integral of the following: a) $\int (2x^2 + x - 1)\,dx$ b) $\int [(x^3 + 1) / x^2]\,dx$ c) $\int (4x - 3\sin x)\,dx$ d) $\int (5\cos x - 2\sec^2 x)\,dx$ e) $\int (\sin^3 x \cos x)\,dx$ f) $\int [x^2 \sqrt{(x^3 + 3)}]\,dx$ g) $\int [x(1 - 3x^2)4]\,dx$ **<u>Answer:</u>** a) $\int (2x^2 + x - 1)\,dx = {}^2/_3\, x^3 + ½\, x^2 - x + C$ b) $\int [(x^3 + 1) / x^2]\,dx = \int [x + ({}^1/_{x^2})]\,dx = ½x^2 - {}^1/_x + C$ c) $\int (4x - 3\sin x)\,dx = 2x^2 + 3\cos x + C$ d) $\int (5\cos x - 2\sec^2 x) = 5\sin x - 2\tan x + C$ e) $\int (\sin^3 x \cos x)\,dx = ¼ \sin^4 x + C$ f) $u = x^3 + 3$, $du = 3x^2\,dx$ $\int [x^2 \sqrt{(x^3 + 3)}]\,dx = {}^1/_3 \int (x^3 + 3)^{½}\, 3x^2\,dx$ $= {}^2/_9\, (x^3 + 3)^{3/2} + C$ g) $u = x^3 + 3$, $du = 3x^2\,dx$

- The general solution of a differential equation is dy=f(x) dx. The operation of finding all solutions of this equation is called antidifferentiation (or indefinite integration) and is denoted by an integral sign, the integrand ∫
- The general solution is denoted by
  $y = \int f(x)\,dx = F(x) + C$ for various integer values of C.
- The expression $\int f(x)\,dx$ is read as the antiderivative of f with respect to x. So the differential dx serves to identify x as the variable of integration. The term indefinite integral is a synonym for antiderivative.
- Basic Integration Rules:
  $\int 0\,dx = C$
  $\int k\,dx = kx + C$
  $\int k f(x)\,dx = k \int f(x)\,dx$
  $\int [f(x) \pm g(x)]\,dx = \int f(x)\,dx \pm \int g(x)\,dx$
  $\int x^n\,dx = \frac{x^{n+1}}{n+1} + C,\ n \neq 1$
  $\int \cos x\,dx = \sin x + C$
  $\int \sin x\,dx = -\cos x + C$
  $\int \sec^2 x\,dx = \tan x + C$
  $\int \sec x \tan x\,dx = \sec x + C$

$\int [x(1 - 3x^2)^4]\,dx$
$= \int (x^3 + 3)^{-1/2} x^2\,dx$
$= \frac{1}{3} \int (x^3 + 3)^{-1/2} 3x^2\,dx$
$= \frac{2}{3} (x^3 + 3)^{1/2} + C$

Task 2:
Use upper and lower sums to approximate the area of the region using the indicated number of subintervals of equal width.

$y = \frac{10}{x^2 + 1}$



Answer:

$y = \frac{10}{x^2+1},\ \Delta x = \frac{1}{2},\ n = 4$

$S(n) = S(4) = \frac{1}{2}\left[\frac{10}{1} + \frac{10}{(1/2)^2+1} + \frac{10}{(1)^2+1} + \frac{10}{(3/2)^2+1}\right]$

≈ 13.0385

$s(n) = s(4) = \frac{1}{2}\left[\frac{10}{(1/2)^2+1} + \frac{10}{1+1} + \frac{10}{(3/2)^2+1} + \frac{10}{2^2+1}\right]$

≈ 9.0385

$9.0385 < \text{Area of Region} < 13.0385$

$\int \csc^2 x\, dx = -\cot x + C$

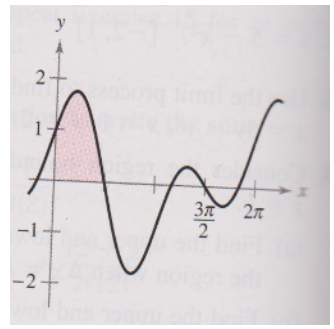
$\int \csc x \cot x\, dx = -\csc x + C$

- To find a particular solution, you need only know the value of y = F(x) for one value of x. This information is called an initial condition.
- The sum of n terms $a_1, a_2, a_3, \dots, a_n$ is written as $\sum a_i = a_1 + a_2 + a_3 + \dots + a_n$ Where I is the index of summation, a_1 is the ith term of the sum, and the upper and lower bounds of summation are n and 1.
- A second classic problem in calculus is finding the area of a plane region that is bounded by the graphs of functions. Consider the region bounded by the graph of the function y = f(x), the x-axis, and the vertical lines x = a and x = b. You can approximate the area of the region with several rectangular regions. As you increase the number of rectangles, the approximation tends to become better and better because the amount of area missed by the rectangle decreases.

Task 3:

Find the area of the region. Use a graphing utility to verify your result.

$\int_0^{\pi/2} [\cos x + \sin(2x)]\, dx$



Answer:

$$\int_0^{\pi/2} (\cos x + \sin(2x))\, dx = \left[\sin x - \frac{1}{2}\cos(2x)\right]_0^{\pi/2}$$
$$= \left(1 + \frac{1}{2}\right) - \left(0 - \frac{1}{2}\right) = 2$$

***Task 4:**
Interdisciplinary SCI

A diesel generator runs continuously, consuming oil at a gradually increasing rate until it must be temporarily shut down to have the filters replaced.

Day	Oil Consumption Rate (liters/hour)
Sun	0.019

- The sum of the areas of the inscribed rectangles is called a lower sum, and the sum of the areas of the circumscribed rectangles is called an upper sum.
- The lower sum is less than or equal to the upper sum. Moreover, the actual area of the region lies between these two sums.
- Let f be continuous and nonnegative on the interval [a,b]. The limits as $n \to \infty$ of both the lower and upper sums exist and are equal to each other. That is
  $\lim_{n\to\infty} s(n)$
  $= \lim_{n\to\infty} \sum f(m_i)\Delta x$
  $= \lim_{n\to\infty} \sum f(M_i)\Delta x$
  $= \lim_{n\to\infty} S(n)$
  Where $\Delta x = (b-a)/n$ and $f(m_i)$ and $f(M_i)$ are the minimum and maximum values of f on the subinterval.
- Let f be continuous and nonnegative on the interval [a,b]. The area of the region bounded by the graph of f, the x-axis, and the vertical lines x = a and x = b is

Mon	**0.020**
Tue	**0.021**
Wed	**0.023**
Thu	**0.025**
Fri	**0.028**
Sat	**0.031**
Sun	**0.035**

a) Give an upper estimate and a lower estimate for the amount of oil consumed by the generator during that week.
b) Use the Trapezoidal Rule to estimate the mount of oil consumed by the generator during that week.

Answer:
a) Upper = 4.392 L; lower = 4.008 L
b) 4.2 L

Modifications and/or Accommodations:
- **Special Education:** Utilize a multi-sensory (VAKT) approach during instruction, provide alternate presentations of skills by varying the method (repetition, simple explanations, additional examples, modeling, etc.), modify test content and/or format, allow students to retake test for additional credit, provide additional times and preferential seating as needed, review, restate and repeat directions, provide study guides, and/or break assignments into segments of shorter tasks.
- **English Language Learners:** Extend time requirements, preferred seating, positive reinforcement, check often for understanding/review, oral/visual directions/prompts when necessary, supplemental materials including use of online bilingual dictionary, and modified assessment and/or rubric.
- **Students at Risk of School Failure:** Deliver instruction utilizing varied

	Area = $\lim_{n\to\infty} \sum f(c_i)\Delta x$ Where $\Delta x = (b-a)/n$. ● Let f be defined on the closed interval [a,b], and let Δ be a partition of [a,b] given by $A = x_0 < x_1 < x_2 < \ldots < x_{n-1} < x_n = b$ Where Δx_1 is the width of the ith subinterval. If c_1 is any point in the ith subinterval, then the sum $\sum f(c_1)\, \Delta x_1$ is called the Riemman Sum of f for the partition Δ. ● If f is defined on the closed interval [a,b] and the limit $\text{Lim}_{\lVert\Delta\rVert\to 0} \sum f(c_1)\, \Delta x_i$ exists, then f is integrable on [a,b] and the limit is denoted by $\lim_{\lVert\Delta\rVert\to 0} \sum f(c_1)\, \Delta x_i = \int_a^b f(x)\, dx$. The limit is called the definite integral of f from a to b. The number a is the lower limit of integration, and the number b is the upper limit of integration. ● If a function f is continuous on the closed interval [a,b], then f is integrable on [a,b]. ● If f is defined at x = a, then we define $\int_a^a f(x)\, dx = 0$	learning styles including audio, visual, and tactile/kinesthetic, provide individual instruction as needed, modify assessments and/or rubrics, repeat instructions as needed. ● **Gifted Students:** Create an enhanced set of introductory activities, integrate active teaching/learning opportunities, incorporate authentic components, propose interest-based extension activities, and connect student to related talent development opportunities **Spot Light On:** *Acknowledge every student's comment or response, even if it's incorrect.* **Teacher Resources** online achievethecore resource online learnzillion resource online khanacademy resource online desmos resource online ixl resource

	• If f is integrable on [a, b], then we define $\int_b^a f(x)\,dx = -\int_a^b f(x)\,dx$. • The Additive Interval Property: If f is integrable on the three closed intervals determined by a, b, and c, then $\int_a^b f(x)\,dx = \int_a^c f(x)\,dx + \int_c^b f(x)dx$. • Properties of Indefinite Integrals: If f and g are integrable on [a,b] and k is a constant, then the functions of kf and f± are integrable on [a,b], and $\int_a^b kf(x)\,dx = k\int_a^b f(x)\,dx$ $\int_a b\,[f(x) \pm g(x)]\,dx = \int_a b\,f(x)\,dx \pm \int_a b\,g(x)\,dx$. • The Fundamental Theorem of Calculus: If a function f is continuous on the closed interval [a,b] and F is an antiderivative of f on the interval [a,b], then $\int_a^b f(x)\,dx = F(b) - F(a)$. • The Mean Value Theorem for Integrals: If f is continuous on the closed interval [a,b], then there exists a number c in the closed interval [a,b] such that $\int_a^b f(x)\,dx = f(c)(b - a)$.	

	● The Average Value of a Function on an Interval: If f is integrable on the closed interval [a,b], then the average value of f on the interval is (1/(b-a))$\int^{ab}$ f(x) dx. ● The Second Fundamental Theorem of Calculus: If f is continuous on an open interval I containing a, then, for every x in the interval, d/dx [$\int^{ax}$ f(t) dt] = f(x). ● The techniques for integrating composite functions are pattern recognition and change of variables. Both techniques involve a u-subsitution. ● Antidifferentiation of a Composite Function: Let g be a function whose range is an interval I, and let f be a function that is continuous on I. If g is differentiable on its domain and F is an antiderivative of f on I,then $\int$ f(g(x))g'(x) dx = F(g(x)) + C. If u = g(x), then du = g'(x) dx and $\int$f(u) du = F(u) + C.	

- The General Power Rule for Integration: If g is a differentiable function of x, then
  $\int [g(x)]^n g'(x)\,dx = \frac{[g(x)]^{n+1}}{n+1} + C, n \neq 1$
  Equivalently, if u = g(x) then
  $\int u^n\,du = \frac{u^{n+1}}{n+1} + C, n \neq -1$
- Change of Variables for Definite Integrals: If the function u = g(x) has a continuous derivative on the closed interval [a,b] and f is continuous on the range of g, then
  $\int_a^b f(g(x))\,g'(x)dx = \int_{g(a)}^{g(b)} f(u)\,du.$
- Integration of Even and Odd Functions: Let f be integrable on the closed interval [-a, a].
  If f is an even function, then $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)\,dx.$
  If f is an odd function, then
  $\int_{-a}^{a} f(x)\,dx = 0$

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Content Statements	**Cumulative Progress Indicators**	
Students will know… ● How to write the general solution of a differential equation. ● How to apply basic integration rules. ● How to solve a vertical motion problem ● How to apply the summation formula ● How to find the area of a plane region ● How to find the upper and lower sums for a region ● How to find area by the limit definition ● How to evaluate a definite integral as a limit ● How to find the areas of common geometric figures ● How to apply properties of definite integrals ● How to apply the Fundamental Theorem of Calculus to find area. ● How to apply the Mean Value Theorem for integrals.	● Tests ● Quizzes ● Practice problems for homework ● Workbook pages ● Worksheets	

● How to find the average value of a function ● How to apply the Second Fundamental Theorem of Calculus ● How to use substitution and integrate ● How to make a change of variables ● How to use the General Power Rule for Integration ● How to integrate even and odd functions ● How to approximate with the Trapezoidal Rule ● How to approximate error in the Trapezoidal Rule.		
Desired Results		
● **Estimating with finite sums** ● **Definite integrals** ● **Definite integrals and antiderivatives** ● **Fundamental theorem of Calculus** ● **Trapezoidal Rule**		

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Standards for Mathematical Practices	

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1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others**
4. **Model with mathematics**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

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LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*

Kate Hutton - “Earthquake Kate.” Staff seismologist at the California Institute of Technology.

The mission is to ensure that every student is able to see themselves in our rich and diverse history.

Social and Emotional Learning: *Competencies*	**Social and Emotional Learning: *Sub-Competencies***
Self-Awareness Social Awareness Self-Management Relationship Skills Responsible Decision-Making	• Recognizing the importance of self-confidence in handling daily tasks and challenges. • Demonstrate an awareness of the expectations for social interactions in a variety of ways. • Demonstrate an understanding of the need for mutual respect when viewpoints differ. • Recognize the skills needed to establish and achieve personal and educational goals. • Utilize positive communication and social skills to interact effectively with others. • Develop, implement, and model effective problem solving and critical thinking skills.

New Jersey Legislative Statutes and Administrative Code (place an “X” before each law/statute if/when present within the curriculum map)									
	Amistad Law: *N.J.S.A. 18A 52:16A-88*		Holocaust Law: *N.J.S.A. 18A:35-28*	**x**	LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*	**x**	Diversity & Inclusion: *N.J.S.A. 18A:35-4.36a*		Standards in Action: *Climate Change*