

Marking Period: 1	Unit Title: Limits and Continuity	Recommended Instruction Days: 25 - 30
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Standard-New Jersey Student Learning Standards: F-IF

Strand:
F-IF: Interpreting Functions

Understand the concept of a function and use function notation

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of the context

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes.

Analyze functions using different representations

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
 - a. Graph linear and quadratic functions and show intercepts, maxima, and minima
 - b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
 - c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
 - d. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
 - e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing

LGBT and Disabilities Law: *N.J.S.A. 18A:35-4.35*

Alan Turing - English mathematician, computer scientist, logician, cryptanalyst, philosopher, and theoretical biologist. Turing was highly influential in the development of theoretical computer science.

The mission is to ensure that every student is able to see themselves in our rich and diverse history.

Social and Emotional Learning: <i>Competencies</i>	Social and Emotional Learning: <i>Sub-Competencies</i>
<p>Self-Awareness</p> <p>Social Awareness</p> <p>Self-Management</p> <p>Relationship Skills</p> <p>Responsible Decision-Making</p>	<ul style="list-style-type: none"> ● Recognizing the importance of self-confidence in handling daily tasks and challenges. ● Demonstrate an awareness of the expectations for social interactions in a variety of ways. ● Demonstrate an understanding of the need for mutual respect when viewpoints differ. ● Recognize the skills needed to establish and achieve personal and educational goals. ● Utilize positive communication and social skills to interact effectively with others. ● Develop, implement, and model effective problem solving and critical thinking skills.

**Recommended Activities, Investigations,
Interdisciplinary Connections, and/or Student
Experiences to Explore NJSLS-CLKS within Unit**

Essential Questions	Progress Indicators	Activity Description
<ul style="list-style-type: none"> ● What are the graphical, numerical, and analytical ways to determine a limit of a function at a given value and its continuity at a given point? 	<ul style="list-style-type: none"> ● Tests ● Quizzes ● Practice problems for homework ● Worksheets ● Leveled assessments ● Projects 	<ul style="list-style-type: none"> ❖ Estimating limits from graphs <ul style="list-style-type: none"> One-sided limits Two-sided limits ❖ Estimating limits from tables ❖ Properties of limits ❖ Limits by direct substitution ❖ Limits using algebraic manipulation

- How do limits serve as a means to better understanding functions and their behaviors?
- What does it mean to be continuous?
- What factors cause a function to be discontinuous?
- What is infinitesimal and how does this concept relate to continuity and limits?

- ❖ Continuity and Discontinuity
- ❖ Infinite Limits
- ❖ Limits at infinity
- ❖ Intermediate Value Theorem
- ❖ Example Tasks Below

Interdisciplinary Connections:

Science Domain: Motion and Stability: Forces and Interactions

Let F be the gravitational force exerted by the Earth on an object of mass m placed a distance $r \geq 0$ from the center of the Earth. Assuming that the Earth is a perfect sphere of radius R and that it has a mass M , Newton's Universal Law of Gravity tells us that

$$F(r) = \begin{cases} \frac{GMmr}{R^3} & \text{if } r < R \\ \frac{GMm}{r^2} & \text{if } r \geq R \end{cases}$$

where $G > 0$ is a constant (the gravitational constant)

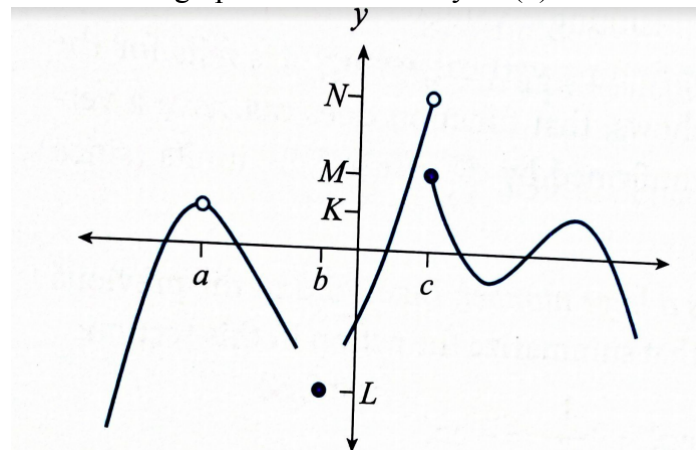
- Briefly explain how the gravitational force felt by the mass m changes as r changes.
- Calculate $\lim_{r \rightarrow R^-} F(r)$ and $\lim_{r \rightarrow R^+} F(r)$
- Is $F(r)$ continuous at $r = R$? Briefly explain.
- On what interval(s) is F continuous?

Answer:

- As r increases from zero, the gravitational force increases (linearly). When $r = R$ (the radius of the Earth), it reaches its maximum value. As r increases past R , the gravitational force decreases.
- Both limits are equal to $\frac{GMm}{R^2}$
- Yes, because the left- and right-hand limits exist, are equal, and they are equal to $F(R)$ (and all of these quantities are real numbers)
- $[0, \infty)$

Task

Below is the graph of the function $y = f(x)$.



(a) Evaluate the limit, or explain why it does not exist.

(i) $\lim_{x \rightarrow a^-} f(x)$ (ii) $\lim_{x \rightarrow a^+} f(x)$

(iii) $\lim_{x \rightarrow b^-} f(x)$ (iv) $\lim_{x \rightarrow b^+} f(x)$

(v) $\lim_{x \rightarrow c^-} f(x)$ (vi) $\lim_{x \rightarrow c^+} f(x)$

(vii) $\lim_{x \rightarrow a} f(x)$ (viii) $\lim_{x \rightarrow b} f(x)$

(ix) $\lim_{x \rightarrow c} f(x)$

(b) True or False: The function is continuous at $x = c$.

Answer

- a. (i) K, (ii) K, (iii) DNE (no portion of graph directly to the left of $x = c$), (iv) DNE (no portion of graph directly to the right of $x = c$), (v) N, (vi) M, (vii) K, (viii) DNE (one-sided limits DNE), (ix) DNE (one-sided limits exist but are not equal).
- b. False, because the one-sided limits are unequal

Task

Evaluate the limit with the help of algebra, if the limit exists

a. $\lim_{x \rightarrow 0} (x^2 - 2x + 1)$

b. $\lim_{x \rightarrow 9} (\sqrt{x} - 3)$

c. $\lim_{x \rightarrow -1} (\sqrt{x^2 + 1})$

d. $\lim_{x \rightarrow 0} \frac{x^3 - x}{x}$

e. $\lim_{x \rightarrow 1^+} \frac{1}{x-1}$

Answer

- a. 1
- b. 0
- c. $\sqrt{2}$
- d. -1
- e. DNE

Mathematical Practices

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.

3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
5. Use appropriate tools strategically.
6. Attend to precision.
7. Look for and make use of structure.
8. Look for and express regularity in repeated reasoning.

Assessments (Formative) <i>To show evidence of meeting the standard/s, students will successfully engage within:</i>		Assessments (Summative) <i>To show evidence of meeting the standard/s, students will successfully complete:</i>	
<p><u>Formative Assessment:</u></p> <ul style="list-style-type: none"> ● Entry and Exit Slips ● Quizzes ● Self Assessments ● Focus Packets 		<p><u>Benchmarks:</u></p> <ul style="list-style-type: none"> ● Chapter Tests ● Projects <p><u>Summative Assessments:</u></p> <ul style="list-style-type: none"> ● District assessments 	
<p>Differentiated Student Access to Content: Teaching and Learning Resources/Materials</p>			
Core Resources	Alternate Core Resources <i>IEP/504/At-Risk/ESL</i>	ELL Core Resources	Gifted & Talented Core Resources
online albert resource online achievethecore resource online learnzillion resource online khanacademy resource online desmos resource online edulastic resource	Reteaching worksheets Skill building workbook Math manipulatives Leveled practice worksheets	Dictionary for native language Video tutorial in native language Success for English Learners worksheets Leveled Strategies for English Learners	Enrichment worksheets Art of Problem Solving Leveled assessments

		Linguistic Support	
Supplemental Resources			
<ul style="list-style-type: none"> • Technology: Chromebooks, Graphing Calculators, Smartboards, • Other: Zoom and Google Meets, Schoology, Google Classroom 			
Differentiated Student Access to Content: Recommended <i>Strategies & Techniques</i>			
Core Resources	Alternate Core Resources <i>IEP/504/At-Risk/ESL</i>	ELL Core Resources	Gifted & Talented Core Resources
Deliver instruction utilizing varied learning styles including audio, visual, and tactile/kinesthetic, provide individual instruction as needed, modify assessments and/or rubrics, repeat	Utilize a multi-sensory (VAKT) approach during instruction, provide alternate presentations of skills by varying the method (repetition, simple explanations, additional examples, modeling, etc.), modify test content and/or format, allow students to retake test for additional credit, provide additional times and preferential seating as needed, review, restate and repeat directions, provide study guides, and/or break assignments into segments of shorter tasks.	Extend time requirements, preferred seating, positive reinforcement, check often for understanding/review, oral/visual directions/prompts when necessary, supplemental materials including use of an online bilingual dictionary, and modified assessment and/or rubric.	Create an enhanced set of introductory activities, integrate active teaching/learning opportunities, incorporate authentic components, propose interest-based extension activities, and connect student to related