Calculus Summer Review

For students who have *completed* Precalculus and are **entering** Calculus

Reviewing key concepts from Algebra 2 and Precalculus is an excellent way to be fully prepared for concepts in Calculus that require prerequisite skills. The following packet will help you practice and self-assess any concepts that you may want to spend extra time on before the start of school.

You should expect to submit this completed packet on the first day of school and plan to be assessed on these skills at the beginning of the school year. An answer key is provided; however, ensure your work is shown to receive full credit. If you would like additional resources to support your practice, we recommend Khan Academy as a great first step.

Revised 3/25/2025 FULL Name ___________________________________

You should be able to recite the following facts from memory:

1. To find x-intercepts, set $y = 0$ and solve the equation for x

2. To find y-intercepts, set $x = 0$ and solve the equation for y

3. The slope m of a non-vertical line passing through the points (x_1, y_1) and (x_2, y_2) is

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

4. Summary – Equations of Lines

   a. General form: $Ax + By + C = 0$

   b. Vertical line: $x = a$

   c. Horizontal line: $y = b$

   d. Point-slope form: $y - y_1 = m(x - x_1)$

   e. Slope-intercept form: $y = mx + b$

5. Summary – Functions can be defined:

   a. Implicitly: $x^2 + 2y = 2$

   b. Explicitly (solved for y): $y = \frac{2 - x^2}{2}$

   c. Using function notation: $f(x) = \frac{2 - x^2}{2}$

6. Basic types of transformations $(c > 0)$

   Original graph: $y = f(x)$

   Horizonal shift c units to the right: $y = f(x - c)$

   Horizonal shift c units to the left: $y = f(x + c)$

   Vertical shift c units downward: $y = f(x) - c$

   Vertical shift c units upward: $y = f(x) + c$

   Reflection about the y-axis: $y = f(-x)$

7. Terms: polynomial, degree of a polynomial, coefficient, leading coefficient, constant term, rational function,

8. Definition of a composite function: Let f and g be functions. Then function given by $f(g(x))$ is called the composition of f with g, and can also be written $(f \circ g)(x)$. The domain of the composition is the set of all x in the domain of g such that $g(x)$ is in the domain of f.

9. The distance between two points (x_1, y_1) and (x_2, y_2) in a plane is given by

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

10. The midpoint between two points (x_1, y_1) and (x_2, y_2) in a plane is given by

$$\left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$$

11. Standard form of the equation of a circle with center (h, k) and radius r:

$$(x - h)^2 + (y - k)^2 = r^2$$

12. Definitions of the trigonometric functions:

a. In a right triangle:

$$\sin\theta = \frac{opp}{hyp}, \cos\theta = \frac{adj}{hyp}, \tan\theta = \frac{opp}{adj}$$

b. In the unit circle

$$x = \cos\theta,\ y = \sin\theta, \frac{y}{x} = \tan\theta$$

c. Circular functions with radius r

$$r = \sqrt{x^2 + y^2}, y = r\sin\theta, x = r\cos\theta$$

$$\tan\theta = \frac{y}{x}$$

13. Pythagorean Identities

$$\sin^2\theta + \cos^2\theta = 1 \qquad \tan^2\theta + 1 = \sec^2\theta \qquad 1 + \cot^2\theta = \csc^2\theta$$

14. Sum and Difference Formulas

$$\sin(\alpha \pm \beta) = \sin\alpha\cos\beta \pm \cos\alpha\sin\beta \qquad \cos(\alpha \pm \beta) = \cos\alpha\cos\beta \mp \sin\alpha\sin\beta$$

$$\tan(\alpha \pm \beta) = \frac{\tan\alpha \pm \tan\beta}{1 \mp \tan\alpha\tan\beta}$$

15. Double Angle Formulas

$$\sin(2\theta) = 2\sin\theta\cos\theta$$

$$\begin{aligned}\cos(2\theta) &= \cos^2\theta - \sin^2\theta \\ &= 2\cos^2\theta - 1 \\ &= 1 - 2\sin^2\theta\end{aligned}$$

16. Reciprocal Identities

$$\csc\theta = \frac{1}{\sin\theta} \qquad \sec\theta = \frac{1}{\cos\theta} \qquad \cot\theta = \frac{1}{\tan\theta}$$

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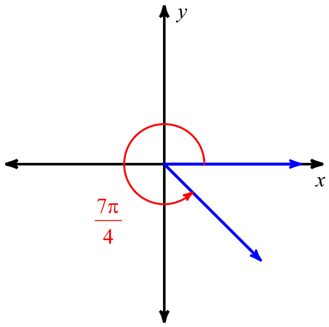
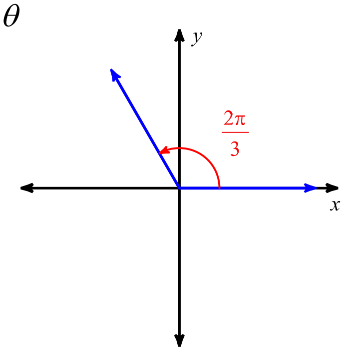
Summer Assignment

Find the exact value of each trigonometric function.

1) $\sec \theta$



2) $\cot \theta$



Find the exact value of each expression.

3) $\cos^{-1} 1$

4) $\tan^{-1} \left(-\sqrt{3}\right)$

Solve each equation for $0 \le \theta < 2\pi$.

5) $3 + \sin \frac{\theta}{2} = \frac{6 - \sqrt{3}}{2}$

6) $-\cos^2 \theta = -3\cos \theta + 2 - \sin^2 \theta$

7) $0 = \sqrt{3}\sin \theta + 2\sin^2 \theta$

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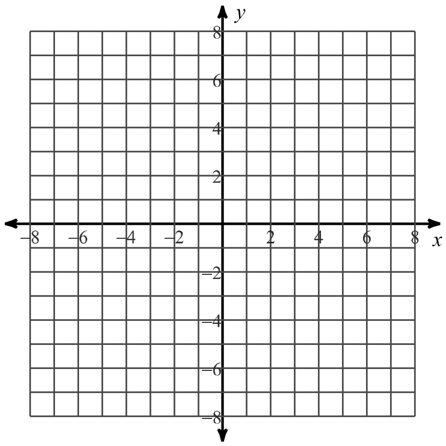
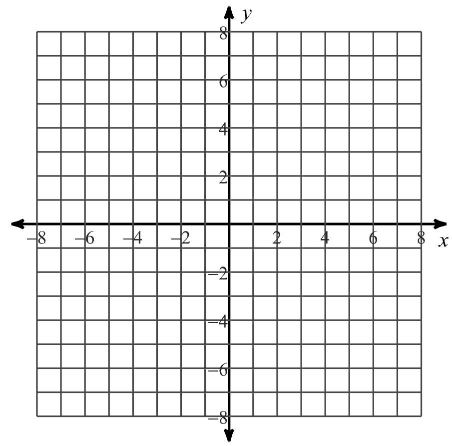
Sketch the graph of each function.

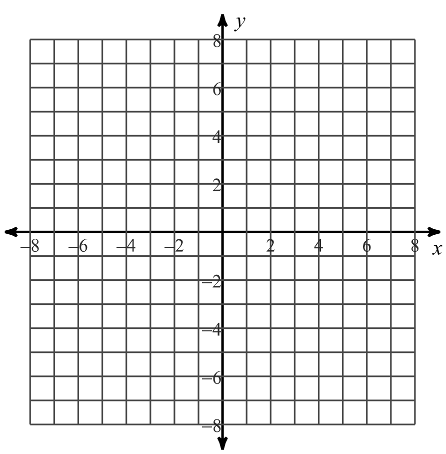
8) $f(x) = \begin{cases} (x-4)^2, & x \neq 4 \\ \dfrac{1}{x-5}, & x = 4 \end{cases}$



9) $f(x) = \begin{cases} 4 - x^2, & x \leq 3 \\ \sqrt{2x}, & x > 3 \end{cases}$

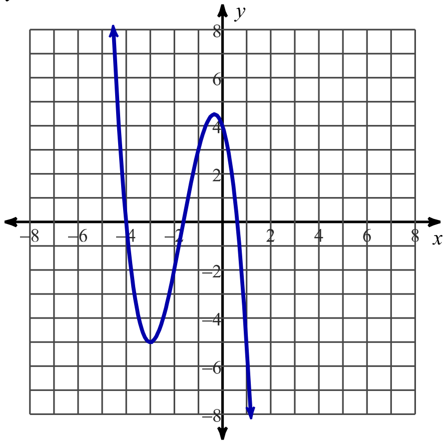


10) $g(x) = \begin{cases} -3, & x < 0 \\ \dfrac{1}{x-1}, & x = 0 \\ -2x + 1, & x > 0 \end{cases}$

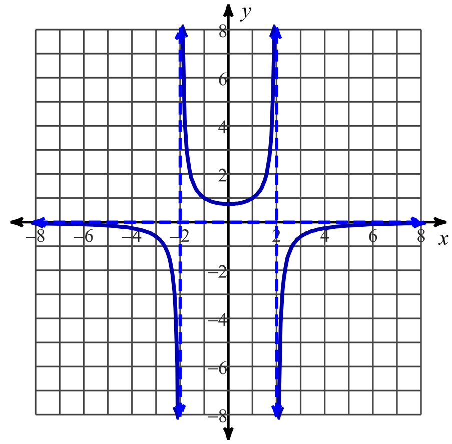


Approximate the intervals where each function is increasing and decreasing.

11) $y = -x^3 - 5x^2 - 3x + 4$

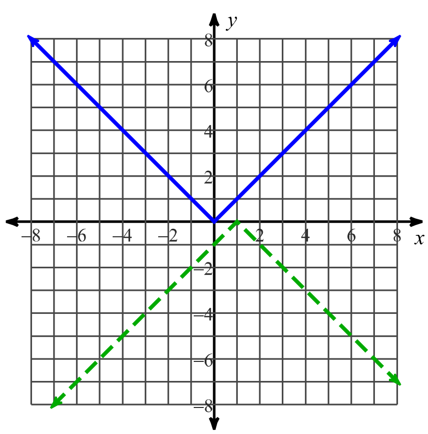


12) $y = -\dfrac{3}{x^2 - 4}$



Describe the transformations necessary to transform the graph of $f(x)$ (solid line) into that of $g(x)$ (dashed line).

13)



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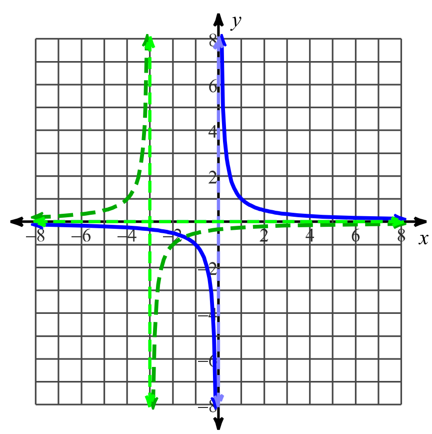

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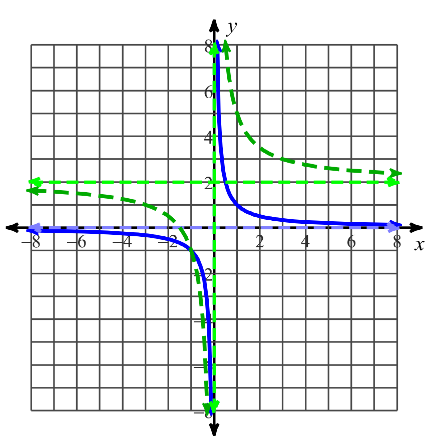
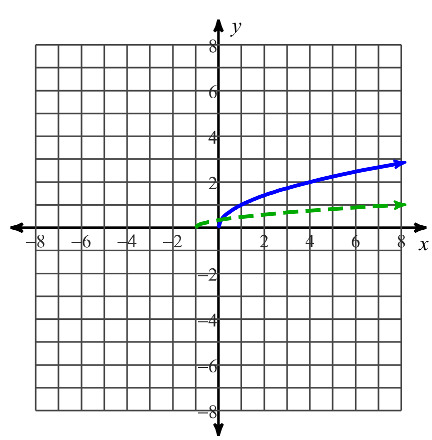


16)



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Perform the indicated operation.

17) $f(n) = 4n + 5$
$g(n) = n^3 + 4n^2$
Find $f(g(2))$

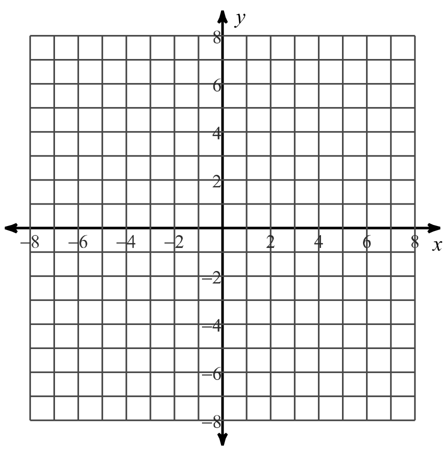
18) $h(x) = 2x - 4$
Find $h(h(-4))$

19) $g(x) = x + 4$
$f(x) = 3x - 2$
Find $g(f(x))$

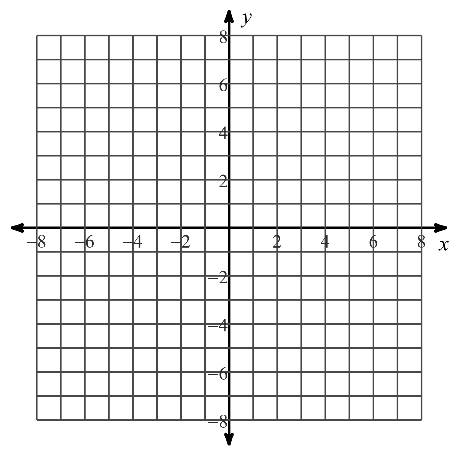
20) $h(x) = 4x$
$g(x) = 2x^3 - 4x^2$
Find $h(g(x))$

For each function, determine the real zeros and state the multiplicity of any repeated zeros and sketch the graph.

21) $f(x) = x^3 - x^2$



22) $f(x) = x^4 - 2x^3 - x^2 + 2x$



Expand each logarithm.

23) $\log_9 (u^6 \cdot v)^3$

24) $\ln (z^5 \sqrt[3]{x})$

25) $\log_5 \sqrt[3]{a \cdot b \cdot c}$

26) $\log_8 \left(\frac{x}{y^6}\right)^4$

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Answers to Summer Assignment

1) $\sqrt{2}$

2) $-\frac{\sqrt{3}}{3}$

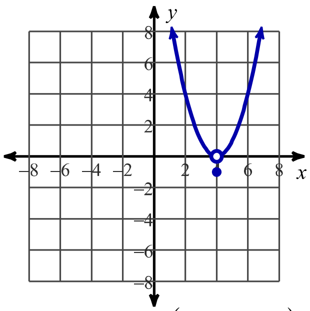
3) 0

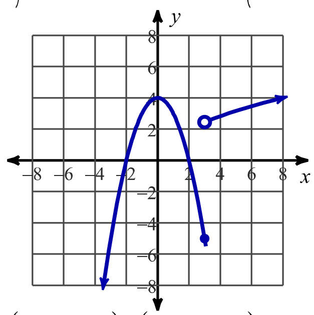
4) $-\frac{\pi}{3}$

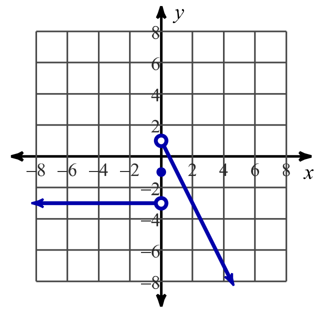
5) No solution.

6) $\left\{0, \frac{\pi}{3}, \frac{5\pi}{3}\right\}$

7) $\left\{0, \pi, \frac{4\pi}{3}, \frac{5\pi}{3}\right\}$

8) 

9) 

10) 

11) Increasing: $(-3, -0.3)$ Decreasing: $(-\infty, -3), (-0.3, \infty)$

12) Increasing: $(0, 2), (2, \infty)$ Decreasing: $(-\infty, -2), (-2, 0)$

13) reflect across the x-axis
translate right 1 unit

14) reflect across the x-axis
translate left 3 units

15) expand vertically by a factor of 3
translate up 2 units

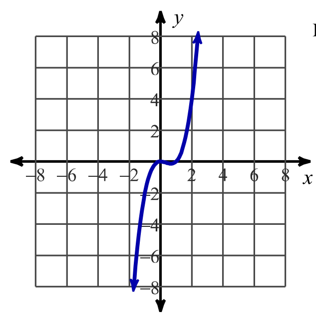
16) compress vertically by a factor of 3
translate left 1 unit

17) 101

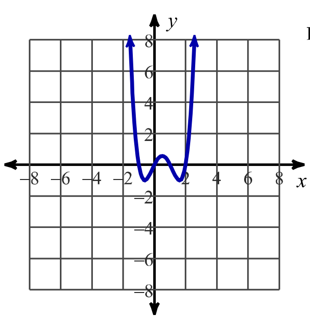
18) -28

19) $3x + 2$

20) $8x^3 - 16x^2$

21) 

Real zeros: {0 mult. 2, 1}

22) 

Real zeros: {0, 2, 1, −1}

23) $18\log_9 u + 3\log_9 v$

24) $5\ln z + \frac{\ln x}{3}$

25) $\frac{\log_5 a}{3} + \frac{\log_5 b}{3} + \frac{\log_5 c}{3}$

26) $4\log_8 x - 24\log_8 y$

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