Calculus Unit 2 - Derivatives	25 to 30 days Established 14-15 Revised 20-21 Revised Nov 2021 Revised August 2023

Standard – NJSLS: F.IF, G.C, and G.GMD
Derivatives (Chapter 3)

Strand
F-IF: Functions: Interpreting Functions
Interpret functions that arise in applications in terms of the context.

4. **For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★**
5. **Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★**
6. **Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★**

Aanalyze functions using different representations.

7. **Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★**
    a. **Graph linear and quadratic functions and show intercepts, maxima, and minima.**

G-C: Geometry: Circles
Understand and apply theorems about circles

4. **(+) Construct a tangent line from a point outside a given circle to the circle.**

G-GMD: Geometry: Geometric Measurement & Dimension
Explain volume formulas and use them to solve problems

3. ***Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.***

Curriculum aligned with: 2009 New Jersey Core Curriculum Content Standards for 21st Century Skills (9.1 A-F)

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21st Century Theme: *Global Awareness ☐, Financial, economic, business and entrepreneurial literacy ☐, Civic literacy ☐, Health literacy ☐Environmental Literacy ☐,*

21st Century Skills: *Critical Thinking & Problem Solving☐, Creativity and Innovation ☐, Collaboration, Teamwork and Leadership ☐, Cross-Cultural Understanding and Interpersonal Communications ☐, Communication and Media Fluency ☐, Accountability, Productivity and Ethics ☐*

Interdisciplinary Connection: Math=MA, English=ELA, Science=SCI, Social Studies=SS, Physical Education=PE, Art=ART, Music=MU, Technology=TECH, World Language=WL, Business = BU

Essential Questions	Enduring Understandings	Activities, Investigation, and Student Experiences
1. How does the derivative represent an instantaneous rate of change? 2. How do you determine that a function is continuous and/or differentiable? 3. What are some ways we can visualize the graph of derivatives?	• Calculus grew out one of four major problems – determining the slope of a tangent line to a curve at a point. • The limit definition is used to define the derivative of a function • There is a relationship between differentiability and continuity • There are basic differentiation rules such as the Constant Rule, Power Rule, Constant Multiple Rule, Sum and Difference Rule, Product Rule, Quotient Rule, Chain Rule, and General Power Rule. • Calculus can be used to find the derivative of any function including trigonometric functions. • Derivatives can be used to find rate of change.	**Task 1:** Find the derivative of the following function a) $f(x) = x^3 - 3x^2$ b) $h(x) = \sqrt{x} \sin x$ c) $y = \frac{x^2}{\cos x}$ d) $f(s) = (s^2 - 1)^{5/2}(s^3 + 5)$ **Answer:** a) $f(x) = x^3 - 3x^2$ $f'(x) = 3x^2 - 6x$ $= 3x(x-2)$ b) $h(x) = \sqrt{x} \sin x = x^{1/2} \sin x$ $h'(x) = \frac{1}{2\sqrt{x}} \sin x + \sqrt{x} \cos x$ c)

	• There are higher-order derivatives of a function (i.e. second derivative, third derivative, etc). • Implicit differentiation can be used to find the derivative of a function. • Rates can be used to solve real-life problem.	$y = \frac{x^2}{\cos x}$ $y' = \frac{\cos x(2x) - x^2(-\sin x)}{\cos^2 x} = \frac{2x\cos x + x^2 \sin x}{\cos^2 x}$ d) $f(s) = (s^2 - 1)^{5/2}(s^3 + 5)$ $f'(s) = (s^2 - 1)^{5/2}(3s^2) + (s^3 + 5)(5/2)(s^2 - 1)^{3/2}(2s)$ $= s(s^2 - 1)^{3/2}[3s(s^2 - 1) + 5(s^3 + 5)]$ $= s(s^2 - 1)^{3/2}(8s^3 - 3s + 25)$
Content Statements	**Cumulative Progress Indicators**	
Students will know… • How to define derivative using limits • How to find the slope of the tangent line to a curve at a point • How to find the derivative of the function by using the definition of the derivative and basic differentiation rules such as the Constant Rule, Power Rule, Constant Multiple Rule, Sum and Difference Rule, Product Rule, Quotient Rule, Chain Rule, and General Power Rule.	• Tests • Quizzes • Practice problems for homework • Workbook pages • Worksheets	***Task 2**: Interdisciplinary: SCI The speed of a car in miles per hour and the stopping distance in feet are recorded in the table

Speed, x	20	30	40	50	60
Stopping Distance, y	25	55	105	188	300

a) Use the regression capabilities of a graphing utility to find a quadratic model for the data.
b) Use a graphing utility to plot the data and graph the model.
c) Use a graphing utility to graph *dy/dx*.
d) Use the model to approximate the stopping distance at a speed of 65 miles per hour

- How to describe the relationship between differentiability and continuity
- How to use derivatives to find rates of change.
- How to find the higher-order derivative of a function (i.e. second derivative, third derivative)
- How to simplify the derivative of a function using algebra.
- How to find the derivative of a trigonometric function using the Chain Rule.
- How to use implicit differentiation to find the derivative of a function
- How to find related rates and use them to solve real life problems

Desired Results

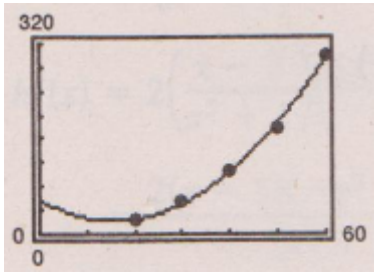
- **Derivative of a function**
- **Differentiability**
- **Rules for differentiation**
- **Velocity and other rates of change**
- **Derivatives of trigonometric functions**
- **Chain rule**
- **Implicit Differentiation**

e) Use the graphs in parts (b) and (c) to explain the change in stopping distance as the speed increases

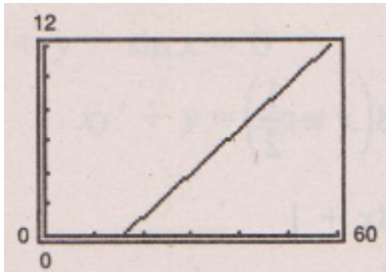
Answer:

a) $y = 0.14x^2 - 4.43x + 58.4$

b)



c)



d) If x = 65, y ≈ 362 feet

e) As the speed increases, the stopping distance increases at an increasing rate.

***Task 3:**
Interdisciplinary: SCI

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- **Derivatives of inverse trigonometric functions**
- **Derivatives of exponential and logarithmic functions**

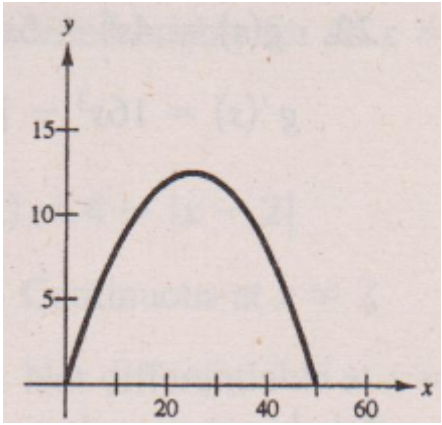
Standards for Mathematical Practices

A ball thrown follows a path described by $y = x - 0.02x^2$.

a) Sketch a graph of the path.
b) Find the total horizontal distance the ball is thrown.
c) At what x – value does the ball reach its maximum height? (Use the symmetry of the path.)
d) Find an equation that gives the instantaneous rate of change of the height of the ball with respect to the horizontal change. Evaluate the equation at x =0, 10, 25, 30, and 50.
e) What is the instantaneous rate of change of the height when the ball reaches its maximum height?

Answer:

a)



1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others**
4. **Model with mathematics**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

b) $0 = x - 0.02x^2$

$0 = x\left(1 - \frac{x}{50}\right)$ implies $x = 50$

c) Ball reaches maximum height when x = 25.

d) $y = x - 0.02x^2$

$y' = 1 - 0.04x$

$y'(0) = 1$

$y'(10) = 0.6$

$y'(25) = 0$

$y'(30) = -0.2$

$y'(50) = -1$

e) $y'(25) = 0$

Modifications and/or Accommodations:

- **Special Education:** Utilize a multi-sensory (VAKT) approach during instruction, provide alternate presentations of skills by varying the method (repetition, simple explanations, additional examples, modeling, etc.), modify test content and/or format, allow students to retake test for additional credit, provide additional times and preferential seating as needed, review, restate and repeat directions, provide study guides, and/or break assignments into segments of shorter tasks.
- **English Language Learners:** Extend time requirements, preferred seating, positive reinforcement, check often for understanding/review, oral/visual directions/prompts when necessary, supplemental materials including use of online bilingual dictionary, and modified assessment and/or rubric.
- **Students at Risk of School Failure:** Deliver instruction utilizing varied learning styles including audio, visual, and tactile/kinesthetic, provide individual instruction as needed, modify assessments and/or rubrics, repeat

instructions as needed.

- **Gifted Students:** Create an enhanced set of introductory activities, integrate active teaching/learning opportunities, incorporate authentic components, propose interest-based extension activities, and connect student to related talent development opportunities

Spot Light On: *Use random response strategies.*

Teacher Resources

online achievethecore resource

online learnzillion resource

online khanacademy resource

online desmos resource

online ixl resource

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LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*

- Alan Turning: English mathematician, computer scientist, logician, cryptanalyst, philosopher, and theoretical biologist. Turning was highly influential in the development of theoretical computer science.

The mission is to ensure that every student is able to see themselves in our rich and diverse history.

Social and Emotional Learning: ***Competencies***	**Social and Emotional Learning:** ***Sub-Competencies***
Self-Awareness Social Awareness Self-Management Relationship Skills Responsible Decision-Making	• Recognizing the importance of self-confidence in handling daily tasks and challenges. • Demonstrate an awareness of the expectations for social interactions in a variety of ways. • Demonstrate an understanding of the need for mutual respect when viewpoints differ. • Recognize the skills needed to establish and achieve personal and educational goals. • Utilize positive communication and social skills to interact effectively with others. • Develop, implement, and model effective problem solving and critical thinking skills.

New Jersey Legislative Statutes and Administrative Code (place an "X" before each law/statute if/when present within the curriculum map)									
	Amistad Law: *N.J.S.A. 18A 52:16A-88*		Holocaust Law: *N.J.S.A. 18A:35-28*	x	LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*	x	Diversity & Inclusion: *N.J.S.A. 18A:35-4.36a*		Standards in Action: *Climate Change*