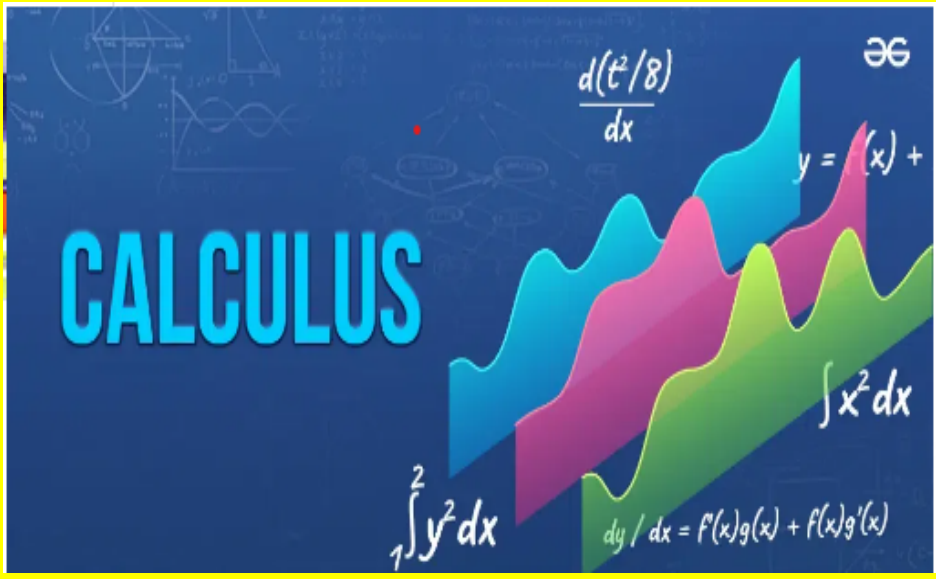


AP CALCULUS AB CURRICULUM

	
Grade Level(s): 11-12	Curriculum Author(s): Laurenn Bertoglio
<p>Course Description: AP Calculus AB is an advanced placement course designed to be the equivalent of a first-semester college calculus course. It covers topics in differential and integral calculus, emphasizing understanding the concepts, methods, and applications. Key topics include limits and continuity, derivatives, integrals and differential equations.</p> <p>Course Objectives:</p> <ul style="list-style-type: none">• Develop a deep understanding of calculus concepts.• Apply calculus in various contexts, including physical, social, and biological sciences.• Prepare for the AP Calculus AB exam, which can lead to college credit or advanced placement. <p>Skills Emphasized:</p> <ul style="list-style-type: none">• Analytical thinking and problem-solving.• Mathematical reasoning.	

- Conceptual understanding and fluency with the language and symbols of calculus.

Prerequisites:

- Successful completion of Honors Pre-Calculus or an equivalent course.

AP Calculus AB is rigorous and demands a strong foundation in algebra and trigonometry. The course is suitable for students who plan to pursue STEM fields in college.

Year At A Glance

Unit Title	Overarching Essential Question	Overarching Enduring Understanding	Vision of A Learner “I Can” Statements
Precalculus Review	How does previous mathematical knowledge apply to calculus?	Mathematical rigor and precision in understanding and applying concepts are crucial for success in calculus. Students must develop a strong foundation in algebraic and trigonometric principles, which will enable them to approach calculus problems systematically and confidently.	TCC3(9-12); TCC2(9-12); TI2(9-12)
Limits and Continuity	How do limits help us understand the behavior of a function near points of interest, such as where the function is undefined or where it approaches infinity?	Limits, continuity, and rate of change are deeply interconnected concepts in calculus. Limits help us understand continuity by determining whether a function behaves consistently as it approaches specific points. Continuity, in turn, is necessary for defining and interpreting the rate of change, especially in the context of derivatives. Together, these concepts form the foundation of differential calculus, enabling us to analyze and model change in various fields.	AA1(9-12); CCE3(9-12); CCE4(9-12); TCC2(9-12)
Introduction to Derivatives	What does the derivative of a function represent, and how is it connected to the concept of the rate of change?	The derivative is a mathematical tool that measures how a function changes as its input changes. It provides a precise way to quantify rates of change, which is fundamental for understanding and modeling dynamic processes in various fields.	AA1(9-12); TCC3(9-12); CCE4(9-12)

Derivatives	How do different rules and techniques for finding derivatives, such as the product rule, chain rule, and implicit differentiation, expand our ability to solve complex problems involving non-linear relationships and composite functions?	Mastery of the chain rule, implicit differentiation, and logarithmic differentiation equips students with a flexible and adaptable approach to differentiation. These techniques allow students to handle a wide variety of functions and equations, expanding their problem-solving toolkit in calculus.	AA1(9-12); TCC3(9-12); CCE4(9-12)
The Derivative as a Tool for Analysis	What is the relationship between a function and its derivative, and how does understanding this relationship allow us to predict and analyze the behavior of the function, including its critical points, intervals of increase and decrease, and concavity?	The derivative provides critical information about the behavior of functions, including identifying intervals of increase and decrease, finding local maxima and minima, and determining points of inflection. This understanding is key to analyzing and graphing functions.	AA1(9-12); TCC3(9-12); CCE4(9-12)
Applications and Extensions of the Derivative	How do derivatives help us understand and describe the behavior of changing quantities in real-world situations, and why are they a critical tool in modeling and solving problems across various disciplines?	Solving problems in optimization, related rates, and linearization requires critical thinking and careful mathematical reasoning. Mastery of these concepts develops students' problem-solving skills and their ability to apply mathematical reasoning to real-world situations.	TCC1(9-12); CCE3(9-12); TI3(9-12)
The Integral	What is an integral, and how does it relate to the concept of accumulation and the area under a curve?	Integration represents the process of accumulating quantities, such as area under a curve, total distance traveled, or accumulated volume. This understanding helps students grasp how integration extends the concept of summation to continuous functions and provides a way to measure quantities that vary over an interval.	CCE4(9-12); TCC2(9-12); CCE2(9-12)
Differential Equations and Mathematical	How do differential equations and slope fields help us model, analyze,	Analytical solutions to differential equations provide precise	CCE2(9-12); CCE3(9-12); TCC1(9-12)

Modeling	and predict the behavior of dynamic systems, and how can we use both graphical and analytical methods to understand the relationships between changing quantities?	mathematical expressions for the behavior of a system, while graphical methods, such as slope fields, offer a visual understanding of these solutions. Both approaches are interconnected and complement each other in solving and interpreting differential equations.	
Applications of Definite Integrals	How can the concept of integration be used to model and solve problems involving accumulation, area, and volume, and how do these applications help us understand and describe real-world phenomena across diverse fields such as physics, economics, and biology?	Integrals can be used to find geometric quantities such as the area under a curve, the area between curves, and the volume of solids formed by revolution or with known cross-sectional areas. Understanding integrals as geometric constructs provides a concrete foundation for their application in various fields.	CCE4(9-12); AA1(9-12); TCC3(9-12)

Unit 1 - Precalculus Review

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards:

- **N.CN.C.7** Solve quadratic equations with real coefficients that have complex solutions.
- **A.SSE.B.3.A** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A.REI.A.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- **A.REI.B.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- **A.REI.B.4** Solve quadratic equations in one variable.
- **A.REI.B.4.B** Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b .
- **A.REI.D.10** Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
- **F.IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
- **F.IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F.IF.C.7.A** Graph linear and quadratic functions and show intercepts, maxima, and minima.*
- **F.IF.C.7.B** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
- **F.BF.B.3** Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- **F.BF.B.4** Find inverse functions.
- **F.BF.B.4.B** Verify by composition that one function is the inverse of another.
- **F.BF.B.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- **F.TF.A.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **F.TF.A.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- **F.TF.A.3** . Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.
- **G.CO.A.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

- **G.SRT.C.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- **G.GPE.B.5** Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... (“I can” statements to be demonstrated)

- TCC2(9-12): I can evaluate evidence from multiple perspectives, and recognize their limitations and implications, in order to justify new conclusions.
- TCC3(9-12): I can integrate relevant information to produce multiple valid solutions.
- TI2(9-12): I can evaluate my objectives and a variety of credible resources to find the best solutions for any challenge.

Understandings: Students will understand that...

- Functions are essential tools for modeling relationships between quantities in various contexts. Understanding different types of functions (linear, polynomial, rational, exponential, logarithmic, and trigonometric) and their properties allows students to represent and analyze real-world situations effectively.
- The behavior of a function is determined by its key characteristics, such as its rate of change, asymptotes, and end behavior. Understanding how functions behave and how they can be transformed provides a foundation for analyzing more complex mathematical relationships in calculus.
- Algebraic and trigonometric functions are deeply interconnected, and a strong understanding of these concepts is necessary for solving complex problems in calculus. Mastery of algebraic manipulation and trigonometric identities enables students to simplify expressions and solve equations efficiently.
- Mathematical rigor and precision in understanding and applying

Essential Questions:

- How does previous mathematical knowledge apply to calculus?
- How can different types of functions (e.g., linear, polynomial, rational, exponential, logarithmic, trigonometric) be represented, analyzed, and interpreted?
- How do transformations (e.g., translations, reflections, stretches, compressions) affect the graph of a function?
- How do the properties of the unit circle help in understanding and applying trigonometric functions?
- How are exponential and logarithmic functions related, and what are their key applications?

<p>concepts are crucial for success in calculus. Students must develop a strong foundation in algebraic and trigonometric principles, which will enable them to approach calculus problems systematically and confidently.</p>	
<p>Students will know...</p> <ul style="list-style-type: none"> • Parallel lines have the same slope and perpendicular lines have opposite reciprocal slopes • There are three forms of linear equations: standard, slope intercept, and point slope • The parent functions for quadratic, square root, cube root, and cubic functions and how to use transformations on them • Interval notation to represent domain and range • Exponential equations are solved using logarithms and logarithmic equations are solved using exponential equations • Logarithmic equations can have extraneous solutions • The unit circle values for all six trigonometric functions • A trigonometric equation can have infinite solutions which is why we restrict the domain • The meaning of finding the exact value of a trigonometric function 	<p>Students will be able to...</p> <ul style="list-style-type: none"> • Write equations of lines in slope intercept and point slope form including parallel and perpendicular lines • Graph functions using transformations including quadratic, square root, cube root, and cubic and state their domain and range in interval notation • Graph a piecewise function • Perform function operations including composition and inverse • Solve exponential and logarithmic equations and applications • Find values of the six trigonometric functions using the unit circle along with inverse trigonometric functions • Solve trigonometric equations
<p>Key Vocabulary: Slope intercept and point slope form of a linear equation, parallel and perpendicular lines, x and y intercept, function, domain, range, interval notation, piecewise function, function notation, inverse function, composition function, logarithmic function, exponential function, extraneous solution, trigonometric function and equation, sine, cosine, tangent, cosecant, secant, cotangent, unit circle</p>	
<p style="text-align: center;">Assessment Evidence</p>	
<p>Performance Tasks:</p> <p>Summative Unit Assessment: TCC3(9-12) Topics include:</p> <ul style="list-style-type: none"> • Write equations of lines in slope intercept and point slope form including parallel and perpendicular lines • Graph functions using transformations including quadratic, square root, cube root, and cubic and state their domain and range in interval notation • Graph a piecewise function 	<p>Other Evidence:</p> <p>This test is given the first week of school after reviewing the summer packet which students completed over the summer. Students have multiple opportunities to ask questions and review before the test.</p>

- Perform function operations including composition and inverse
- Solve exponential and logarithmic equations and applications
- Find values of the six trigonometric functions using the unit circle along with inverse trigonometric functions
- Solve trigonometric equations

Learning Plan

This is a short unit that is meant to review the concepts learned in Precalculus that will be necessary throughout the year in Calculus. Students complete a summer packet, we discuss student questions from their packet, and a unit test is given to prove that the students have the necessary prerequisite skills.

Vision of A Learner Attributes: TCC2(9-12), TCC3(9-12), TI2(9-12)

- Students will submit completed summer work for the course. Class discourse on correct solutions. Students will ask questions and we will compare strategies for solving, including discussing the most efficient method.
- Individual/group in class practice on concepts that will be measured on the assessment.

Teacher Resources: Teacher created summer packet, mini white boards

Unit 2 - Limits and Continuity

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards:

- **N.CN.A.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- **A.SSE.B.3.A** Factor a quadratic expression to reveal the zeros of the function it defines.
- **F.IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
- **A.APR.D.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- **TCC2(9-12):** I can evaluate evidence from multiple perspectives, and recognize their limitations and implications, in order to justify new conclusions.
- **CCE3(9-12):** I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.
- **CCE4(9-12):** I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.
- **AA1(9-12):** I can evaluate different approaches and justify the best pathway to success.

Understandings: Students will understand that...

- The concept of a limit is fundamental to calculus, serving as the basis for defining both the derivative and the integral. Understanding limits allows us to explore the behavior of functions as they approach specific values, which is essential for

Essential Questions:

- What is the concept of a limit, and how does it describe the behavior of a function as it approaches a particular value?
- What does it mean for a function to be continuous at a point, and how can continuity be determined both graphically and

<p>analyzing change and accumulation.</p> <ul style="list-style-type: none"> • Continuity is a crucial property of functions that ensures they behave predictably without sudden jumps or breaks. A function is continuous if it is uninterrupted and smooth at every point in its domain, and this continuity is a prerequisite for applying many calculus techniques. Understanding continuity enables us to make accurate predictions about the behavior of functions and to apply calculus in a wide range of contexts. • The rate of change is a core concept in calculus that measures how one quantity changes in relation to another. The average rate of change provides insights into how a function behaves over an interval, while the instantaneous rate of change, captured by the derivative, reveals how the function behaves at a specific point. Mastering the concept of rate of change is essential for understanding motion, growth, and other dynamic processes. • Limits, continuity, and rate of change are deeply interconnected concepts in calculus. Limits help us understand continuity by determining whether a function behaves consistently as it approaches specific points. Continuity, in turn, is necessary for defining and interpreting the rate of change, especially in the context of derivatives. Together, these concepts form the foundation of differential calculus, enabling us to analyze and model change in various fields. • The study of limits, continuity, and rate of change requires careful attention to mathematical rigor and precision. Developing a deep understanding of these concepts involves not just computational skills but also the ability to reason logically and critically about how functions behave and how calculus can be used to describe and solve complex problems. 	<p>analytically?</p> <ul style="list-style-type: none"> • What are the implications of a function being discontinuous at a point, and how do different types of discontinuities (removable, jump, infinite) affect the behavior of a function? • What do limits at infinity tell us about the end behavior of a function, and how are they related to horizontal asymptotes?
<p>Students will know...</p> <ul style="list-style-type: none"> • The limit of a function $f(x)$ as x approaches a specific value c is the value that $f(x)$ gets closer to as x gets closer to c. • For the limit $\lim_{x \rightarrow c} f(x)$ to exist, the left-hand limit and the right-hand limit must both exist and be equal. • All the methods to manipulate a function algebraically to evaluate a limit 	<p>Students will be able to...</p> <ul style="list-style-type: none"> • Express limits symbolically with proper notation • Interpret limits • Estimate limits numerically and graphically • Evaluate limits algebraically • Interpret behavior of functions using limits

<ul style="list-style-type: none"> • Vertical and horizontal asymptotes are crucial for finding limits graphically and algebraically • There are 3 types of discontinuities they will need to know in this course: removable, jump, and infinite • The average rate of change gives the slope over an interval of time while the instantaneous rate of change gives the slope at an exact time • The meaning of all limit notations including one sided limits • The difference quotient finds the instantaneous rate of change at a certain x value 	<ul style="list-style-type: none"> • Analyze functions for points of discontinuity and identify the type of discontinuity • Find the slope of a curve at a certain point using the difference quotient and find the equation of the tangent and normal line at that point • Find the average rate of change of a given function over a given interval
Key Vocabulary: Limit, secant line, tangent line, normal line, asymptote, continuous function (continuity), discontinuity, average rate of change, removable, jump, infinite discontinuities, point slope form of a linear equation, average velocity, instantaneous velocity	
Assessment Evidence	
Performance Tasks: Summative Unit Assessment: AA1(9-12), TCC3(9-12), CCE4(9-12) Topics include: <ul style="list-style-type: none"> • Express limits symbolically with proper notation • Interpret limits • Estimate limits numerically and graphically • Evaluate limits algebraically • Interpret behavior of functions using limits • Analyze functions for points of discontinuity and identify the type of discontinuity • Find the slope of a curve at a certain point using the difference quotient and find the equation of the tangent and normal line at that point • Find the average rate of change of a given function over a given interval 	Other Evidence: Textbook problems from sections: 2.1: Rates of change and limits 2.2: Limits involving infinity 2.3 Continuity 2.4: Rates of change and tangent lines Any AP practice questions from textbook, AP Classroom, or other resources Activity: TCC2(9-12), CCE3(9-12), AA1(9-12) Groups will solve limit problems taken from UConn professor and compare solutions with their peers
Learning Plan	
In this unit students are introduced to the concept of a limit and the proper limit notation. They learn to evaluate limits of a function using algebraic, graphical, and numerical methods. They work with limits involving infinity and learn the properties of limits. They will learn what it means for a function to be continuous and how to find points of discontinuity and the different types of discontinuities. Most importantly, students will be	

introduced to the difference quotient and how it relates to the slope of a tangent/normal line and limits. They will learn the difference between the average and instantaneous rate of change.

Vision of A Learner Attributes: TCC2(9-12), CCE3(9-12), CCE4(9-12), AA1(9-12)

- Students will complete textbook practice problems working in groups during class and individually outside of class
- Class discourse on correct solutions to textbook problems - compare strategies and ask questions
- Group practice with UConn limit problems
- AP practice problems- Class discourse on correct solutions- compare strategies and ask questions
- Individual/group in class practice on concepts that will be measured on the assessment

Teacher Resources: Calculus textbook, UConn extra practice, AP Classroom

Unit 3 - Introduction to Derivatives

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards:

- **N.RN.A.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $[5^{1/3}]^3 = 5^{[(1/3) \times 3]}$ to hold, so $[5^{1/3}]^3$ must equal 5.
- **N.RN.A.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- **N.Q.A.2** Define appropriate quantities for the purpose of descriptive modeling.
- **A.SSE.B.3.A** Factor a quadratic expression to reveal the zeros of the function it defines.
- **F.IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F.IF.B.6** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph
- **F.BF.A.1** Write a function that describes a relationship between two quantities.
- **F.TF.A.3** . Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi - x$, $\pi + x$, and $2\pi - x$ in terms of their values for x , where x is any real number.

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- **TCC3(9-12):** I can integrate relevant information to produce multiple valid solutions.
- **CCE4(9-12):** I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.
- **AA1(9-12):** I can evaluate different approaches and justify the best pathway to success.

Understandings: Students will understand that...

Essential Questions:

<ul style="list-style-type: none"> • The derivative is a mathematical tool that measures how a function changes as its input changes. It provides a precise way to quantify rates of change, which is fundamental for understanding and modeling dynamic processes in various fields. • The derivative of a function at a specific point represents the slope of the tangent line to the function's graph at that point. • Differentiation involves a set of rules and techniques (such as the power rule, product rule, and quotient rule) that enable us to find the derivative of a wide variety of functions. Mastery of these techniques is essential for solving complex problems in calculus. • Derivatives have meaningful interpretations in different contexts, such as velocity in physics, marginal cost in economics, and rates of change in biology. Understanding these interpretations helps students see the practical applications of derivatives in the real world. • Higher-order derivatives, such as the second derivative, offer additional insights into the behavior of functions. • Not all functions are differentiable at every point. Situations where a function has sharp corners, cusps, or vertical tangents reveal important characteristics about the function and highlight the limits of differentiability. Understanding these cases is crucial for a complete grasp of the concept. • Derivatives are a cornerstone of calculus, deeply connected to other fundamental concepts like limits and integrals. They play a vital role in both theoretical and applied mathematics, making them an essential tool for solving a wide range of problems in science, engineering, economics, and beyond. 	<ul style="list-style-type: none"> • What does the derivative of a function represent, and how is it connected to the concept of the rate of change and how is it used in the context of physics (velocity) and economics (marginal cost)? • What are the basic rules and techniques for differentiating functions, such as the power rule, product rule, and quotient rule? • What are higher-order derivatives, and what do they tell us about the behavior of a function, such as acceleration in the context of motion? • How does the concept of the derivative apply to functions that are not linear, and what does it tell us about the slope of the tangent line at a point on the graph of a non-linear function?
<p>Students will know...</p> <ul style="list-style-type: none"> • When to use the the power rule, product rule, and quotient rule to find a derivative • Higher order derivatives represent a rate of change of a rate of change • Derivative rules are just a quicker, easier way to find an instantaneous rate of change • The derivative represents the slope of the tangent line at a 	<p>Students will be able to...</p> <ul style="list-style-type: none"> • Find derivatives using the proper rules • Find higher order derivatives • Calculate position, displacement, average and instantaneous velocity, acceleration, and speed of an object over time • Calculate derivatives involving the trigonometric functions and use trig identities to manipulate them

<p>certain point and can be used to write the equation of the tangent line or the normal line</p> <ul style="list-style-type: none"> • The difference between position, displacement, average and instantaneous velocity, acceleration, and speed of an object over time • That a function will not have a derivative at a cusp, corner, or vertical tangent. • Keywords in an application problem indicate to find the derivative. 	<ul style="list-style-type: none"> • Analyze the continuity of a function including piecewise functions • Represent derivatives graphically and describe the motion of an object based on a graph • Write the equation of a tangent/normal line to a curve using the derivative. • Use a derivative to interpret a real life situation
<p>Key Vocabulary: Derivative, displacement, position, average velocity, instantaneous velocity, acceleration, speed, marginal cost or revenue, differentiability, discontinuity, cusp, corner, product rule, power rule, quotient rule</p>	
<p style="text-align: center;">Assessment Evidence</p>	
<p>Performance Tasks: Summative Unit Assessment: AA1(9-12), TCC3(9-12), CCE4(9-12) Topics include:</p> <ul style="list-style-type: none"> • Find derivatives using the proper rules • Find higher order derivatives • Calculate position, displacement, average and instantaneous velocity, acceleration, and speed of an object over time • Calculate derivatives involving the trigonometric functions and use trig identities to manipulate them • Analyze the continuity of a function including piecewise functions • Represent derivatives graphically and describe the motion of an object based on a graph • Write the equation of a tangent/normal line to a curve using the derivative. • Use a derivative to interpret a real life situation 	<p>Other Evidence: Textbook problems from sections: 3.1 Derivative of a function 3.2 Differentiability 3.3 Rules for differentiation 3.4 Velocity and other rates of change</p> <p>Students work individually and in groups on extra practice problems from the Larson Calculus textbook.</p> <p>Any AP practice questions from textbook, AP Classroom, or other resources</p>
<p style="text-align: center;">Learning Plan</p>	
<p>In this unit, students are introduced to the concept of a derivative and how it is related to limits. They learn basic derivative rules including those for products, quotients, and trigonometric functions. They will see how derivatives are related to motion problems involving position, velocity, and acceleration, and other application problems. They learn how to analyze a function for continuity and differentiability. They will find equations of</p>	

tangent/normal lines using the new derivative techniques. They will begin to analyze graphs and represent motion graphically.

Vision of A Learner Attributes: TCC3(9-12), CCE4(9-12), AA1(9-12)

- Students will complete textbook practice problems working in groups during class and individually outside of class
- Class discourse on correct solutions to textbook problems - compare strategies and ask questions
- Group/individual practice with Larson problems
- AP practice problems- Class discourse on correct solutions including comparing strategies and asking questions
- Individual/group in class practice on concepts that will be measured on the assessment

Teacher Resources: Finney calculus textbook, Larson textbook, AP Classroom, teacher created extra practice

Unit 4 - Derivatives

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

- **N.RN.A.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- **A.APR.D.7** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- **F.BF.A.1.C** Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
- **F.BF.B.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- **TCC3(9-12):** I can integrate relevant information to produce multiple valid solutions.
- **CCE4(9-12):** I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.
- **AA1(9-12):** I can evaluate different approaches and justify the best pathway to success.

Understandings: Students will understand that...

- The chain rule is essential for finding the derivative of a composite function, where one function is nested inside another. Understanding the chain rule allows students to differentiate more complex functions by systematically applying the rule to each layer of the function.
- Implicit differentiation is a powerful tool for finding the derivative of equations where the dependent and independent variables

Essential Questions:

- What is the chain rule and how can it be applied to differentiate functions that are composed of multiple layers of functions (e.g., trigonometric, exponential, or logarithmic functions within other functions)?
- In what situations is the chain rule the most effective method for finding the derivative, and how does it simplify the differentiation process?

<p>are intertwined and cannot be easily separated. This method extends the ability to differentiate beyond explicitly defined functions, making it essential for handling more complex equations.</p> <ul style="list-style-type: none"> • Implicit differentiation provides insights into the relationships between variables that are not easily solved for one variable in terms of the other. This technique is particularly useful in scenarios where the relationship between variables is defined implicitly, such as in circles, ellipses, and other non-linear curves. • Logarithmic differentiation is an effective method for differentiating functions that involve complex products, quotients, or powers. By taking the natural logarithm of both sides of an equation, this technique simplifies differentiation, especially when dealing with functions that are difficult to differentiate using standard rules. • Mastery of the chain rule, implicit differentiation, and logarithmic differentiation equips students with a flexible and adaptable approach to differentiation. These techniques allow students to handle a wide variety of functions and equations, expanding their problem-solving toolkit in calculus. • Applying these differentiation techniques requires careful attention to detail and precision in mathematical reasoning. Understanding when and how to use each technique fosters a deeper appreciation for the rigor involved in calculus and prepares students for more advanced mathematical studies. • Parametric equations provide a powerful way to represent curves that cannot be easily described using a single function in Cartesian form. By expressing both the x and y coordinates as functions of a third parameter (often time, t), parametric equations allow for the modeling of more complex and dynamic relationships between variables. 	<ul style="list-style-type: none"> • What is implicit differentiation, and why is it necessary for differentiating equations where the variables are not explicitly solved for one in terms of the other? • How do we find the derivative of a parametric curve, and what does this derivative represent in terms of the curve's slope or rate of change?
<p>Students will know...</p> <ul style="list-style-type: none"> • The chain rule is necessary to find the derivative of a composite function • To use implicit differentiation when an equation is not solved for 	<p>Students will be able to...</p> <ul style="list-style-type: none"> • Use the chain rule to evaluate derivatives, including higher order derivatives

<ul style="list-style-type: none"> a variable All the derivatives of inverse trig functions and exponential and logarithmic functions To use logarithmic differentiation when the function has an exponent raised to another exponent Recognize a parametric equation when they see equations for x and y in terms of t. To find the equation of a tangent line you need a slope, given by the derivative which may include the chain rule, and a point on the curve. 	<ul style="list-style-type: none"> Find derivatives using implicit differentiation, including higher order derivatives Evaluate derivatives involving the inverse trigonometric functions and exponential and logarithmic functions Find a derivative using logarithmic differentiation Find the derivative of a parametric equation Find the equation of the line tangent to the curve where using the chain rule is necessary
Key Vocabulary: Chain rule, parametric equation, implicit differentiation, logarithmic differentiation	
Assessment Evidence	
Performance Tasks: Summative Unit Assessment: AA1(9-12), TCC3(9-12), CCE4(9-12) Topics include: <ul style="list-style-type: none"> Use the chain rule to evaluate derivatives, including higher order derivatives Find derivatives using implicit differentiation, including higher order derivatives Evaluate derivatives involving the inverse trigonometric functions and exponential and logarithmic functions Find a derivative using logarithmic differentiation Find the derivative of a parametric equation Find the equation of the line tangent to the curve where using the chain rule is necessary 	Other Evidence: Textbook problems from sections: <ul style="list-style-type: none"> 4.1 Chain rule 4.2 Implicit differentiation 4.3 Derivatives of inverse trigonometric functions 4.4 Derivatives of exponential and logarithmic functions Any AP practice questions from textbook, AP Classroom, or other resources
Learning Plan	
<p>In this unit, students are introduced to more difficult methods of differentiation including the chain rule, implicit differentiation, inverse trigonometric functions, and exponential/logarithmic derivatives. Higher order derivatives and parametric equations are also included. Practice is crucial for these methods.</p> <p>Vision of A Learner Attributes: TCC3(9-12), CCE4(9-12), AA1(9-12)</p> <ul style="list-style-type: none"> Students will complete textbook practice problems working in groups during class and individually outside of class Class discourse on correct solutions to textbook problems - compare strategies and ask questions 	

- AP practice problems- Class discourse on correct solutions- compare strategies and ask questions
- Individual/group in class practice on concepts that will be measured on the assessment

Teacher Resources: Finney Calculus textbook, AP Classroom, teacher created extra practice problems

Unit 5 - The Derivative as a Tool for Analysis

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

- **A.SSE.B.3.A** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A.APR.B.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **A.REI.A.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- **F.IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
- **F.IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **F.IF.C.7.E** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- **TCC3(9-12):** I can integrate relevant information to produce multiple valid solutions.
- **CCE4(9-12):** I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.
- **AA1(9-12):** I can evaluate different approaches and justify the best pathway to success.

<p>Understandings: Students will understand that...</p> <ul style="list-style-type: none"> • The derivative of a function at a specific point represents the slope of the tangent line to the function's graph at that point. This connection allows us to understand the local behavior of functions, such as whether they are increasing or decreasing. • The derivative provides critical information about the behavior of functions, including identifying intervals of increase and decrease, finding local maxima and minima, and determining points of inflection. This understanding is key to analyzing and graphing functions. • Higher-order derivatives, such as the second derivative, offer additional insights into the behavior of functions. For example, the second derivative indicates concavity and helps identify the nature of critical points, which is important for understanding the overall shape of a function's graph. • The MVT (Mean Value Theorem) guarantees that if a function is continuous on a closed interval and differentiable on the open interval, there exists at least one point where the derivative (instantaneous rate of change) matches the average rate of change over the entire interval. This concept is key to understanding the behavior of functions, particularly in identifying where changes occur most significantly. • The EVT (Extreme Value Theorem) shows that maximum and minimum values of a function can occur either at critical points (where the derivative is zero or undefined) or at the endpoints of the interval. Understanding this helps students develop strategies for finding extreme values and analyzing the overall behavior of a function. 	<p>Essential Questions:</p> <ul style="list-style-type: none"> • What is the relationship between the derivative and the graph of a function, particularly in terms of identifying local maxima, minima, and points of inflection? • How can the second derivative be used to analyze the concavity of a function and the nature of critical points? • What does the Mean Value Theorem reveal about the relationship between the average rate of change and the instantaneous rate of change of a function, and how can this understanding be applied to analyze the behavior of functions in real-world contexts? • How does the Extreme Value Theorem guarantee the existence of maximum and minimum values for continuous functions on a closed interval, and why is this important for understanding the behavior of functions in both theoretical and practical applications?
<p>Students will know...</p> <ul style="list-style-type: none"> • The meaning of the extreme value theorem and the mean value theorem and how it can help to analyze a graph • How to use the derivative to analyze and interpret the properties of a function including intervals of increasing/decreasing, extreme values, intervals of concavity and inflection points. • When and how to use a sign chart • The meaning derived from the second derivative test • How to analyze a given graph 	<p>Students will be able to...</p> <ul style="list-style-type: none"> • Apply the Mean Value Theorem to describe the behavior of a function over an interval • Use the extreme Value Theorem to analyze properties of a function including extrema • Use the derivative to analyze a function including extrema, intervals of increasing/decreasing, concavity, and points of inflection, which will include making a sign chart

	<ul style="list-style-type: none"> Analyze a given graph in order to find intervals of increasing/decreasing and concavity.
Key Vocabulary: Extrema, critical point, concavity, point of inflection, sign chart, 2nd derivative test, mean value theorem, extreme value theorem	
Assessment Evidence	
Performance Tasks: Summative Unit Assessment: AA1(9-12), TCC3(9-12), CCE4(9-12) Topics include: <ul style="list-style-type: none"> Apply the Mean Value Theorem to describe the behavior of a function over an interval Use the extreme Value Theorem to analyze properties of a function including extrema Use the derivative to analyze a function including extrema, intervals of increasing/decreasing, concavity, and points of inflection, which will include making a sign chart Analyze a given graph in order to find intervals of increasing/decreasing and concavity. 	Other Evidence: Textbook problems from sections: 5.1 Extreme values of functions 5.2 Mean value theorem 5.3 Connecting f' and f'' with the graph of f Any AP practice questions from textbook, AP Classroom, or other resources
Learning Plan	
<p>In this unit, we drive deeper into the connections between the derivatives and their graphs. We compare the function with its first and second derivative graphs. The students are introduced to two very important theorems: the extreme value and the mean value theorems, and how they can be used to justify an answer. Most importantly, students learn how to use derivatives to analyze functions and find their extrema, intervals of increasing/decreasing, concavity, and points of inflection.</p> <p>Vision of A Learner Attributes: TCC3(9-12), CCE4(9-12), AA1(9-12)</p> <ul style="list-style-type: none"> Students will complete textbook practice problems working in groups during class and individually outside of class Class discourse on correct solutions to textbook problems - compare strategies and ask questions AP practice problems- Class discourse on correct solutions- compare strategies and ask questions Individual/group in class practice on concepts that will be measured on the assessment 	
Teacher Resources: Finney Calculus textbook, AP Classroom, teacher created extra practice problems	

Unit 6 - Applications and Extensions of the Derivative

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

- **A.SSE.B.3.A** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A.APR.B.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **A.REI.A.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- **G.GMD.A.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- **G.MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder)
- **G.MG.A.3** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost)

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- **TCC1(9-12):** I can ask purposeful, insightful questions to find a variety of innovative solutions.
- **CCE3(9-12):** I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.
- **CCE4(9-12):** I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.
- **TI3(9-12):** I can formulate and investigate probing questions to further my learning.

Understandings: Students will understand that...

- Optimization involves finding the best possible solution within given constraints, whether it's maximizing or minimizing a particular quantity. This process is crucial for decision-making in real-world scenarios, where resources are often limited and efficiency is key.
- In optimization, critical points—where the derivative is zero or

Essential Questions:

- How do we identify an optimization problem, and what strategies in calculus can we use to find the maximum or minimum value of a function in a given context?
- What are some real-world examples of optimization problems, and how does understanding the underlying mathematics help in making informed decisions?

<p>undefined—are essential for identifying potential maximum or minimum values. Additionally, analyzing boundary values within a domain is crucial for determining the optimal solution.</p> <ul style="list-style-type: none"> • Optimization problems are prevalent across various fields, including economics, engineering, and biology. Understanding how to formulate and solve these problems using calculus is fundamental to applying mathematical concepts in real-world situations. • Related rates problems highlight how the rate of change of one quantity is connected to the rate of change of another, often through implicit relationships. This understanding is key to solving problems where multiple variables change simultaneously. • Implicit differentiation is a powerful tool for solving related rates problems, as it allows for the differentiation of equations where variables are interdependent. Mastery of this technique is essential for analyzing systems where direct differentiation is not possible. • Related rates problems are deeply rooted in real-world scenarios, such as the changing dimensions of a physical object, the speed of a moving object, or the growth rates in biological systems. • Linearization provides a way to approximate the behavior of a function near a given point using the tangent line. This method is valuable for making quick, reasonably accurate predictions about a function's value without requiring complex calculations. • Solving problems in optimization, related rates, and linearization requires critical thinking and careful mathematical reasoning. Mastery of these concepts develops students' problem-solving skills and their ability to apply mathematical reasoning to real-world situations. 	<ul style="list-style-type: none"> • How can related rates be used to model and analyze situations where two or more quantities are changing over time, such as the flow of water into a tank or the speed of a shadow moving across the ground? • What is linearization, and how can it be used to approximate the value of a function near a given point?
<p>Students will know...</p> <ul style="list-style-type: none"> • The steps to solving an optimization or related rates problem. • The derivative is used to find maximum or minimum values in an application problem • A linearization problem will be solved just like finding the equation of a tangent line 	<p>Students will be able to...</p> <ul style="list-style-type: none"> • Solve application problems involving optimization and related rates • Model a real world situation with calculus and explain how the calculus can be used to solve a problem • Find the linearization of a function at a certain point

<ul style="list-style-type: none"> • Basic geometry formulas needed to solve these application problems • The importance of drawing a figure in these problems • All the methods of differentiation needed to solve these • The importance of answering the question at the end of a problem 	
Key Vocabulary: Optimization, linearization, related rate	
Assessment Evidence	
Performance Tasks: Summative Unit Assessment: AA1(9-12), CCE4(9-12) Topics include: <ul style="list-style-type: none"> • Solve application problems involving optimization and related rates • Model a real world situation with calculus and explain how the calculus can be used to solve a problem • Find the linearization of a function at a certain point 	Other Evidence: Textbook problems from sections: 5.4 Modeling and optimization 5.5 Linearization 5.6 Related rates Any AP practice questions from textbook, AP Classroom, or other resources Activity: TCC1(9-12), TCC2(9-12) CCE3(9-12), TI3(9-12) Groups will solve optimization and related rates problems taken from UConn professor and compare solutions with their peers
Learning Plan	
<p>In this unit, students use derivatives to solve real life application problems. First are modeling and optimization problems such as designing a can that uses the least amount of material, or maximizing profit. Secondly, they learn related rates problems which use derivatives to find the rate of change of a quantity over time, such as how fast the volume of a balloon changes based on the rate of change of the radius. Students also solve linearization problems which is a new way of using the derivative to find the equation of a tangent line.</p> <p>Vision of A Learner Attributes: TCC1(9-12), CCE3(9-12), CCE4(9-12), TI3(9-12)</p> <ul style="list-style-type: none"> • Students will complete textbook practice problems working in groups during class and individually outside of class • Class discourse on correct solutions to textbook problems - compare strategies and ask questions • Extra in class group practice from UConn professor and Larson book-compare strategies and ask questions • AP practice problems- Class discourse on correct solutions- compare strategies and ask questions • Individual/group in class practice on concepts that will be measured on the assessment 	
Teacher Resources: Finney Calculus textbook, extra practice from Larson textbook, AP Classroom, UConn practice problems and guided note	

taking sheets



Unit 7 - The Integral

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

- **A.SSE.B.3.A** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A.APR.B.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **G.MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder)
- **F.IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- **TCC2(9-12):** I can evaluate evidence from multiple perspectives, and recognize their limitations and implications, in order to justify new conclusions.
- **CCE2(9-12):** I can give and receive actionable and relevant feedback with openness to be able to determine meaningful revisions for success.
- **CCE4(9-12):** I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.

Understandings: Students will understand that...

- Integration represents the process of accumulating quantities, such as area under a curve, total distance traveled, or accumulated volume. This understanding helps students grasp how integration extends the concept of summation to continuous functions and provides a way to measure quantities that vary

Essential Questions:

- What is an integral and how can we interpret the definite integral as the total accumulation of quantities, such as distance traveled or area, and what does this reveal about the behavior of functions?
- How are integration and differentiation related through the

<p>over an interval.</p> <ul style="list-style-type: none"> • The Fundamental Theorem of Calculus links integration and differentiation, showing that integration can be understood as the reverse process of differentiation. This connection reveals how finding antiderivatives and computing definite integrals are interconnected, deepening students' understanding of both operations. • The definite integral calculates the total accumulation over a specific interval, while the indefinite integral represents the general antiderivative of a function. Understanding the distinction and relationship between these two types of integrals is crucial for applying integration techniques to a wide range of problems. • Methods such as the Trapezoidal Rule and Riemann sums provide ways to approximate definite integrals when an exact solution is difficult to obtain. These numerical methods highlight the practical importance of integration and the role of approximation in real-world applications. • The definition of a definite integral involves taking the limit of Riemann sums as the number of subintervals approaches infinity. This concept underscores the role of limits in calculus and the transition from discrete approximations to continuous integration. • The integral can be interpreted graphically as the area under a curve or the net area between the curve and the x-axis. This visual understanding aids in conceptualizing how integration represents the accumulation of quantities and helps in solving problems involving areas and net change. 	<p>Fundamental Theorem of Calculus, and what does this relationship tell us about the inverse nature of these operations?</p> <ul style="list-style-type: none"> • What are some examples of how integrals are applied in fields such as physics, engineering, and economics, and how does understanding these applications enhance our ability to solve practical problems? • How do Riemann sums and the Trapezoidal rule provide a method for approximating the area under a curve, and how does this approach lead to the definition of the definite integral?
<p>Students will know...</p> <ul style="list-style-type: none"> • That a Riemann sum approximates an integral • That an integral represents the area under a curve • The difference between a definite and indefinite integral • Some integrals can be evaluated using geometry • The properties of integrals • The process of integration and differentiation are inverses of each other 	<p>Students will be able to...</p> <ul style="list-style-type: none"> • Calculate a Riemann sum (left, right, or midpoint) to approximate the value of an integral or a real life situation • Use proper notation with integrals • Evaluate some integrals using geometry • Evaluate a definite integral using the antiderivative • Find the average value of a function over an interval using an integral • Approximate the area under a curve using the Trapezoidal Rule.

<ul style="list-style-type: none"> • The trapezoidal rule is the most accurate approximation of the area under a curve • The average value of a function is found by dividing the value of the integral by the interval • The position, velocity, and acceleration of a particle over time by using the concept of an integral and what it looks like on a graph 	<ul style="list-style-type: none"> • Approximate integrals that are represented graphically, numerically, algebraically, and verbally. • Solve motion application problems using the concept of the integral including position, velocity, and acceleration
Key Vocabulary: Integral, Riemann sum, antiderivative, trapezoidal rule, indefinite and definite integral, summation notation	
Assessment Evidence	
Performance Tasks: Summative Unit Assessment: AA1(9-12), TCC3(9-12), CCE4(9-12) Topics include: <ul style="list-style-type: none"> • Calculate a Riemann sum (left, right, or midpoint) to approximate the value of an integral or a real life situation • Evaluate some integrals using geometry • Evaluate a definite integral using the antiderivative • Find the average value of a function over an interval using an integral • Approximate the area under a curve using the Trapezoidal Rule. • Approximate integrals that are represented graphically, numerically, algebraically, and verbally. • Solve motion application problems using the concept of the integral including position, velocity, and acceleration 	Other Evidence: Textbook problems from sections: 6.1 Estimating with finite sums 6.2 Definite integrals 6.3 Definite integrals and antiderivatives 6.4 Fundamental Theorem of calculus 6.5 Trapezoidal Rule Any AP practice questions from textbook, AP Classroom, or other resources
Learning Plan	
<p>In this unit, students are introduced to the concept of the integral and proper use of the notation. They start by finding ways to estimate the value of an integral using rectangles and how this concept is related to the limit. They are then introduced to a definite integral and ways to calculate it including geometry and some simple integral rules and properties. They are introduced to the term antiderivative and the concept of average value of a function. They also learn the fundamental theorem of calculus in this unit and start to analyze integrals that are represented graphically, numerically, algebraically, and verbally. They will connect the concept of the integral to motion problems including position, velocity, and acceleration. At the end of the unit, they will approximate integrals using the Trapezoidal Rule.</p> <p>Vision of A Learner Attributes: TCC2(9-12), CCE2(9-12), CCE4(9-12)</p> <ul style="list-style-type: none"> • Use guided note-taking sheets from UConn professor for initial concepts • Students will complete textbook practice problems working in groups during class and individually outside of class 	

- Class discourse on correct solutions to textbook problems - compare strategies and ask questions
- AP practice problems- Class discourse on correct solutions- compare strategies and ask questions
- Individual/group in class practice on concepts that will be measured on the assessment

Teacher Resources: Finney Calculus textbook, also problems from previous edition of Finney textbook, guided note taking sheet from UConn professor on Riemann Sums, AP Classroom

Unit 8 - Differential Equations and Mathematical Modeling

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

- **N.RN.A.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- **A.REI.A.2** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- **F.BF.B.5** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- **TCC1(9-12):** I can ask purposeful, insightful questions to find a variety of innovative solutions.
- **TCC3(9-12):** I can integrate relevant information to produce multiple valid solutions.
- **CCE2(9-12):** I can give and receive actionable and relevant feedback with openness to be able to determine meaningful revisions for success.
- **CCE3(9-12):** I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.
- **CCE4(9-12):** I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.

Understandings: Students will understand that...

- Slope fields provide a graphical representation of the solutions to a first-order differential equation, allowing students to visualize the direction and behavior of solution curves even when explicit solutions are difficult to obtain.
- Slope fields are closely related to the differential equation they represent, showing how the slope of the solution curve at any

Essential Questions:

- What is a slope field, how does it visually represent the solutions to a differential equation, and how can it help us understand the general behavior of solutions to a differential equation, particularly when an explicit solution is difficult or impossible to find?
- What is a differential equation, how does it represent the

given point corresponds to the value of the derivative at that point. Understanding this relationship reinforces the connection between graphical and analytical approaches to differential equations.

- Differential equations are powerful tools for modeling a wide range of dynamic systems in mathematics, science, and engineering. They describe how a quantity changes over time or space based on its current state, providing insights into the behavior of physical, biological, and economic systems.
- Various methods exist for solving differential equations but this course will focus on separation of variables. Understanding this method allows students to find both general and particular solutions.
- Initial conditions play a crucial role in determining the specific solution to a differential equation from a family of possible solutions. Understanding how initial conditions influence the outcome of a differential equation is key to accurately modeling real-world scenarios.
- Analytical solutions to differential equations provide precise mathematical expressions for the behavior of a system, while graphical methods, such as slope fields, offer a visual understanding of these solutions. Both approaches are interconnected and complement each other in solving and interpreting differential equations.
- U-substitution is a powerful method for simplifying integrals, particularly those involving composite functions. By substituting part of the integrand with a single variable (u), students can transform a complex integral into a simpler one, making it easier to evaluate.
- U-substitution is the reverse process of the chain rule in differentiation. This connection reinforces the idea that integration and differentiation are inverse operations, and it provides a systematic approach to handling integrals that involve composite functions.
- U-substitution is applicable to both definite and indefinite integrals. In the case of definite integrals, understanding how to adjust the limits of integration after substitution is crucial for obtaining the correct result.

relationship between a function and its derivatives and how can they be used to model real-world situations, such as population growth, motion, or chemical reactions?

- What are the methods for solving first-order differential equations, such as separation of variables and integrating factors, and how do these methods apply to different types of differential equations?
- In what types of integrals is u-substitution most useful, and how can this technique be applied to solve integrals involving trigonometric, exponential, or logarithmic functions?

<ul style="list-style-type: none"> • The process of choosing an appropriate substitution requires critical thinking and pattern recognition. Students must analyze the structure of the integrand to identify the best substitution, which fosters deeper problem-solving skills and mathematical intuition. • After integrating with respect to u, it is essential to revert the integral back to the original variable. This step ensures that the solution is expressed in terms of the original problem, emphasizing the importance of understanding the relationship between the substitution and the original function. 	
<p>Students will know...</p> <ul style="list-style-type: none"> • Only certain types of differential equations can be solved by separation of variables • The difference between a general and initial value differential equation • A slope field is constructed by graphing the slope of the differential equation at particular points in the x-y plane • When to use u-substitution to solve an integral • All the steps to solve an integral using u-substitution and be familiar with the many strategies they have practiced • Differential equations can be used to solve application problems related to exponential growth and decay • When solving an indefinite integral you must add the constant of integration to the solution 	<p>Students will be able to...</p> <ul style="list-style-type: none"> • Analyze differential equations to obtain general and specific solutions • Construct a slope field for a given differential equation. • Solve an integral (definite and indefinite) using integration by u-substitution, including many different strategies • Model an exponential growth/decay situation with a differential equation and solve • Match a slope field to the appropriate differential equation
<p>Key Vocabulary: Differential equation, slope field, indefinite integral, constant of integration, u-substitution, separation of variables</p>	
<p style="text-align: center;">Assessment Evidence</p>	
<p>Performance Tasks: Summative Unit Assessment: AA1(9-12), TCC3(9-12), CCE4(9-12) Topics include:</p> <ul style="list-style-type: none"> • Analyze differential equations to obtain general and specific solutions • Construct a slope field for a given differential equation. • Solve an integral (definite and indefinite) using integration by u-substitution, including many different strategies 	<p>Other Evidence: Textbook problems from sections: 7.1 Slope Fields 7.2 Antidifferentiation by substitution 7.4 Exponential growth and decay</p> <p>U-substitution practice: TCC1(9-12), CCE2(9-12), CCE3(9-12) Students will work in groups to solve integrals by u-substitution</p>

<ul style="list-style-type: none"> • Model an exponential growth/decay situation with a differential equation and solve • Match a slope field to the appropriate differential equation 	Any AP practice questions from textbook, AP Classroom, or other resources
<p style="text-align: center;">Learning Plan</p> <p>In this unit students are introduced to differential equations and how to obtain general and specific solutions to them as well as slope fields and how they represent an equation. Most importantly, they are introduced to the concept of u-substitution to solve more difficult integrals. This involves many different strategies and types of problems. They will learn how to solve differential equations by separation of variables and to solve application problems involving exponential growth and decay, for example, finding the amount of time it will take for a radioactive material to decay to a certain amount of its original value.</p> <p>Vision of A Learner Attributes: TCC1(9-12),TCC3(9-12), CCE2(9-12), CCE3(9-12), CCE4(9-12)</p> <ul style="list-style-type: none"> • Students will complete textbook practice problems working in groups during class and individually outside of class • Class discourse on correct solutions to textbook problems - compare strategies and ask questions • Extra practice from Larson book- work in groups-class discourse on correct solutions- compare strategies and ask questions • AP practice problems- Class discourse on correct solutions- compare strategies and ask questions • Individual/group in class practice on concepts that will be measured on the assessment <p>Teacher Resources: Finney Calculus textbook, Larson textbook, teacher created extra practice and review, AP Classroom</p>	

Unit 9 - Applications of Definite Integrals

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

- **N.RN.A.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents.
- **N.Q.A.2** Define appropriate quantities for the purpose of descriptive modeling.
- **A.SSE.B.3.A** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A.CED.A.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. For example, rearrange Ohm's law $V = IR$ to highlight resistance R .
- **F.IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
- **G.GMD.A.3** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
- **G.GMD.B.4** Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
- **G.MG.A.1** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- **CCE4(9-12):** I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.
- **AA1(9-12):** I can evaluate different approaches and justify the best pathway to success.
- **TCC3(9-12):** I can integrate relevant information to produce multiple valid solutions.

Understandings: Students will understand that...

- The definite integral is a powerful tool for accumulating

Essential Questions:

- How can the concept of definite integrals be used to find the

<p>quantities, representing the total accumulation of a rate of change over an interval. It can model real-world phenomena such as distance traveled, area, volume, and other quantities that can be interpreted as the accumulation of a changing rate.</p> <ul style="list-style-type: none"> Integrals can be used to find geometric quantities such as the area under a curve, the area between curves, and the volume of solids formed by revolution or with known cross-sectional areas. Understanding integrals as geometric constructs provides a concrete foundation for their application in various fields. Integrals can be represented and interpreted in multiple ways, including graphically, algebraically, and numerically. Understanding these different representations enables students to approach problems from various angles and apply the most effective method for a given situation. In applications of integrals, the limits of integration are crucial for defining the interval over which accumulation occurs. Whether calculating net change, total area, or volume, understanding the significance of these limits is essential for correctly setting up and interpreting the integral. 	<p>total accumulation of a quantity such as distance, area, and volume?</p> <ul style="list-style-type: none"> How can integrals be used to find the area of a region bounded by multiple functions? What is the difference between the disk method and the washer method, and when is each appropriate to use? How do you set up an integral to find the volume of a solid when rotating a curve about the x-axis or y-axis? How do you set up an integral to calculate the volume of a solid with known cross sections, such as squares, rectangles, triangles, or semicircles?
<p>Students will know...</p> <ul style="list-style-type: none"> The definite integral can be used to express information about accumulation and net change in many applied contexts Definite integrals can be used to find the area between two curves and finding the points of intersection is a necessary part of the problem The shortcuts for finding cross-sections of squares, semi-circles, and equilateral triangles Volumes of solids with known cross sections, including discs and washers, can be calculated with definite integrals How to distinguish a disk versus a washer volume problem Drawing a picture is an important first step when trying to visualize a volume of revolution or a cross section. The type of integral that represents parts of the motion application problem 	<p>Students will be able to...</p> <ul style="list-style-type: none"> Use a definite integral to solve application problems involving accumulation and net change Use a definite integral to find the area between two curves Visualize a three-dimensional object formed by the revolution of a function about a vertical or horizontal axis and use a definite integral to calculate the volume of the object, including disks and washers Use a definite integral to find the volume of a three dimensional shape formed by cross sections between two curves Solve motion problems with integrals including finding the direction of a particle, the displacement, and the distance traveled, given a graph or equation

Key Vocabulary: Solid of revolution, cross section, displacement

Assessment Evidence

Performance Tasks:

Summative Unit Assessment: AA1(9-12), TCC3(9-12), CCE4(9-12)

Topics include:

- Use a definite integral to solve application problems involving accumulation and net change
- Use a definite integral to find the area between two curves
- Visualize a three-dimensional object formed by the revolution of a function about a vertical or horizontal axis and use a definite integral to calculate the volume of the object, including disks and washers
- Use a definite integral to find the volume of a three dimensional shape formed by cross sections between two curves
- Solve motion problems with integrals including finding the direction of a particle, the displacement, and the distance traveled, given a graph or equation

Other Evidence:

Textbook problems from sections:

- 8.1 Accumulation and net change
- 8.2 Areas in the plane
- 8.3 Volumes

Any AP practice questions from textbook, AP Classroom, or other resources

Learning Plan

In this last unit, students will solve application problems using integrals. They will solve motion problems using integrals, for example finding displacement and total distance traveled, and using integrals to represent accumulation such as consumption of oil in the US over a certain time. Next they will find the area under a curve using an integral, given a graph or an equation. Lastly, they will learn to visualize a three-dimensional object formed by the revolution of a function about a vertical or horizontal axis and use a definite integral to calculate the volume of the object, including disks and washers. This will also include finding the volume of a three dimensional shape formed by cross sections between two curves.

Vision of A Learner Attributes: AA1(9-12), TCC3(9-12), CCE4(9-12)

- Students will complete textbook practice problems working in groups during class and individually outside of class
- Class discourse on correct solutions to textbook problems - compare strategies and ask questions
- Extra practice from Larson book- work in groups-class discourse on correct solutions- compare strategies and ask questions
- AP practice problems- Class discourse on correct solutions- compare strategies and ask questions
- Individual/group in class practice on concepts that will be measured on the assessment

Teacher Resources: Finney Calculus textbook, Larson textbook, teacher created extra practice and review, AP Classroom