Topic: ALGEBRA I

Grade Level:	9- 10-11-12
Length:	1 Year
Period(s) Per Day:	1
Credit:	1

Course Description: Algebra I is a comprehensive Algebra course that includes detailed work with various number systems, solving equations and inequalities, operations with polynomials, functions, graphing, systems of equations, radicals, probability, statistics, and other topics as time permits.

Essential Understanding

This course covers the real number system, solving equations and inequalities, operations with polynomials, functions, graphing, systems of equations, radical expressions and equations, rational expressions and equations, probability, and statistics.

Theme Samples

1. Number Sense
2. Algebraic Expressions
3. Linear Equations and Inequalities
4. Linear Functions
5. Non-Linear Functions
6. Non-Linear Equations
7. Geometry
8. Data, Statistics, and Probability

Course Objectives and Expectations

1. To understand variables, simplify expressions, and manipulate and simplify real numbers.
2. To solve equations and inequalities.
3. To understand slope, rate of change, and to write and graph linear equations.
4. To solve linear systems in two variables by graphing, substitution, and elimination methods.
5. To manipulate and solve quadratic functions by graphing, factoring, finding square roots, and using the Quadratic Formula.
6. To manipulate exponents and exponential functions.
7. To factor polynomials and perform mathematical operations on polynomial expressions.
8. To simplify radical expressions and solve radical equations.
9. To simplify rational expressions, perform mathematical operations on rational expressions, and solve rational equations.
10. To understand data analysis and probability.

Student Objectives

1. I can understand variables, simplify expressions, and manipulate and simplify real numbers.
2. I can solve equations and inequalities.
3. I can understand slope, rate of change, and write and graph linear equations.
4. I can solve linear systems in two variables by graphing, substitution, and elimination methods.
5. I can manipulate and solve quadratic functions by graphing, factoring, finding square roots, and using the Quadratic Formula.
6. I can manipulate exponents and exponential functions.
7. I can factor polynomials and perform mathematical operations on polynomial expressions.
8. I can simplify radical expressions and solve radical equations.
9. I can simplify rational expressions, perform mathematical operations on rational expressions, and solve rational equations.
10. I can understand data analysis and probability.

Pacing: Semester 1	***Montana Common Core Standard***
Semester 1	
Unit 1- Foundations for Algebra	N-RN, N-NQ, A-SSE
Unit 2- Solving Equations	A-CED, A-REI
Unit 3- Solving Inequalities	A-CED, A-REI
Unit 4- An Introduction to Functions	F-IF, F-BF
Unit 5- Linear Functions	F-LE
Unit 6- Systems of Equations and Inequalities	A-REI

Course Outline: Semester 1

1st Semester

Foundations for Algebra

A. Variables and Expressions
B. Order of Operations and Evaluating Expressions
C. Real Numbers and the Number Line
D. Properties of Real Numbers
E. Adding, Subtracting, Multiplying, and Dividing Real Numbers
F. The Distributive Property

Solving Equations

A. Solving Two-Step Equations
B. Solving Multi-Step Equations
C. Solving Equations With Variables on Both Sides
D. Literal Equations and Formulas
E. Ratios, Rates, and Conversions
F. Solving Proportions
G. Proportions and Similar Figures
H. Percent's

I. Change Expressed as a Percent

Solving Inequalities

A. Inequalities and Their Graphs
B. Solving Two-Step Inequalities
C. Solving Multi-Step Inequalities
D. Working With Sets
E. Compound Inequalities
F. Absolute Value Equations and Inequalities
G. Unions and Intersections of Sets

An Introduction to Functions

A. Using Graphs to Relate Two Quantities
B. Patterns, Equations, and Graphs
C. Graphing a Function Rule
D. Writing a Function Rule
E. Formalizing Relations and Functions
F. Sequences and Functions

Linear Functions

A. Rate of Change and Slope
B. Direct Variation
C. Slope-Intercept Form
D. Point-Slope Form
E. Standard Form
F. Parallel and Perpendicular Lines

Systems of Equations and Inequalities

A. Solving Systems by Graphing
B. Solving Systems by Substitution
C. Solving Systems by Elimination
D. Applications of Linear Systems
E. Linear Inequalities
F. Systems of Linear Inequalities

Pacing: Semester 2

Semester 2

Unit 7- Exponents and Exponential Functions	A-SSE, F-LE
Unit 8- Polynomials and Factoring	A-APR
Unit 9- Quadratic Functions and Equations	A-CED, A-REI, F-BF, F-LE
Unit 10- Radical Expressions and Equations	A-SSE, A-REI
Unit 11- Rational Expressions and Equations	A-APR, G-SRT
Unit 12- Data Analysis and Probability	S-ID, S-IC, S-CP

Course Outline: Semester 2

2nd Semester

Exponents and Exponential Functions

A. Zero and Negative Exponents
B. Multiplying Powers With the Same Base
C. More Multiplication Properties of Exponents
D. Division Properties of Exponents
E. Exponential Functions
F. Exponential Growth and Decay

Polynomials and Factoring

A. Adding and Subtracting Polynomials
B. Multiplying and Factoring
C. Multiplying Binomials
D. Multiplying Special Cases
E. Factoring $x^2 + bx + c$
F. Factoring $ax^2 + bx + c$
G. Factoring Special Cases

Quadratic Functions and Equations

A. Quadratic Graphs and Their Properties
B. Quadratic Functions
C. Solving Quadratic Equations
D. Factoring to Solve Quadratic Equations
E. The Quadratic Formula and the Discriminant
F. Linear, Quadratic, and Exponential Models
G. Systems of Linear and Quadratic Equations

Radical Expressions and Equations

A. The Pythagorean Theorem
B. Simplifying Radicals
C. Operations with Radical Expressions
D. Solving Radical Equations
E. Trigonometric Ratios

Rational Expressions and Functions

A. Simplifying Rational Expressions
B. Multiplying and Dividing Rational Expressions
C. Adding and Subtracting Rational Expressions
D. Solving Rational Equations

Data Analysis and Probability

A. Organizing Data Using Matrices
B. Frequency and Histograms
C. Measures of Central Tendency and Dispersion
D. Box-and-Whisker Plots
E. Samples and Surveys
F. Permutations and Combinations
G. Theoretical and Experimental Probability
H. Probability of Compound Events

Timeline

Unit 1	(1 week to cover)
Unit 2	(2 weeks to cover)
Unit 3	(2 weeks to cover)
Unit 4	(2 ½ weeks to cover)
Unit 5	(2 ½ weeks to cover)
Unit 6	(2 ½ weeks to cover)
Unit 7	(2 ½ weeks to cover)
Unit 8	(3 weeks to cover)
Unit 9	(3 weeks to cover)
Unit 10	(2 ½ weeks to cover)
Unit 11	(2 ½ weeks to cover)
Unit 12	(2 ½ weeks to cover)

Pertinent Montana Content Standards:

Montana Standards for Algebra 1

The Real Number System N-RN

Use properties of rational and irrational numbers.

3. Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational.

Quantities N-Q

Reason quantitatively and use units to solve problems.

1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
2. Define appropriate quantities for the purpose of descriptive modeling.
3. Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Seeing Structure in Expressions A-SSE

Interpret the structure of expressions.

1. Interpret expressions that represent a quantity in terms of its context.★
   a. Interpret parts of an expression, such as terms, factors, and coefficients.
   b. Interpret complicated expressions by viewing one or more of their parts as a single entity. *For example, interpret P*(1+*r*)n *as the product of P and a factor not depending on P.*
2. Use the structure of an expression to identify ways to rewrite it. *For example, see* $x^4 - y^4$ *as* $(x^2)^2 - (y^2)^2$, *thus recognizing it as a difference of squares that can be factored as* $(x^2 - y^2)(x^2 + y^2)$.

Write expressions in equivalent forms to solve problems.

3. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.★
    a. Factor a quadratic expression to reveal the zeros of the function it defines.-
    ~~b.~~ Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines~~.~~
    c. Use the properties of exponents to transform expressions for exponential functions. *For example the expression* 1.15t *can be rewritten as* $(1.151/12)12t \approx 1.01212t$ *to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.*

Arithmetic with Polynomials and Rational Expressions A-APR

Perform arithmetic operations on polynomials.

1. Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.

Understand the relationship between zeros and factors of polynomials.

3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Rewrite rational expressions.

6. Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system.

Creating Equations A-CED

Create equations that describe numbers or relationships.

1. Create equations and inequalities in one variable and use them to solve problems. *Include equations arising from linear and quadratic functions, and simple rational and exponential functions.*
2. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
3. Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or nonviable options in a modeling context. *For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.*
4. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. *For example, rearrange Ohm's law V = IR to highlight resistance R.*

Reasoning with Equations and Inequalities A-REI

Understand solving equations as a process of reasoning and explain the reasoning.

1. Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.

2. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

Solve equations and inequalities in one variable

3. Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
4. Solve quadratic equations in one variable.
    a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
    b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.

Solve systems of equations

5. Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
6. Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
7. Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.

Represent and solve equations and inequalities graphically.

10. Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
11. Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
12. Graph the solutions to a linear inequality in two variables as a halfplane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Interpreting Functions F-IF

Understand the concept of a function and use function notation.

1. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$.
2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.

3. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1)\ for\ n \geq 1$.

Interpret functions that arise in applications in terms of the context.

4. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.★
5. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.★
6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.★

Analyze functions using different representations.

7. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
    a. Graph linear and quadratic functions and show intercepts, maxima, and minima.
    b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.
    c. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
    d. (+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
    e. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
8. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
    a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
    b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t, y = (0.97)^t, y = (1.01)12^t, y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.
9. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). *For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.*

Building Functions F-BF

Build a function that models a relationship between two quantities.

1. Write a function that describes a relationship between two quantities.★
    a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
    b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
    c. (+) Compose functions. For example, if $T(y)$ is the temperature in the atmosphere as a function of height, and $h(t)$ is the height of a weather balloon as a function of time, then $T(h(t))$ is the temperature at the location of the weather balloon as a function of time.
2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.★

Build new functions from existing functions.

3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

Linear, Quadratic, and Exponential Models F-LE

Construct and compare linear, quadratic, and exponential models and solve problems.

1. Distinguish between situations that can be modeled with linear functions and with exponential functions.
    a. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
    b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
    c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
2. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
3. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function.
4. For exponential models, express as a logarithm the solution to $ab^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.

Interpret expressions for functions in terms of the situation they model.

5. Interpret the parameters in a linear or exponential function in terms of a context.

Similarity, Right Triangles, and Trigonometry G-SRT

Define trigonometric ratios and solve problems involving right triangles.

6. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
7. Explain and use the relationship between the sine and cosine of complementary angles.
8. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.★

Interpreting Categorical and Quantitative Data S-ID

Summarize, represent, and interpret data on a single count or measurement variable.

1. Represent data with plots on the real number line (dot plots, histograms, and box plots).
2. Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
3. Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

Summarize, represent, and interpret data on two categorical and quantitative variables.

5. Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
6. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. *Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.* b. Informally assess the fit of a function by plotting and analyzing residuals. c. Fit a linear function for a scatter plot that suggests a linear association.

Interpret linear models.

7. Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
8. Compute (using technology) and interpret the correlation coefficient of a linear fit.
9. Distinguish between correlation and causation.

Making Inferences and Justifying Conclusions S-IC

Understand and evaluate random processes underlying statistical experiments.

1. Understand statistics as a process for making inferences about population parameters based on a random sample from that population.
2. Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. *For example, a model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?*

Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

3. Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each.
4. Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling.

5. Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant.
6. Evaluate reports based on data.

Conditional Probability and the Rules of Probability S-CP

Understand independence and conditional probability and use them to interpret data.

1. Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
2. Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

Use the rules of probability to compute probabilities of compound events in a uniform probability model.

7. Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.

Resources:

Textbook: Algebra 1
Textbook Resources: Algebra 1, Teacher Resources
HPS Technology Standards
Montana Common Core State Standards