Calculus Unit 3 Applications of Derivatives	25 to 30 days Established 14-15 Revised 20-21 Revised Nov 2021 Revised August 2023

Standard – NJSLS: A.APR, A.REI, F.IF, F.TF, and G.GMD
Applications of Derivatives (Chapter 4)

Strand

A-APR: Algebra: Arithmetic with Polynomials and Rational Expressions

Perform arithmetic operations on polynomials

1. **Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.**

Understand the relationship between zeros and factors of polynomials.

3. **Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial**

A-REI: Algebra: Reasoning with Equations & Inequalities

Represent and solve equations and inequalities graphically

11. **Explain why the x-coordinates of the points where the graphs of the equations y = f(x) andy = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★**

F-IF: Functions: Interpreting Functions

Interpret functions that arise in applications in terms of the context.

4. **For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★**
5. **Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★**

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6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Aanalyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★

- **a. Graph linear and quadratic functions and show intercepts, maxima, and minima.**
- **b. *Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.***
- **e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. .**

F-TF: Functions: Trigonometric Functions

Prove and apply trigonometric identities

8. *Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(theta), cos(theta), or tan(theta) given sin(theta), cos(theta), or tan(theta) and the quadrant of the angle.*

G-GMD: Geometry: Geometric Measurement & Dimension

Explain volume formulas and use them to solve problems

3. *Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.*

Curriculum aligned with: 2009 New Jersey Core Curriculum Content Standards for 21st Century Skills (9.1 A-F)

21st Century Theme: *Global Awareness ☐, Financial, economic, business and entrepreneurial literac☐y, Civic literacy ☐, Health literacy ☐Environmental Literacy ☐*

21st Century Skills: *Critical Thinking & Problem Solving☐, Creativity and Innovation ☐, Collaboration, Teamwork and Leadership ☐, Cross-Cultural Understanding and Interpersonal Communications ☐, Communication and Media Fluency ☐, Accountability, Productivity and Ethics ☐*

Interdisciplinary Connection: Math=MA, English=ELA, Science=SCI, Social Studies=SS, Physical Education=PE, Art=ART, Music=MU, Technology=TECH, World Language=WL, Business = BU

Essential Questions	Enduring Understandings	Activities, Investigation, and Student Experiences

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1. How relationships in "life" similar to the relationships between are functions, 1st and 2nd derivatives?
2. Why is it important to be able to determine the characteristics of a function?
3. When is a function not differentiable?

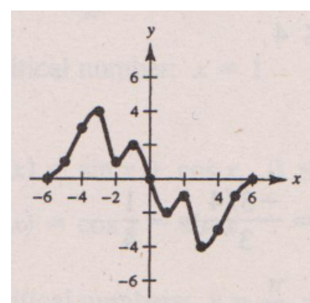
- A functions' first and second derivatives can be compared to your family's first and second generation. Without the first generation (i.e. parents) there wouldn't be a second generation (i.e offspring). Also, from the second generation, you can determine the previous generations after that.
- The extrema of a function f are values on an interval where f is either a maximum or a minimum.
- If there is an open interval containing c on which f(c) is a maximum, then f(c) is called a relative maximum of f, or f has a relative maximum at (c, f(c)).
- If there is an open interval containing c on which f(c) is a minimum, then f(c) is called a relative minimum of f or f has a relative minimum at (c, f(c)).
- The Extreme Value Theorem: If f is continuous on a closed interval [a,b] then f has both a minimum and a maximum on the interval.

Task 1:
Consider the odd function f that is continuous and differentiable and has the functional values shown in the table:

x	-5	-4	-1	0	2	3	6
$f(x)$	1	3	2	0	-1	-4	0

a) Determine $f(4)$.
b) Determine $f(-3)$.
c) Plot the points and make a possible sketch of the graph of f on the interval [-6, 6]. What is the smallest number of critical points in the interval? Explain.
d) Does there exist at least one real number c in the interval (-6, 6) where $f'(c) = -1$? Explain.
e) Is it possible that $\lim_{x\to 0} f(x)$ does not exist? Explain.
f) Is it necessary that $f'(x)$ exists at x = 2? Explain.

Answer:
a) $f(4) = -f(-4) = -3$
b) $f(-3) = -f(3) = -(-4) = 4$
c)



- Let f be defined at c. If f'(c)=0 or if f is not differentiable at c, then c is a critical number of f.
- If f has a relative minimum or relative maximum at x = c, then c is a critical number of f.
- To find the extrema of a continuous function f on a closed interval [a,b], first you must find the critical numbers of f in (a,b), then evaluate f at each critical number in (a,b), evaluate f at each endpoint of [a, b], and finally the least of these values is the minimum and likewise, the greatest of these values is the maximum.
- Rolle's Theorem: Let f be continuous on the closed interval [a,b] and differentiable on the open interval (a,b). If f(a) = f(b), then there is at least one number c in (a,b) such that f'(c) = 0.
- The Mean Value Theorem: If f is continuous on the closed interval [a, b] and differentiable on the open interval (a,b), then there exists a number c in (a,b) such that $f'(c) = \frac{f(b) - f(a)}{b - a}$

d) Yes. Since $f(-2) = -f(2) = -(-1) = 1$ and $f(1) = -f(-1) = -2$, the Mean Value says that there exists at least one value c in (-2, 1) such that
$$f'(c) = \frac{f(1) - f(-2)}{1 - (-2)} = \frac{-2 - 1}{1 + 2} = -1$$

e) No, $\lim_{x \to 0} f(x)$ exists because f is continuous at (0, 0).

f) Yes, f is differentiable at x = 2.

Task 2:

Use the following tests to find any relative extrema of the function.

a) First Derivative Test: $h(t) = \frac{1}{4}t^4 - 8t$

b) First Derivative Test: $g(x) = \frac{3}{2}\sin\left(\frac{\pi x}{2} - 1\right)$, [0, 4]

c) Second Derivative Test: $g(x) = 2x^2\,(1 - x^2)$

d) Second Derivative Test: $h(t) = t - 4\sqrt{(t + 1)}$

Answer:

1. $h(t) = \frac{1}{4}t^4 - 8t$
   $h'(t) = t^3 - 8 = 0$ when t = 2.
   Relative minimum: (2, -12)

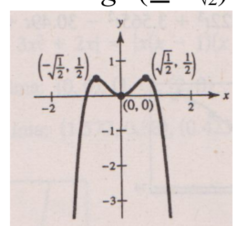
Test Interval:	$-\infty < t < 2$	$2 < t < \infty$
Sign of $h'(t)$:	$h'(t) < 0$	$h'(t) > 0$
Conclusion:	Decreasing	Increasing

2. $g(x) = \frac{3}{2}\sin\left(\frac{\pi x}{2} - 1\right)$, [0, 4]
   $g'(x) = \frac{3}{2}\left(\frac{\pi}{2}\right)\cos\left(\frac{\pi x}{2} - 1\right) = 0$
   when $x = 1 + \frac{2}{\pi}, 3 + \frac{2}{\pi}$
   Relative maximum: $\left(1 + \frac{2}{\pi}, \frac{3}{2}\right)$
   Relative minimum: $\left(3 + \frac{2}{\pi}, \frac{-3}{2}\right)$

- Analyzing the characteristics of a graph gives us a visual memory of a function by using algebra and geometry combined.
- A function f is increasing on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) < f(x_2)$.
- A function f is decreasing on an interval if for any two numbers x_1 and x_2 in the interval, $x_1 < x_2$ implies $f(x_1) > f(x_2)$
- Test for Increasing and Decreasing Functions: Let f be a function that is continuous on the closed interval [a,b] and differentiable on the open interval (a,b). If f'(x) > 0 for all x in (a,b), then f is increasing on [a,b]. If f'(x) < 0 for all x in (a,b), then f is decreasing on [a,b]. If f'(x)=0 for all x in (a,b), then f is constant on [a,b]
- The First Derivative Test: Let c be a critical number of a function f that is continuous on an open interval I containing c. IF f is differentiable on the interval, except possibly at c, then f has a relative minimum at (c, f(c)). If f'(x) changes from

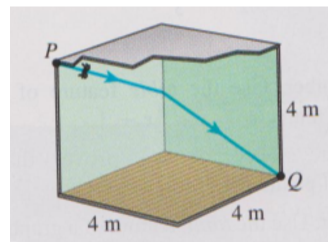
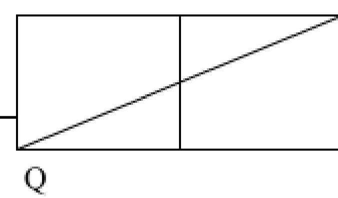
Test Interval	$0 < x < 1 + {}^{2}/_{\pi}$	$1 + {}^{2}/_{\pi} < x < 3 + {}^{2}/_{\pi}$	$3 + {}^{2}/_{\pi} < x < 4$
Sign of $g'(x)$	$g'(x) > 0$	$g'(x) < 0$	$g'(x) > 0$
Conclusion:	Increasing	Decreasing	Increasing

3. $g(x) = 2x^2(1 - x^2)$
   $g'(x) = -4x(2x^2 - 1)$ Critical numbers: $x = 0, \pm {}^{1}/_{\sqrt{2}}$
   $g''(x) = 4 - 24x^2$
   $g''(0) = 4 > 0$, Relative minimum at (0, 0)
   $g''(\pm {}^{1}/_{\sqrt{2}}) = -8 < 0$, Relative maximum at $(\pm {}^{1}/_{\sqrt{2}}, ½)$



4. $h(t) = t - 4\sqrt{t + 1}$, Domain: $[-1, \infty)$
   $h'(t) = 1 - \dfrac{2}{\sqrt{t + 1}} = 0$
   $t = 3$
   $h''(t) = \dfrac{1}{(t + 1)^{3/2}}$
   $h''(3) = 1/8 > 0$ (3, -5) is a relative minimum.

Task 3:

	positive to negative at c, then f has a relative maximum at (c, f(c)). If f'(x) is positive on both sides of c or negative on both sides of c, then f(c) is neither a relative minimum nor a relative maximum. ● Let f be differentiable on an open interval I. The graph of f is concave upward on I if f' is increasing on the interval and concave downward on I if f' is decreasing on the interval. ● Test for Concavity: Let f be a function whose second derivative exists on an open interval I. If f''(x) > 0 for all x in I, then the graph of f is concave upward in I. If f''(x)<0 for all x in I, then the graph of f is concave downward in I. ● Let f be a function that is continuous on an open interval and let c be a point in the interval. If the graph of f has a tangent line at this point (c, f(c)), then this point is a point of inflection of the graph of f if the concavity of f changes from upward to downward (or	Consider a room in the shape of a cube, 4 meters on each side. A bug at point P wants to walk to point Q at the opposite corner, as shown in the figure below.  Use calculus to determine the shortest path. Can you solve the problem without Calculus? **Answer:** Distance = √42 + x2 + √(4 – x)2 + 42 = f(x) $f'(x) = \frac{x}{\sqrt{4^2 + x^2}} - \frac{4 - x}{\sqrt{(4-x)^2 + 4^2}} = 0$ $x\sqrt{(4-x)^2 + 4^2} = -(x-4)\sqrt{4^2 + x^2}$ $x^2[16 - 8x + x^2 + 16] = (x^2 - 8x + 16)(16 + x^2)$ $32x^2 - 8x^3 + x^4 = x^4 - 8x^3 + 32x^2 - 128x + 256$ $128x = 256$ $x = 2$ The bug should head towards the midpoint of the opposite side. Without Calculus: Imagine opening up the cube: 

	downward to upward) at that point. ● If (c, f(c)) is a point of inflection of the graph of f, then either f''(c) = 0 or f'' does not exist at x = c. ● Second Derivative Test: Let f be a function such that f''(c)>0, then f has a relative minimum at (c, f(c)). If f''(c)< 0, then f has a relative maximum at (c, f(c)). If f''(c)=0, the test fails. That is, f may have a relative maximum, a relative minimum, or neither. In such cases, you can use the First Derivative Test. ● One of the most common applications of calculus involves the determination of minimum and maximum values. We can apply calculus to find the minimum and maximum volume, distance, area, and length. ● Consider a function f that is differentiable at c. The equation for the tangent line at point (c, f(c)) is given by: $y - f(c) = f'(c)\,(x - c)$ $y = f(c) + f'(c)(x - c)$	The shortest distance is the line PQ, passing through the midpoint. ***Task 4:** **Interdisciplinary SCI** **The diameter of a tree was 10 in. During the following year, the circumference increased 2 in. About how much did the tree's diameter increase? The tree's cross section area?** **Answer:** The diameter grew $2/\pi = 0.6366$ in. The cross section area grew about 10 in^2. **Modifications and/or Accommodations:** ● **Special Education:** Utilize a multi-sensory (VAKT) approach during instruction, provide alternate presentations of skills by varying the method (repetition, simple explanations, additional examples, modeling, etc.), modify test content and/or format, allow students to retake test for additional credit, provide additional times and preferential seating as needed, review, restate and repeat directions, provide study guides, and/or break assignments into segments of shorter tasks. ● **English Language Learners:** Extend time requirements, preferred seating, positive reinforcement, check often for understanding/review, oral/visual directions/prompts when necessary, supplemental materials including use of online bilingual dictionary, and modified assessment and/or rubric. ● **Students at Risk of School Failure:** Deliver instruction utilizing varied learning styles including audio, visual, and tactile/kinesthetic, provide individual instruction as needed, modify assessments and/or rubrics, repeat instructions as needed. ● **Gifted Students:** Create an enhanced set of introductory activities, integrate active teaching/learning opportunities, incorporate authentic

	• Let y = *f*(x) represent a function that is differentiable on an open interval containing x. The differential of x (denoted by *dx*) is any nonzero real number. The differential of y (denoted *dy*) is $dy=f'(x)\,dx$. • The differential formulas are: Let u and v be differentiable functions of x. Constant Multiple: $d[cu]=c\,du$ Sum or difference: $d[u \pm v] = du \pm dv$ Product: $d[uv] = u\,dv + v\,du$ Quotient: $d[u/v] = \frac{v\,du - u\,dv}{v^2}$ • A function is not differentiable when its behavior from the left and right, at a sharp point, and when it is oscillating.	components, propose interest-based extension activities, and connect student to related talent development opportunities **Spot Light On:** *Ask challenging questions equitably of all students.* **Teacher Resources** online achievethecore resource online learnzillion resource online khanacademy resource online desmos resource online ixl resource

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Content Statements	**Cumulative Progress Indicators**	

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Students will know… ● How to find an extrema on a closed interval. ● How to use Rolle's Theorem ● How to apply the Mean Value Theorem by finding a tangent line and finding an instantaneous rate of change ● How to test for increasing and decreasing functions ● Students will apply the first derivative test ● How to test for concavity ● How to find points of inflection ● How to apply the second derivative test ● How to solve minimum and maximum problems such as finding maximum volume, minimum distance, minimum area, minimum length, and endpoint maximum. ● How to use tangent line approximation. ● How to compare Δy and differential of y. ● How to estimate error ● How to calculate differentials	● Tests ● Quizzes ● Practice problems for homework ● Workbook pages ● Worksheets	

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Desired Results

- **Extreme values of functions**
- **Mean Value Theorem**
- **Connecting f' and f" with the Graph of f**
- **Modeling and optimization**
- **Linearization and Newton's Method**
- **Related rates.**

Standards for Mathematical Practices

1. **Make sense of problems and persevere in solving them.**
2. **Reason abstractly and quantitatively.**
3. **Construct viable arguments and critique the reasoning of others**
4. **Model with mathematics**
5. **Use appropriate tools strategically.**
6. **Attend to precision.**
7. **Look for and make use of structure.**
8. **Look for and express regularity in repeated reasoning.**

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LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35* Troy Lee Hudson - openly gay engineer at NASA's Jet Propulsion Laboratory The mission is to ensure that every student is able to see themselves in our rich and diverse history.	
Social and Emotional Learning: ***Competencies***	**Social and Emotional Learning:** ***Sub-Competencies***
Self-Awareness Social Awareness Self-Management Relationship Skills Responsible Decision-Making	• Recognizing the importance of self-confidence in handling daily tasks and challenges. • Demonstrate an awareness of the expectations for social interactions in a variety of ways. • Demonstrate an understanding of the need for mutual respect when viewpoints differ. • Recognize the skills needed to establish and achieve personal and educational goals. • Utilize positive communication and social skills to interact effectively with others. • Develop, implement, and model effective problem solving and critical thinking skills.

New Jersey Legislative Statutes and Administrative Code (place an "X" before each law/statute if/when present within the curriculum map)									
	Amistad Law: *N.J.S.A. 18A 52:16A-88*		Holocaust Law: *N.J.S.A. 18A:35-28*	**x**	LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*	**x**	Diversity & Inclusion: *N.J.S.A. 18A:35-4.36a*		Standards in Action: *Climate Change*