Calculus Unit 1 Limits and Continuity	25 to 30 days Established 14-15 Revised 20-21 Revised Nov 2021 Revised August 2023

Standard – NJSLS: A.APR, F.IF, F.BF, and F.TF
Limits and Continuity (Chapter 2)

Strand

A-APR: Algebra: Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials

1. ***Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.***

F-IF: Functions: Interpreting Functions

Interpret functions that arise in applications in terms of the context

4. **For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★**
5. **Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.**

Analyze functions using different representations

7. **Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.**
   - **a.** ***Graph linear and quadratic functions and show intercepts, maxima, and minima.***
   - **b.** ***Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.***
   - **e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.**
9. **Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.**

F-BF: Functions: Building Functions

Build a function that models a relationship between two quantities

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1. Write a function that describes a relationship between two quantities.

c. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

F-TF: Functions: Trigonometric Functions

Model periodic phenomena with trigonometric functions

5. *Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*

Prove and apply trigonometric identities

8. *Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(theta), cos(theta), or tan(theta) given sin(theta), cos(theta), or tan(theta) and the quadrant of the angle.*

Curriculum aligned with: 2009 New Jersey Core Curriculum Content Standards for 21st Century Skills (9.1 A-F)

21st Century Theme: *Global Awareness ☐, Financial, economic, business and entrepreneurial literacy☐, Civic literacy ☐, Health literacy ☐Environmental Literacy ☐,*

21st Century Skills: *Critical Thinking & Problem Solving☐, Creativity and Innovation ☐, Collaboration, Teamwork and Leadership ☐, Cross-Cultural Understanding and Interpersonal Communications ☐, Communication and Media Fluency ☐, Accountability, Productivity and Ethics ☐*

Interdisciplinary Connection: Math=MA, English=ELA, Science=SCI, Social Studies=SS, Physical Education=PE, Art=ART, Music=MU, Technology=TECH, World Language=WL, Business = BU

Essential Questions	Enduring Understandings	Activities, Investigation, and Student Experiences

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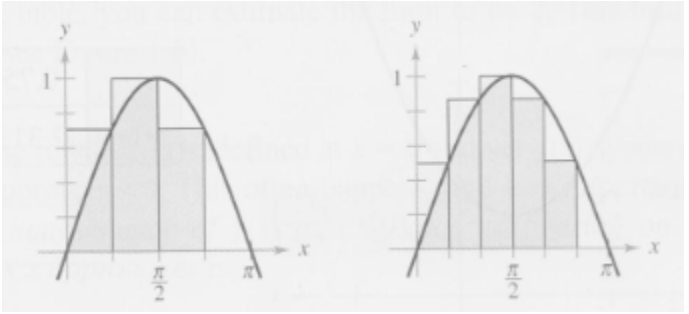
1. How do we use calculus to model and solve real-life problems? 2. How can we evaluate limits of functions analytically, graphically, and numerically?	• The tangent line and area problem are basic to calculus. • A limit can be estimated using a numerical or graphical approach. • There are different ways that a limit can fail to exist. • You can evaluate limits using properties and techniques such as dividing out and rationalizing. • Calculus is used to model a variety of real-world situations.	**Task 1:** a) Use the rectangles in each graph to approximate the area of the region bounded by y = sin x, y = 0, x = 0, and x = π  b) Describe how you could continue this process to obtain a more accurate approximation of the area. **Answer:** **a)** For the figure on the left, each rectangle has width π/4 Area ≈ π/4 (sin π/4 + sin π/2 + sin 3 π/4 + sin π) ≈ π/4 (√2/2 + 1 + √2/2) ≈ $\frac{\sqrt{2}+1}{4}$ *(π) ≈ 1.8961 For the figure on the right, each rectangle has width π/6 Area ≈ π/6 (sin π/6 + sin π/3 + sin π/2 + sin 5π/6 + sin π) ≈ π/6 (½ + √3/2 + 1 + √3/2 + ½) ≈ $\frac{\sqrt{3}+2}{6}$ * (π) ≈ 1.9541 b) Answers may vary. You could obtain a more accurate approximation by using more rectangles.

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Content Statements	**Cumulative Progress Indicators**
Students will know… ● How to find the area of the shaded region ● How to find the slope of a secant line ● How to find limits graphically and numerically ● How to use properties of limits ● How to evaluate limits using dividing out and rationalizing technique ● How to discuss continuity of each function ● How to determine infinite limits from a graph ● How to find vertical asymptotes of the graph of the function ● How to determine limits at infinity	● Tests ● Quizzes ● Practice problems for homework ● Workbook pages ● Worksheets

Desired Results

- **Rates of Change and limits**
- **Limits involving infinity**
- **Continuity**
- **Rates of change and Tangent Lines**

Task 2:

Let P(3, 4) be a point on the circle $x^2 + y^2 = 25$

a) What is the slope of the line joining P and O (0,0)?
b) Find the equation of the tangent line to the circle at P.
c) Let Q(x, y) be another point on the circle in the first quadrant. Find the slope, m_x, of the line joining P and Q in terms of x.
d) Calculate the $\lim_{x \to 3} m_x$. How does this number relate to the answer in part (b)?

Answer:

Standards for Mathematical Practices

(a) Slope $= \frac{4-0}{3-0} = \frac{4}{3}$

(b) Slope $= -\frac{3}{4}$ Tangent line: $y - 4 = -\frac{3}{4}(x - 3)$

$$y = -\frac{3}{4}x + \frac{25}{4}$$

(c) Let $Q = (x, y) = \left(x, \sqrt{25 - x^2}\right)$

$$m_x = \frac{\sqrt{25 - x^2} - 4}{x - 3}$$

(d) $$\lim_{x \to 3} m_x = \lim_{x \to 3} \frac{\sqrt{25 - x^2} - 4}{x - 3} \cdot \frac{\sqrt{25 - x^2} + 4}{\sqrt{25 - x^2} + 4}$$

$$= \lim_{x \to 3} \frac{25 - x^2 - 16}{(x - 3)\left(\sqrt{25 - x^2} + 4\right)}$$

$$= \lim_{x \to 3} \frac{(3 - x)(3 + x)}{(x - 3)\left(\sqrt{25 - x^2} + 4\right)}$$

$$= \lim_{x \to 3} \frac{-(3 + x)}{\sqrt{25 - x^2} + 4} = \frac{-6}{4 + 4} = -\frac{3}{4}$$

This is the slope of the tangent line at P.

Task 3:

Sketch the graph of the function

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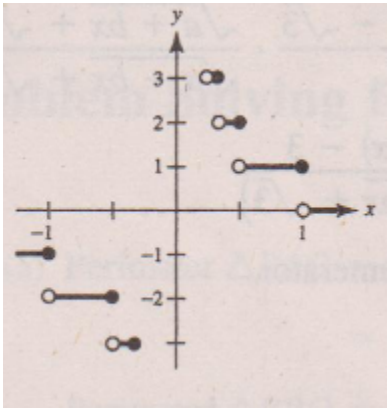
1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others**
4. **Model with mathematics**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

$$f(x) = \left[\left[\frac{1}{x}\right]\right]$$

a) Evaluate f(1/4)
b) Evaluate f(3)
c) Evaluate f(1).
d) Evaluate the limits: $\lim_{x\to 1^-} f(x)$, $\lim_{x\to 1^+} f(x)$, $\lim_{x\to 0^-} f(x)$, and $\lim_{x\to 0+} f(x)$.

Discuss the continuity of the function

Answer:



a) $f(1/4) = [[4]] = 4$
b) $f(3) = [[1/3]] = 0$
c) $f(1) = [[1]] = 1$
d) $\lim_{x\to 1^-} f(x) = 1$
$\lim_{x\to 1^+} f(x) = 0$
$\lim_{x\to 0^-} f(x) = -\infty$

$\lim_{x\to 0^+} f(x) = \infty$

e) f is continuous for all real numbers except x = 0 , ± 1, $\pm 1/2$, $\pm 1/3$, ...

***Task 4:**
Interdisciplinary SCI

A water balloon dropped from a window high above the ground falls $y = 4.9t^2$m in t sec. Find the balloon's

a) average speed during the first 3 sec of fall
b) speed at the instant t = 3.

Answer:

a) 14.7 m/sec
b) 29.4 m/sec

****Please modify your activities based on the needs of your students.*

Modifications and/or Accommodations:

- **Special Education:** Utilize a multi-sensory (VAKT) approach during instruction, provide alternate presentations of skills by varying the method (repetition, simple explanations, additional examples, modeling, etc.), modify test content and/or format, allow students to retake test for additional credit, provide additional times and preferential seating as needed, review, restate and repeat directions, provide study guides, and/or break assignments into segments of shorter tasks.
- **English Language Learners:** Extend time requirements, preferred seating, positive reinforcement, check often for understanding/review, oral/visual directions/prompts when necessary, supplemental materials including use of online bilingual dictionary, and modified assessment and/or rubric.
- **Students at Risk of School Failure:** Deliver instruction utilizing varied learning styles including audio, visual, and tactile/kinesthetic, provide

individual instruction as needed, modify assessments and/or rubrics, repeat instructions as needed.

- **Gifted Students:** Create an enhanced set of introductory activities, integrate active teaching/learning opportunities, incorporate authentic components, propose interest-based extension activities, and connect student to related talent development opportunities

Spot Light On: *Seek multiple perspectives and different answers to questions.*

Teacher Resources

online achievethecore resource

online learnzillion resource

online khanacademy resource

online desmos resource

online ixl resource

LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*

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Stephen Hawking - Despite living with amyotrophic lateral sclerosis, Stephen Hawking is a world-renowned physicist who is credited with groundbreaking discoveries involving quantum theory and general relativity, among others.

The mission is to ensure that every student is able to see themselves in our rich and diverse history.

Social and Emotional Learning: ***Competencies***	**Social and Emotional Learning:** ***Sub-Competencies***
Self-Awareness Social Awareness Self-Management Relationship Skills Responsible Decision-Making	• Recognizing the importance of self-confidence in handling daily tasks and challenges. • Demonstrate an awareness of the expectations for social interactions in a variety of ways. • Demonstrate an understanding of the need for mutual respect when viewpoints differ. • Recognize the skills needed to establish and achieve personal and educational goals. • Utilize positive communication and social skills to interact effectively with others. • Develop, implement, and model effective problem solving and critical thinking skills.

New Jersey Legislative Statutes and Administrative Code (place an "X" before each law/statute if/when present within the curriculum map)								
	Amistad Law: *N.J.S.A. 18A 52:16A-88*		Holocaust Law: *N.J.S.A. 18A:35-28*	x	LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*	x	Diversity & Inclusion: *N.J.S.A. 18A:35-4.36a*		Standards in Action: *Climate Change*