

Marietta City Schools

	Advanced Algebra: Concepts & Connections		
Unit title	Unit 6 (DOE Unit 5): Investigating Linear Algebra and Matrices	Unit duration (hours)	13.5

Mastering Content and Skills through INQUIRY (Establishing the purpose of the Unit): What will students learn?

GA DoE Standards

<u>Standards</u>

AA.PAR.6: Represent data with matrices, perform mathematical operations, and solve systems of linear equations leading to real-world linear programming applications.

AA.PAR.6.1 Use matrices to represent data, and perform mathematical operations with matrices and scalars, demonstrating that some properties of real numbers hold for matrices, but that others do not.

Fundamentals

- This is the first formal reference to matrices in the K-12 mathematics standards progression.
- Students should be able to perform operations with matrices that include the identity matrix and the zero matrix.
- Students should recognize that matrix multiplication is not commutative.

Strategies and Methods

• Students should be able to calculate the following matrix operations without the use of technology: scalar multiplication, addition, subtraction, multiplication of 2x2 matrices, calculate the determinant of a 2x2 matrix, and the inverse of an invertible 2x2 matrix. Students should have opportunities to utilize technology to perform the same calculations with matrices of greater dimension.

• Students may use technology in calculations with matrices of greater dimension than 2x2.

AA.PAR.6.2 Rewrite a system of linear equations using a matrix representation.

Relevance and Application

• A system of linear equations in standard form can be represented as an equation of a coefficient matrix multiplied by a variable matrix, equal to a constant matrix. **Example**

• Given the system of linear equations, $\begin{cases}
a_1x + b_1y + c_1z = d_1 \\
a_2x + b_2y + c_2z = d_2 \\
a_3x + b_3y + c_3z = d_3 \\
students should represent it as:
\begin{bmatrix}
a_1 & b_1 & c_1 \\
a_2 & b_2 & c_2 \\
a_3 & b_3 & c_3
\end{bmatrix}
\begin{bmatrix}
x \\
y \\
z
\end{bmatrix} = \begin{bmatrix}
d_1 \\
d_2 \\
d_3
\end{bmatrix}$

AA.PAR.6.3 Use the inverse of an invertible matrix to solve systems of linear equations.

Relevance and Application • Students may use technology for matrices of dimension 2 x 2 or higher to calculate the inverse of an invertible matrix.

AA.PAR.6.4 Utilize linear programming to represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret data points as solutions or non-solutions under the established constraints in real-world problems.

Relevance and Application

• Food and Agriculture, Engineering, and Manufacturing optimization problems would be appropriate for this learning objective. Other contexts may be used, as well.

AA.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.

AA.MM.1.1 Explain applicable, mathematical problems using a mathematical model.

Fundamentals

- Students should be provided with opportunities to learn mathematics in the context of culturally relevant problems.
- Mathematically applicable problems are problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (i.e., model with mathematics).

AA.MM.1.2 Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts. Fundamentals

• Mathematically proficient students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.

AA.MM.1.3 Using abstract and quantitative reasoning, make decisions about information and data from a mathematical, applicable situation.

Fundamentals

• Students should be able to:

o analyze functions, graphs, tables, and equations and make decisions about the real-life situations they describe based upon their understanding of mathematical functions. o analyze statistical results to decide the best course of action or approach to a problem.

Example

Given a rectangle with length = (x - 2) and width = (2x + 3), a student could discover and articulate that the area = (x - 2)(2x + 3) = 2x2 - x - 6. From the student's understanding of parabolas, a student would know that the parabola that represents all possible areas of this rectangle opens upwards and that there is no maximum area possible for this rectangle.
 AA.MM.1.4 Use various mathematical representations and structures to represent and solve real-life problems.

Fundamentals

- Students should be able to generate models, graphs, charts, and equations, to represent real-world phenomena in order to solve problems.
- Students should be provided opportunities to generate representations of real-world phenomena utilizing technology to show these phenomena and to solve problems.

Concepts/Skills to support mastery of standards

- Mastery of matrix operations: addition, subtraction, scalar multiplication, and matrix multiplication.
- Ability to rewrite systems of linear equations as matrix equations and vice versa.

- Proficiency in finding the inverse of a matrix and using it to solve systems of equations.
- Understanding linear programming basics: objective functions, decision variables, and constraints.
- Application of mathematical concepts to real-world problems, including data interpretation and problem-solving within constraints.

Vocabulary

Associative	Commutative	Column	Consistent System	Constraint	Decision Variable				
Determinant (A)	Diagonal of a Matrix	Extreme Point	Feasible Region	Identity	Identity Matrix				
Inconsistent System	Inverse	Inverse of a Matrix	Invertible Matrix	Line	Linear Function				
Linear Programming	Mapping	Matrix	Matrix Dimensions	Objective Function	Plane				
Reduced Row Echelon Form	Row	Scalar	Systems of Equations	Scalar Multiplication	Transpose				
Vector	Vector Space	Zero Matrix							
Notation									
 Matrices are often represented using uppercase bold letters, such as A,B,C. Scalar multiplication involves multiplying a scalar by each element of a matrix, commonly denoted as kA, where k is the scalar and A is the matrix. [A]+[B] or A + B, for all operations. 									
Essential Questions									

How can matrices be utilized to represent datasets, and what mathematical operations are performed with matrices and scalars?

In what ways do matrices exhibit similarities and differences compared to real numbers, and how do these properties impact mathematical operations?

How can a system of linear equations be transformed into a matrix representation, and what advantages does this provide for solving such systems?

What role does the inverse of an invertible matrix play in solving systems of linear equations, and how is it utilized effectively?

What is linear programming, and how does it enable the representation of constraints in terms of equations or inequalities?

In what ways can linear algebraic concepts and linear programming techniques be applied to real-world scenarios, and what insights do they provide for problem-solving and decision-making processes?

Assessment Tasks

List of common formative and summative assessments.

Formative Assessment(s):

Unit Quiz

Summative Assessment(s):

Unit Test

<u>Learning Experiences</u> Add additional rows below as needed.						
Objective or Content	Learning Experiences	Personalized Learning and Differentiation All information included by PLC in the differentiation box is the responsibility and ownership of the local school to review and approve per Board Policy IKB.				
AA.PAR.6.1 AA.MM.1.1 AA.MM.1.2 AA.MM.1.3 AA.MM.1.4	 Organizing Matrices In this learning plan, students will be introduced to matrices as tools for organizing and storing information. Students will explore a fundraising project where they will analyze cost of materials, time to produce each item, inventory, etc. Dimensions and dimension labels are used to provide rationale for addition and multiplication procedures. Emphasis is placed on interpreting entries as matrices are written, added, multiplied (scalar and regular) and transposed. Learning Goal(s) I can represent data in matrix form and determine the dimensions of matrices. I can perform scalar multiplication on matrices. I can multiply matrices and know when matrix multiplication is defined. I can solve problems using matrix operations. 	Students will be able to work at their own pace in collaborative groups where additional scaffolding is available as needed. Use of technology - graphing calculators and desmos				

Content Resources

Textbook Correlation: enVision A|G|A - Algebra 2

AA.PAR.6.1 - Lessons 10-1, 10-2, Topic 10-Mathematical Modeling in 3 Acts AA.PAR.6.2 - Lessons 1-7, 10-5, Topic 10-Mathematical Modeling in 3 Acts AA.PAR.6.3 - Lessons 1-7, 10-4, 10-5 AA.PAR.6.4 - Lesson 1-6, Topic 1-Mathematical Modeling in 3 Acts