Honors Pre-Calculus

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Teacher Notes

- *Additional mathematics that students should learn in order to take advanced courses such as calculus, advanced statistics, or discrete mathematics is indicated by (+)*

- ***Standards of Mathematical Practice***

  Mathematically proficient students:

  1. *Make sense of problems and persevere in solving them.*
  2. *Reason abstractly and quantitatively.*
  3. *Construct viable arguments and critique the reasoning of others.*
  4. *Model with mathematics*
  5. *Use appropriate tools strategically.*
  6. *Attend to precision.*
  7. *Look for and make use of structure.*
  8. *Look for and express regularity in repeated reasoning.*

Conceptual Category

1. Number and Quantity

Domain

1.1. The Complex Number System

Clusters

- *Perform arithmetic operations with complex numbers*
- *Represent complex numbers and their operations on the complex plane*

Pacing

- *3 days*

Standard Content Statements/Learning Targets

- *(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.*
- *Represent complex numbers and their operations on the complex plane*
- *(+) Represent addition, subtraction, multiplication, and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. For example, $(-1 + \sqrt{3}\, i)3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°*
- *(+) Calculate the distance between numbers in the complex plane as the modulus of the difference, and the midpoint of a segment as the average of the numbers at its endpoints.*

Learning Targets:

I can:

- *Express complex numbers in binomial, rectangular, polar, and trigonometric form.*
- *Add, subtract, multiply, and divide complex numbers.*
- *Prove or verify properties of complex numbers.*
- *Graph complex numbers and verify the Geometric Addition and Geometric Multiplication Theorems.*
- *Use De Moivre's Theorem to graph powers of complex numbers.*
- *Find powers and roots of complex numbers.*

Mathematical Progressions

Number and Quantity, K-5
Number and Quantity, 3-5
Number and Quantity, 6-7
Number and Quantity, 6-8

Content Vocabulary

Real, imaginary, complex conjugate

Academic Vocabulary

Formative Assessment

Conferencing - individual/small group, exit slip/admit slip, white boards, observations, projects, self/peer assessment, questioning, short quizzes, technology response systems.

Summative Assessment

Quizzes, unit assessments, semester exams

Resources

UCSMP FST (s7.5) & UCSMP PDM (s9.1 - s9.3)

Enrichment Strategies

Provide examples of differentiated task demands for each specific standards, create interdisciplinary product demands to elevate learning and efficiently address multiple standards at once, problem based learning, independent study extensions.

Integrations

Intervention Strategies

Instruction on solving word problems that is based on common underlying structures, explicit and systematic instruction of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, frequent cumulative review, guided notes

Conceptual Category

Number and Quantity

Domain

1.2. Vector, Quantities, and Matrices

Clusters

- *Represent and model with vector quantities*
- *Perform operations on vectors*
- *Perform operations on matrices and use matrices in applications*

Pacing

18 days

Standards Content Statements/Learning Targets

- *(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v).*
- *(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.*
- *(+) Solve problems involving velocity and other quantities that can be represented by vectors.*
- *(+) Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction $v - w$ as $v + (-w)$, where $-w$ is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.*
- *(+) Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(vx , vy) = (cvx , cvy)$. b. Compute the magnitude of a scalar multiple cv using $||cv|| = |c|v$. Compute the direction of cv knowing that when $|c|v \neq 0$, the direction of cv is either along v (for $c > 0$) or against v (for $c < 0$).*
- *6. (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.*

- *(+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.*
- *(+) Add, subtract, and multiply matrices of appropriate dimensions.*
- *(+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.*
- *(+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.*
- *(+) Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.*
- *(+) Work with 2 × 2 matrices as transformations of the plane, and interpret the absolute value of the determinant in terms of area.*

Learning Targets:
I can:

- *Find the magnitude and direction of two-dimensional vectors.*
- *Use vectors in a plane to decompose vectors into x- and y-components.*
- *Represent two-dimensional vectors in component or polar form, or as directed segments.*
- *Find sums and opposites of two-dimensional vectors.*
- *Prove or disprove generalizations about vector operations.*
- *Use addition of vectors in a plane to solve problems involving forces or velocities.*
- *Represent addition and subtraction of two-dimensional vectors graphically.*
- *Find scalar products of two-dimensional vectors.*
- *Identify parallel and orthogonal vectors.*
- *Represent scalar multiplication of two-dimensional vectors graphically.*
- *Represent lines in a plane using vector or parametric equations.*
- *Find dot products of two-dimensional vectors.*
- *Find the measure of the angle between two vectors.*

Mathematical Progressions

Number and Quantity, K-5
Number and Quantity, 3-5
Number and Quantity, 6-7
Number and Quantity, 6-8

Content Vocabulary

Magnitude, direction, dot product, cross product, parallel vectors

Academic Vocabulary

Formative Assessment

Conferencing - individual/small group, exit slip/admit slip, white boards, observations, projects, self/peer assessment, questioning, short quizzes, technology response systems.

Summative Assessment

Quizzes, unit assessments, semester exams

Resources

UCSMP PDM (s10.4 - s10.7)

Enrichment Strategies

Provide examples of differentiated task demands for each specific standards, create interdisciplinary product demands to elevate learning and efficiently address multiple standards at once, problem based learning, independent study extensions.

Integrations

Intervention Strategies

Instruction on solving word problems that is based on common underlying structures, explicit and systematic instruction of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, frequent cumulative review, guided notes

Conceptual Category

Number and Quantity

Domain

1.3. Logic

Clusters

- *Identify and write forms of logical statements*
- *Write negations of logical statements*
- *Use logic to prove or disprove arguments*
- *Use the laws of valid argument forms*
- *Use DeMorgan's Laws to negate "and" and "or" statements*

Pacing

- *11 days*

Standards Content Statements/Learning Targets

- *Analyze the two important types of generalizations in mathematics are universal statements (all objects of a particular kind have some property) and existential statements (there is an object of a particular kind that has some property).*
- *Understand that every statement p has a negation not p; negations are not necessarily the same as opposites.*
- *Combine two statements using the words "and" or "or" to create a compound statement.*
- *Recognize that logic gates are based on the three logical functions "not," "and," and "or."*
- *Universal if-then statements are true if there is no situation in which the antecedent is true and the consequent is false; when such a situation exists, it is a counterexample and the statement is false.*
- *Understand that the validity of an argument is determined by its logical form, not by whether or not the conclusion is true.*
- *Combine the laws of logic and premises to come up with conclusions.*
- *Write direct proofs using the Laws of Detachment and Transitivity.*

Learning Targets
<u>I can:</u>

- *Identify forms of logical statements.*
- *Write equivalent forms of logical statements.*
- *Determine the truth value of a logical statement.*
- *Identify the truth properties of logical statements.*
- *Use the Law of Substitution to verify specific statements.*

- *Use logic to prove or disprove statements.*
- *Determine the truth of quantified statements outside of mathematics.*
- *Write the negation of a logical statement.*
- *Use logic to prove or disprove statements.*
- *Write truth tables for logical expressions.*
- *Determine whether arguments are valid or invalid.*
- *Determine whether or not a logical argument outside of mathematics is valid.*
- *Determine whether arguments are valid or invalid.*

Mathematical Progressions

Number and Quantity, K-5
Number and Quantity, 3-5
Number and Quantity, 6-7
Number and Quantity, 6-8

Content Vocabulary

Statements and quantifiers, negations, if-then statements, converse, inverse, contrapositive, truth tables, law of detachment, law of indirect reasoning

Academic Vocabulary

Formative Assessment

Conferencing - individual/small group, exit slip/admit slip, white boards, observations, projects, self/peer assessment, questioning, short quizzes, technology response systems.

Summative Assessment

Quizzes, unit assessments, semester exams

Resources

UCSMP PDM (ch 1)

Enrichment Strategies

Provide examples of differentiated task demands for each specific standards, create interdisciplinary product demands to elevate learning and efficiently address multiple standards at once, problem based learning, independent study extensions.

Integrations

Intervention Strategies

Instruction on solving word problems that is based on common underlying structures, explicit and systematic instruction of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, frequent cumulative review, guided notes

Conceptual Category

Number and Quantity

Domain

1.4. Modular Arithmetic

Clusters

- *Determine the congruence of integers for a given modulus*
- *Use properties of congruence of integers in a given modulus to rewrite sentences*

Pacing

- *6 days*

Standards Content Statements/Learning Targets

- *Given a positive integer d > 1, all the integers can be sorted into d sets based on their remainders when divided by d, and useful arithmetic can be done with these sets.*
- *Every positive integer can be uniquely represented as a polynomial in a positive integer base b whose coefficients are nonnegative integers less than b.*

Learning Targets:
I can:

- *Determine the congruence of integers in a given modulus.*
- *Use the properties of congruence of integers in a given modulus to rewrite sentences.*
- *Use modular arithmetic to solve applied problems.*
- *Represent numbers in other bases and perform addition in base 2.*

Mathematical Progressions

Number and Quantity, K-5
Number and Quantity, 3-5
Number and Quantity, 6-7
Number and Quantity, 6-8

Content Vocabulary

Modulus, modulo, mod, congruence class, remainder

Academic Vocabulary

Formative Assessment

Conferencing - individual/small group, exit slip/admit slip, white boards, observations, projects, self/peer assessment, questioning, short quizzes, technology response systems.

Summative Assessment

Quizzes, unit assessments, semester exams

Resources

UCSMP PDM (s4.6, s4.7)

Enrichment Strategies

Provide examples of differentiated task demands for each specific standards, create interdisciplinary product demands to elevate learning and efficiently address multiple standards at once, problem based learning, independent study extensions.

Integrations

Intervention Strategies

Instruction on solving word problems that is based on common underlying structures, explicit and systematic instruction of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, frequent cumulative review, guided notes

Conceptual Category

Number and Quantity

Domain

1.5. Mathematical Induction

Clusters

- *Present complete and convincing arguments and justifications, using inductive and deductive reasoning, adapted to be effective for various audiences*

Pacing

- *12 days*

Standards Content Statements/Learning Targets

- *The principle of mathematical induction is a theorem of logic that enables you to prove that many statements are true for all integers n greater than or equal to 1 or to another fixed integer.*
- *Mathematical induction can be used to prove that a given integer is a factor of the values of certain expressions f(n) for all n.*
- *Mathematical induction can be used to prove that certain inequalities are true for all positive integers n greater than or equal to 1 or to another fixed integer.*

Learning Targets:
I can:

- *Prove that a recursively defined sequence has a particular explicit formula.*
- *Prove statements involving sums using the Principle of Mathematical Induction.*
- *Evaluate a finite or infinite geometric series.*

Mathematical Progressions

Number and Quantity, K-5
Number and Quantity, 3-5
Number and Quantity, 6-7
Number and Quantity, 6-8

Content Vocabulary

Basic step, inductive step, inductive assumption

Academic Vocabulary

Proof

Formative Assessment

Conferencing - individual/small group, exit slip/admit slip, white boards, observations, projects, self/peer assessment, questioning, short quizzes, technology response systems.

Summative Assessment

Quizzes, unit assessments, semester exams

Resources

UCSMP PDM (s6.3 - s6.5)

Enrichment Strategies

Provide examples of differentiated task demands for each specific standards, create interdisciplinary product demands to elevate learning and efficiently address multiple standards at once, problem based learning, independent study extensions.

Integrations

Intervention Strategies

Instruction on solving word problems that is based on common underlying structures, explicit and systematic instruction of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, frequent cumulative review, guided notes

Conceptual Category

2. Algebra

Domain

2.1. Reasoning with Equations and Inequalities

Clusters

- *Solve systems of equations*
- *Solve absolute value equations*

Pacing

- *10 days*

Standards Content Statements/Learning Targets

- *To solve many equations of the form f(x) = g(x) for x, you can add h(x) to or subtract h(x) from both sides, or multiply or divide both sides by h(x), or apply h to both sides of the equation without affecting its solutions.*
- *By considering mathematical expressions as a single variable, many complicated equations can be solved.*
- *In an equation to be solved for x, substituting x - h for x adds h to any solution; substituting*
- *$\frac{x}{a}$ for x multiplies any solution by a.*
- *To solve many inequalities of the form f(x) < g(x) for x, you can add h(x) to or subtract h(x) from both sides, or multiply or divide both sides by h(x), or apply h to both sides of the inequality without affecting its solutions, but you have to consider the possibility that the sense of the inequality will change.*
- *In solving many inequalities of the form f(x) < g(x) for x, you can use the zeros of the function f - g.*
- *The first step in solving many equations and inequalities involving the absolute value function is to apply the definition of absolute value.*
- *(+) Represent a system of linear equations as a single matrix equation in a vector variable.*
- *(+) Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater).*

Learning Targets:
<u>I can:</u>

- *Solve equations, taking into account nonreversible steps.*
- *Apply equation-solving techniques to real-world problems.*
- *Analyze the reversibility of steps used in solving equations.*

- *Find zeros of function using factoring or chunking.*
- *Apply equation-solving techniques to real-world problems.*
- *Use graphs to approximate zeros of functions and solve equations and inequalities.*
- *Solve inequalities algebraically.*
- *Analyze the reversibility of steps used in solving equations.*
- *Identify and prove properties of inverse functions.*
- *Use inequalities to solve real-world problems.*
- *Solve inequalities algebraically.*
- *Use graphs to approximate zeros of functions and solve equations and inequalities.*
- *Solve equations, taking into account nonreversible steps.*
- *Apply equation-solving techniques to real-world problems.*
- *Solve inequalities algebraically.*
- *Use inequalities to solve real-world problems.*

Mathematical Progressions

Algebra, Grade K-5
Algebra, Grades 6-8
Algebra, High School

Content Vocabulary

Reversible and nonreversible functions, chunking, monotone function, test point

Academic Vocabulary

System of equations

Formative Assessment

Conferencing - individual/small group, exit slip/admit slip, white boards, observations, projects, self/peer assessment, questioning, short quizzes, technology response systems.

Summative Assessment

Quizzes, unit assessments, semester exams

Resources

UCSMP PDM (s3.4 - s3.6, s3.8 - s3.10)

Enrichment Strategies

Provide examples of differentiated task demands for each specific standards, create interdisciplinary product demands to elevate learning and efficiently address multiple standards at once, problem based learning, independent study extensions.

Integrations

Intervention Strategies

Instruction on solving word problems that is based on common underlying structures, explicit and systematic instruction of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, frequent cumulative review, guided notes

Conceptual Category

3. Functions

Domain

3.1. Interpreting and Building Functions

Clusters

- *Analyze functions using different representations*
- *Build a function that models a relationship between two quantities*
- *Build new functions from existing functions*

Pacing

- *16 days*

Standards Content Statements/Learning Targets

- *Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.**
  - *(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.*
- *Write a function that describes a relationship between two quantities.**
  - *(+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
- *Find inverse functions.*
  - *(+) Verify by composition that one function is the inverse of another.*
  - *(+) Read values of an inverse function from a graph or a table, given that the function has an inverse.*
  - *(+) Produce an invertible function from a non-invertible function by restricting the domain.*
- *(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.*

Learning Targets:
<u>I can:</u>

- *Identify the domain and range of functions.*
- *Determine the global and local minimum and maximum values of functions.*
- *Solve max-min problems.*
- *Analyze a function from its graph.*
- *Determine intervals on which functions are increasing or decreasing.*

- *Describe the end behavior of a function.*
- *Use exponential functions as models.*
- *Rewrite exponential and logarithmic expressions and equations.*
- *Use sequences, exponential functions, logistic functions, and logarithmic functions as models.*

Mathematical Progressions

Functions, High School
Modeling, High School

Content Vocabulary

Scale change, translation, domain, range, increasing/decreasing, one-to-one, even and odd functions, symmetries

Academic Vocabulary

Transformations, analyze, interpret

Formative Assessment

Conferencing - individual/small group, exit slip/admit slip, white boards, observations, projects, self/peer assessment, questioning, short quizzes, technology response systems.

Summative Assessment

Quizzes, unit assessments, semester exams

Resources

UCSMP PDM (s2.1 - s2.7)

Enrichment Strategies

Provide examples of differentiated task demands for each specific standards, create interdisciplinary product demands to elevate learning and efficiently address multiple standards at once, problem based learning, independent study extensions.

Integrations

Intervention Strategies

Instruction on solving word problems that is based on common underlying structures, explicit and systematic instruction of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, frequent cumulative review, guided notes

Conceptual Category

Functions

Domain

3.2. Trigonometric Functions

Clusters

- *Extend the domain of trigonometric functions using the unit circle*
- *Model periodic phenomena with trigonometric functions*
- *Prove and apply trigonometric identities*

Pacing

- *10 days*

Standards Content Statements/Learning Targets

- *(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.*
- *(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.*
- *(+) Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.*
- *(+) Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.**
- *(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.*

Learning Targets:
I can:

- *Convert between degrees, radians, and revolutions.*
- *Use the unit circle to find values of sines, cosines, and tangents.*
- *Apply the definitions of the sine, cosine, and tangent functions.*
- *Apply theorems about sines, cosines, and tangents.*
- *Find values of sines, cosines, and tangents.*
- *Draw or interpret graphs of the parent sine, cosine, and tangent functions in degrees or radians.*
- *Identify the amplitude, period, frequency, phase shift, and other properties of trigonometric functions.*
- *Graph and describe transformation images of graphs of trigonometric functions.*
- *Write and solve equations for phenomena described by trigonometric functions.*
- *Find values of sine, cosine, tangent, cotangent, secant, and cosecant.*
- *Find values of sine, cosine, tangent, cotangent, secant, and cosecant.*

- *Describe the end behavior of a function.*
- *Prove trigonometric identities and identify their domains.*
- *Use a graphing calculator to test proposed trigonometric identities.*
- *Use trigonometric identities to express values of trigonometric functions in terms of rational numbers and radicals.*
- *Prove trigonometric identities and identify their domains.*

Mathematical Progressions

Functions, High School
Modeling, High School

Content Vocabulary

Period, phase shift, vertical shift, amplitude, sine, cosine, tangent, cotangent, cosecant, secant, radians vs. degrees, inverse trig functions

Academic Vocabulary

Transformation of functions, translations/scale changes, solving equations using inverse functions, proving identities using substitution

Formative Assessment

Conferencing - individual/small group, exit slip/admit slip, white boards, observations, projects, self/peer assessment, questioning, short quizzes, technology response systems

Summative Assessment

Quizzes, unit assessments, semester exams

Resources

UCSMP FST (ch 4) & UCSMP PDM (s2.8, s2.9, s5.7, s5.8, s5.9)

Enrichment Strategies

Provide examples of differentiated task demands for each specific standards, create interdisciplinary product demands to elevate learning and efficiently address multiple standards at once, problem based learning, independent study extensions.

Integrations

Intervention Strategies

Instruction on solving word problems that is based on common underlying structures, explicit and systematic instruction of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, frequent cumulative review, guided notes

Conceptual Category

Functions

Domain

3.3. Limits

Clusters

- *Find limits graphically*
- *Find limits numerically*
- *Find limits algebraically*

Pacing

- *10 days*

Standards Content Statements/Learning Targets

- *Explore end behavior of various functions.*
- *Calculate limits both graphically and analytically.*
- *Understand the continuity of a function at a point and on a domain.*
- *Understand limits at infinity and limits that approach infinity.*

Learning Targets:
I can:

- *Describe the end behavior of various functions.*
- *Analyze a function from its graph.*
- *Calculate a limit using a table of values*
- *Calculate a limit graphically*
- *Use and understand the precise definition of a limit.*
- *Calculate a limit algebraically using the limit laws.*
- *Calculate a limit that is in an indeterminate form*
- *Discuss the continuity of a function at a point*
- *Discuss the continuity of a function on a domain*
- *Understand and apply the Squeeze Theorem*
- *Calculate limits that approach infinite*
- *Calculate limits at infinity*
- *Understand how limits relate to vertical and horizontal asymptotes*

Mathematical Progressions

Functions, High School
Modeling, High School

Content Vocabulary

Limit, End Behavior

Academic Vocabulary

Solving, Equation of a line, Average Rate of Change, Asymptotes

Formative Assessment

Conferencing - individual/small group, exit slip/admit slip, white boards, observations, projects, self/peer assessment, questioning, short quizzes, technology response systems

Summative Assessment

Quizzes, unit assessments, semester exams

Resources

UCSMP PDM (s2.4) & Larson Calculus (ch 1)

Enrichment Strategies

Provide examples of differentiated task demands for each specific standards, create interdisciplinary product demands to elevate learning and efficiently address multiple standards at once, problem based learning, independent study extensions.

Integrations

Intervention Strategies

Instruction on solving word problems that is based on common underlying structures, explicit and systematic instruction of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, frequent cumulative review, guided notes

Conceptual Category

Functions

Domain

3.4. Differentiation

Clusters

- *Calculate the derivative at a point*
- *Examine and appropriately use the derivative function*
- *Find the equation of a tangent line*
- *Make use of the differentiation rules*
- *Use derivatives to analyze graphs*

Pacing

- *25 days*

Standards Content Statements/Learning Targets

- *Explore the idea that rate of change, average velocity, slope and difference quotient are different aspects of the same idea*
- *Develop the concept of instantaneous rate of change, instantaneous velocity, and the derivative of a function at a point as well as construct the derivative.*
- *Extend geometric ideas about derivative.*
- *Consider functions f of the form $f(x)=ab^x$ and show them to have derivative f' , where $f'(x) = (\ln b)f(x) = a(\ln b)b^x$*
- *Understand that a tangent line represents the instantaneous rate of change of a function at a point.*
- *Understand how the secant line be used to find the value of a tangent line.*
- *Use the rules for derivatives to calculate the derivatives of polynomial, trigonometric, and rational functions.*

Learning Targets:
<u>I can</u>:

- *Compute average rates of change in real situations.*
- *Relate average rates of change to secant line of graphs and functions.*
- *Use the definition of derivative to computer derivatives.*
- *Use derivatives to find the velocity of a moving object.*
- *Estimate derivatives by finding slopes of tangent lines.*
- *Use the second derivative to find the acceleration of a moving object.*
- *Use derivatives to identify properties of functions.*
- *Determine properties of a function and its derivatives from their graphs.*

- *Use derivatives to solve optimization problems.*
- *Find the derivative using the limit definition*
- *Find the derivative's value at a point using the limit definition*
- *Learn the different ways to say "derivative"*
- *Learn the different notations for a derivative*
- *Apply the power rule, constant multiple rule, and sum and difference rules to calculate a derivative*
- *Find the equation of a tangent line*
- *Apply the product and quotient rules to calculate a derivative*
- *Apply the chain rule to derivatives*
- *Calculate higher derivatives*
- *Apply derivatives to Physics: position, velocity, acceleration*
- *Calculate trigonometric derivatives*
- *Calculate a derivative implicitly*
- *Compare rates of change in quantities using related rates*

Mathematical Progressions

Functions, High School
Modeling, High School

Content Vocabulary

Difference Quotient, Instantaneous Rate of Change, Tangent Line, Velocity, Acceleration, Critical Numbers, Absolute Extrema, Relative Extrema, and First Derivative Test

Academic Vocabulary

Solving, Equation of a line, Average Rate of Change

Formative Assessment

Conferencing - individual/small group, exit slip/admit slip, white boards, observations, projects, self/peer assessment, questioning, short quizzes, technology response systems

Summative Assessment

Quizzes, unit assessments, semester exams

Resources

UCSMP PDM (ch 7) & Larson Calculus (chapters 2 and 3)

Enrichment Strategies

Provide examples of differentiated task demands for each specific standards, create interdisciplinary product demands to elevate learning and efficiently address multiple standards at once, problem based learning, independent study extensions.

Integrations

Intervention Strategies

Instruction on solving word problems that is based on common underlying structures, explicit and systematic instruction of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, frequent cumulative review, guided notes

Conceptual Category

4. Geometry

Domain

4.1. Expressing Geometric Properties with Equations

Clusters

- *Translate between the geometric description and the equation for a conic section*

Pacing

- *4 days*

Standards Content Statements/Learning Targets

- *(+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.*

  Learning Targets:
  I can:
  - *Relate geometric properties of polar graphs and polar equations.*
  - *Relate geometric properties of conic sections and rectangular and polar equations.*
  - *Use polar coordinates to model and describe real world problems.*

Mathematical Progressions

K-6, Geometry
***Secondary, Geometry* - Not Yet Available**

Content Vocabulary

Parts of a cone - Axis, Vertex, nappes, Cross section, Eccentricity, Ellipse, Hyperbola, Focus, Directrix,

Academic Vocabulary

Parabola, Standard Form of an Equation of second degree, focal constant

Formative Assessment

Conferencing - individual/small group, exit slip/admit slip, white boards, observations, projects, self/peer assessment, questioning, short quizzes, technology response systems.

Summative Assessment

Quizzes, unit assessments, semester exams

Resources

UCSMP PDM (s8.6, s8.7)

Enrichment Strategies

Provide examples of differentiated task demands for each specific standards, create interdisciplinary product demands to elevate learning and efficiently address multiple standards at once, problem based learning, independent study extensions.

Integrations

Intervention Strategies

Instruction on solving word problems that is based on common underlying structures, explicit and systematic instruction of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, frequent cumulative review, guided notes

Conceptual Category

Geometry

Domain

4.2. Geometric Measurement and Dimension

Clusters

- *Explain volume formulas and use them to solve problems*

Pacing

- *3 days*

Standards Content Statements/Learning Targets

- *(+) Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.*

  Learning Targets:
  I can:
  - *Determine the global and local minimum and maximum values of functions.*
  - *Solve max-min problems.*
  - *Analyze a function from its graph.*

Mathematical Progressions

K-6, Geometry
***Secondary, Geometry* - Not Yet Available**

Content Vocabulary

maximizing area, minimizing surface area using volume and surface area formulas

Academic Vocabulary

maximum and minimum (global & local)

Formative Assessment

Conferencing - individual/small group, exit slip/admit slip, white boards, observations, projects, self/peer assessment, questioning, short quizzes, technology response systems.

Summative Assessment

Quizzes, unit assessments, semester exams

Resources

UCSMP PDM (s2.2)

Enrichment Strategies

Provide examples of differentiated task demands for each specific standards, create interdisciplinary product demands to elevate learning and efficiently address multiple standards at once, problem based learning, independent study extensions.

Integrations

Intervention Strategies

Instruction on solving word problems that is based on common underlying structures, explicit and systematic instruction of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, frequent cumulative review, guided notes

Conceptual Category

Geometry

Domain

4.3. Polar and Parametric Equations

Clusters

- *Use polar coordinates to specify locations on a plane*
- *Sketch graphs of polar equations*
- *Determine suitable parametric equations and graph*

Pacing

- *14 days*

Standards Content Statements/Learning Targets

- *Graph points in the polar coordinate system and convert back and forth from polar coordinates to rectangular coordinates.*
- *Graph the simplest polar equations $r=k$ for $k>0$ (a circle with center at the origin and radius k) and $\theta = k$ (a line containing the origin).*
- *Provide an introduction to polar graphs, concentrating on the circles with equations of the form $r=a \cos \theta$ and $r=a \sin \theta$ and the rose curves of the form $r=a \sin b\theta$ and $r=a \cos b\theta$.*
- *Introduce polar coordinates and discuss the relationships between polar coordinates $[r, \theta]$ and rectangular coordinates (x, y) for the same point.*
- *Discuss the polar graphs of equations whose rectangular coordinate graphs are known to students: simple trigonometric equations of the form $r=a \sin (n\theta)$, linear equations of the form $r = a\theta + b$, and exponential equations of the form $r= ab^{\theta}$ that result in rose curves, spirals of Archimedes and logarithmic spirals, respectively.*
- *Convert an equation for a curve in polar coordinates into an equation for the same curve in rectangular coordinates.*
- *Introduce parametric equations and relate them to functions of the type that have already been studied and to the description of curves.*

Learning Targets:
<u>I can</u>:

- *Graph parametric equations of circles and ellipses.*
- *Plot points in a polar coordinate system.*
- *Given polar coordinates of a point, determine its rectangular coordinate, and vice-versa.*
- *Graph and interpret graphs of polar equations.*
- *Relate geometric properties of polar graphs and polar equations.*

- *Use polar coordinates to model and describe real-world problems.*
- *Sketch graphs of polar equations.*
- *Write rectangular and polar equations for curves.*
- *Convert between polar and rectangular coordinates.*
- *Model two-dimensional motion with parametric equations.*
- *Graph parametric equations.*

Mathematical Progressions

K-6, Geometry

***Secondary, Geometry* - Not Yet Available**

Content Vocabulary

Pole, Polar Axes, Polar Coordinates, Cardioid, Limacon, Parameter, Parametric Equation, Rose Curve, Spiral of Archimedes, Logarithmic Spiral, Cycloid, Witch of Agnesi

Academic Vocabulary

Formative Assessment

Conferencing - individual/small group, exit slip/admit slip, white boards, observations, projects, self/peer assessment, questioning, short quizzes, technology response systems.

Summative Assessment

Quizzes, unit assessments, semester exams

Resources

UCSMP FST (s5.8, s13.3, s13.4) & UCSMP PDM (s8.2 - s8.5, s10.1)

Enrichment Strategies

Provide examples of differentiated task demands for each specific standards, create interdisciplinary product demands to elevate learning and efficiently address multiple standards at once, problem based learning, independent study extensions.

Integrations

Intervention Strategies

Instruction on solving word problems that is based on common underlying structures, explicit and systematic instruction of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, frequent cumulative review, guided notes

Conceptual Category

5. Statistics and Probability

Domain

5.1. Using Probability to Make Decisions

Clusters

- *Calculate expected values and use them to solve problems*
- *Use probability to evaluate outcomes of decisions*

Pacing

- *18 days*

Standards Content Statements/Learning Targets

- *(+) Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions.*
- *(+) Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.*
- *(+) Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. For example, find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.*
- *(+) Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households?*
- *(+) Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values.*
    - *Find the expected payoff for a game of chance. For example, find the expected winnings from a state lottery ticket or a game at a fast- food restaurant.*
    - *Evaluate and compare strategies on the basis of expected values. For example, compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.*

Learning Targets:
<u>I can</u>:

- *Determine relationships and interpret data presented in a table.*

- *Calculate averages with weights, frequencies, and relative frequencies.*
- *Use Σ-notation to represent a sum or mean.*
- *Read, interpret, and draw histograms and population pyramids from data.*
- *Read, interpret, and draw box plots from data.*
- *Calculate and draw line graphs of cumulative frequencies and cumulative relative frequencies from tables of frequencies.*

Mathematical Progressions

K-5, Statistics and Probability
6-8, Statistics and Probability
High School, Statistics and Probability

Content Vocabulary

Statistics, Population, Sample (representative as well), Survey, Census, Sigma, Summation Notation, Index, Distributions, Bins, Population Pyramid,
Cumulative Data and Distribution,

Academic Vocabulary

Data, variable(categorical & numerical), measures of center (mean,median,mode), weighted average, relative frequency, Subscripted Variables, Histograms (frequency, relative frequency), skewed data, symmetric, Box Plot, Minimum, 1st 2nd 3rd Quartiles, Maximum, 5 number Summary, Interquartile Range(IQR), whiskers, outliers, Percentiles, Range, Standard Deviation (Population & Sample), Variance

Formative Assessment

Conferencing - individual/small group, exit slip/admit slip, white boards, observations, projects, self/peer assessment, questioning, short quizzes, technology response systems.

Summative Assessment

Quizzes, unit assessments, semester exams

Resources

UCSMP FST (ch 1)

Enrichment Strategies

Provide examples of differentiated task demands for each specific standards, create interdisciplinary product demands to elevate learning and efficiently address multiple standards at once, problem based learning, independent study extensions.

Integrations

Intervention Strategies

Instruction on solving word problems that is based on common underlying structures, explicit and systematic instruction of proficient problem solving, verbalization of thought processes, guided practice, corrective feedback, frequent cumulative review, guided notes