PRECALCULUS CURRICULUM

(Refer to the Curriculum Template Explanation document while writing curriculum)

Grade Level(s): 11-12

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Course Description:
Pre-Calculus is designed to equip students with the mathematical knowledge and skills necessary for success in calculus and other advanced mathematical disciplines. The course focuses on developing a deep understanding of algebraic, trigonometric, and transcendental functions, as well as their applications.

Year At A Glance			
Unit Title	**Overarching Essential Question**	**Overarching Enduring Understanding**	**Vision of A Learner "I Can" Statements**
Prerequisite	How do foundational algebraic and geometric concepts form the basis for understanding more advanced mathematical topics in Precalculus?	Students will reflect on the importance of mastering foundational concepts in algebra such as functions, equations, and graphing, which are crucial for tackling the more complex ideas in Pre-Calculus.	TCC1(9-12); P3(9-12); TI3(9-12); CCE4(9-12); TCC3(9-12); P1(9-12)
Functions	How do functions and their inverses help us understand and solve real-world problems, and what are the key principles that govern their relationships and transformations?	Functions can be built and represented verbally, graphically, numerically, and algebraically. Students will understand that while many functions behave very differently, transformations can still be applied to each function, and inverse functions essentially reverses the effect of the original function.	TCC1(9-12); CCE4(9-12); TI1(9-12); P1(9-12); AA1(9-12)
Polynomials	How do the algebraic properties and graphical characteristics of polynomial functions help us model, analyze, and solve real-world problems, and how can understanding these properties inform predictions about the behavior of complex systems?	Polynomial functions exhibit predictable patterns and behaviors based on their degree, coefficients, and roots, allowing them to model complex relationships and solve real-world problems. Understanding the algebraic structure and graphical characteristics of polynomials enables students to analyze, predict, and interpret the behavior of these functions in various contexts.	P3 (9-12), TCC3(9-12)

Exponential/Logs (Honors Only)	How do exponential and logarithmic functions model real-world phenomena, and how can understanding these functions help us solve complex problems in various fields?	Exponential and logarithmic functions are fundamental mathematical tools that model and analyze dynamic processes involving growth, decay, and scaling. Understanding these functions enables us to interpret complex relationships, solve practical problems, and make informed predictions across diverse fields such as science, finance, and engineering.	TCC4(9-12); CCE3(9-12); AA1(9-12); TI3(9-12)
Rational Functions (CP Only)	How do rational functions model real-world phenomena and how can understanding these functions and interpreting their graphs help us solve complex problems?	Rational functions represent relationships where the dependent and independent variables are interconnected through a ratio of polynomials. Understanding these functions involves analyzing their asymptotic behavior, intercepts, and domain restrictions, which collectively describe how changes in one variable impact the other. Mastery of rational functions equips students with the tools to model and interpret real-world scenarios involving rates, proportions, and optimization, revealing the underlying patterns and structures of complex relationships.	TCC3(9-12); TCC4(9-12); TCC1(9-12); CCE3(9-12); TI3(9-12); P2(9-12); P3(9-12)
Trigonometric Functions	How do the relationships between angles, triangles, and trigonometric functions allow us to model, solve, and interpret real-world problems?	Trigonometry provides a framework for understanding and modeling the relationships between angles, lengths, and periodic phenomena.	P3 (9-12), TCC3(9-12)

Trigonometric Identities	How can manipulating and solving trigonomic functions help us master skills for solving real-world problems in fields such as physics, engineering, and signal processing?	Trigonometric identities are foundational tools that reveal the inherent relationships between trigonometric functions and their angles. Mastery of these identities enables students to simplify complex trigonometric expressions, solve equations, and analyze periodic phenomena.	TCC3(9-12); CCE2(9-12); CCE3(9-12); CCE4(9-12); DE1(9-12); TI3(9-12); P1(9-12); P3(9-12); AA1(9-12)
Modeling with Law of Sines and Cosines	How can the Law of Sines and the Law of Cosines be used to solve real-world problems involving triangles, and what do these methods reveal about the relationships between the angles and sides of a triangle?	The Law of Sines and the Law of Cosines provide systematic methods for analyzing and solving problems involving non-right triangles, revealing the intrinsic relationships between angles and sides. Mastery of these laws enables the accurate modeling of real-world scenarios and geometrical problems where triangle dimensions need to be determined or inferred.	TCC3(9-12); TCC2(9-12); DE1(9-12); P1(9-12); P3(9-12); AA2(9-12)
Matrices ★	How can matrices be used to solve systems of linear equations, and why is this method sometimes more efficient than traditional methods?	Matrices provide a powerful way to represent and solve systems of linear equations, with each system corresponding to a matrix equation.	P3(9-12), TCC3(9-12)

Unit 1 - Prerequisite

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (* indicates a modeling standard; ★ indicates greater emphasis in the Honors sections)

- **A.SSE.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
  - **A.SSE.3a.** Factor a quadratic expression to reveal the zeros of the function it defines.
  - **A.SSE.3b.** Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
  - **A.SSE.3c.** Use the properties of exponents to transform expressions for exponential functions
- **A.APR.7** (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.★
- **A.CED.1** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple (rational★) and exponential functions.*
- **A.REI.2** Solve simple (rational★) and radical equations in one variable, and give examples showing how extraneous solutions may arise.
- **A.CED.4** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.★
- **A.REI.3** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.
- **A.REI.4** Solve quadratic equation in one variable
  - **A.REI.3a** Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form.
  - **A.REI.3b** Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b.
- **A.REI.11** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solution of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and /or g(x) are linear, polynomial, (rational★), absolute value, exponential, and logarithmic functions.*
- **S.ID.7** Interpret the slope (rate of change) and the intercept of a linear model in the context of data

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.

- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- **TCC1(9-12)**: I can ask purposeful, insightful questions to find a variety of innovative solutions.
- **P3(9-12)**: I can accept constructive feedback and use setbacks to adjust my learning journey in order to reach my goals.
- **TI3(9-12)**: I can formulate and investigate probing questions to further my learning
- **CCE4(9-12)**: I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.
- **TCC3(9-12)**: I can integrate relevant information to produce multiple valid solutions.
- **P1(9-12**): I can self-reflect and seek feedback to build upon my strengths, apply those strengths to overcome obstacles, and share my strategies with others.
- **CCE3(9-12)**: I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.

Understandings: Students will understand that…

- Expressions can be written in multiple ways using the rules of algebra; each version of the expression tells something about the problem it represents.
- There is often an optimal method of manipulating equations to solve a mathematical problem; however other methods, which may not be as efficient, can still provide insight to the problem.
- Algebraic expressions such as polynomials and rational expressions, symbolize numerical relationships and can be manipulated in much the same way as numbers.★

Essential Questions:

- Why do we structure expressions in different ways?
- In what ways can the problem be solved, and why should one method be chosen over another?
- How can the properties of the real number system be useful when working with polynomials and rational expressions?★
- Which function is being modeled in specific real world applications and how can I use acquired knowledge to solve the problem?

Students will know…

- Linear functions may be represented by equations, graphs, tables, and words. The graph of a function is often a useful way of visualizing the relationship the function models.
- For linear applications, the y-intercept is the starting point and the slope is the rate of change.

Students will be able to...

- Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
- Create equations in one variable and use them to solve problems.
- Solve linear equations including equations with coefficients represented by letters.

• The form of the equation determines which solution method (graphing or algebraic) is most efficient or appropriate. • Understand solving equations as a process of reasoning and explain the reasoning • Determine and explain the best solving method for a quadratic equation in one variable. • How to apply solving techniques to interpret and solve mathematical models	• Interpret the slope (rate of change) and the intercept (constant term)of a linear model in the context of the data. • Add, subtract, multiply, simplify rational exponents and radical expressions • Add, subtract, multiply, simplify rational expressions ★ • Solve rational equations.★ • Factor a quadratic expression to reveal the zeros of the function it defines. • Solve quadratic equations by inspection (e.g., for *x*^2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. • Recognize when the quadratic formula gives complex solutions and write them as *a* ± *bi* for real numbers *a* and *b*. • Solve radical equations. • Interpret and solve literal equations ★ • Define extraneous solutions and determine which numbers cannot be solutions of radical and (rational★) equations.

Key Vocabulary:

Linear Equations	Extraneous Solution	Completing the square★
Slope, Y-intercept	Literal Equation★	Quadratic Formula
Radical Equation	Quadratic Equation	Radicand
Rational Exponent	Polynomial	Complex Number
Rational Expression★	Factoring	Imaginary
Rational Equation★	Square root method	Solution

Assessment Evidence

Performance Tasks:
 	
Summative Unit Assessment: TCC3(9-12) Topics will include:
• Simplify: Quadratics, rational exponents, radical expressions, (rational expressions★).
• Solve: Quadratics
  ○ Factoring
  ○ Square Root Method
	Other Evidence:
	
	Activities:
	• Desmos discover and practice activities **P1(9-12**)
	• Around the room solving applications activity
	• Hot Seat game Review★
	• Escape Room Review activities ★ **CCE3**

○ Completing the Square ○ Quadratic Formula ● Solve: Absolute value, (rational★), and radical equations. ● Apply solving skills to mathematical applications including area, perimeter, and pythagorean theorem, calculating the area of a space, determining the trajectory of a projectile (like a ball thrown in sports), optimizing profit in business based on price and quantity, or designing curved structures in engineering, where the path follows a parabolic shape	**Interims:** ● **Interim 1**:**P1(9-12**) ○ Simplify Exponents ○ Simplify radicals ○ Factor quadratics and polynomials ■ Greatest Common Factor ■ Grouping ■ Trinomial a=1 ■ Trinomial a≠1 ■ Difference of Squares ■ Sum/Difference of Cubes★ ● **Interim 2★**: (Pre-Calc CP will address this in a separate unit) ○ Simplify rational expressions ○ Multiply and divide rational expressions ○ Add and subtract rational expressions ● **Interim 3**: (Pre-Calc CP only) **P1(9-12**) ○ Solving quadratics ■ Factoring ■ Square root method ■ Quadratic formula ○ Solving radical equations

Learning Plan

The prerequisite unit for Pre-Calculus focuses on foundational concepts necessary for success in the course.This unit will review solving functions such as: Linear, Absolute Value, Quadratic, Radicals, Rational, and exponential and logs.

- Students will complete textbook practice problems working in groups during class and individually outside of class.
- Class discourse on correct solutions to textbook problems - compare strategies and ask questions. **TCC1(9-12)**
- Desmos Exploratory and review activities **P1(9-12**), **TI3(9-12)**
- Extra practice on graphing rational functions- students will work in groups-class discourse on correct solutions- compare strategies and ask questions. ★**CCE3(9-12)**
- Individual/group in class practice on concepts that will be measured on the summative assessment.**P3(9-12)TCC3(9-12)**

Teacher Resources: Blitzer Precalculus Textbook, TI-84 graphing calculator, Desmos calculator, mini white boards, teacher created worksheet

Unit 2 - Functions

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (* indicates a modeling standard; ★ indicates greater emphasis in the Honors sections)

- **F.IF.A.1** Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
- **F.IF.A.2** Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
- **F.IF.B.4** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.*
- **F.IF.B.5** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.*
- **F.IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
    - **F.IF.C.7.B** Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.*
- **F.BF.A.1** Write a function that describes a relationship between two quantities.*
    - **F.BF.A.1.A** Determine an explicit expression, a recursive process, or steps for calculation from a context.
    - **F.BF.A.1.C** (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
- **F.BF.B.3** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
- **F.BF.B.4** Find inverse functions.
    - **F.BF.4a** Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2(x^3) or f(x) = (x+1)/(x-1) for x ≠ 1 (x not equal to 1).
    - **F.BF.4b** (+) Verify by composition that one function is the inverse of another.
    - **F.BF.4c** (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
    - **F.BF.4d** (+) Produce an invertible function from a non-invertible function by restricting the domain.

Standards for Mathematical Practice

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- MP2: Reason abstractly and quantitatively.

- MP3: Construct viable arguments and critique the reasoning of others.
- MP4: Model with mathematics.
- MP5: Use appropriate tools strategically.
- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- **TCC1(9-12):** I can ask purposeful, insightful questions to find a variety of innovative solutions.
- **CCE4(9-12)**: I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.
- **TI1(9-12)**: I can implement a realistic plan and adapt when necessary to achieve my goals.I can self-reflect and seek feedback to build upon my strengths, apply those strengths to overcome obstacles, and share my strategies with others.
- **AA1(9-12)**: I can evaluate different approaches and justify the best pathway to success.
- **TI3(9-12)**: I can formulate and investigate probing questions to further my learning.

Understandings: Students will understand that...

- A function is a specific type of relation where each input is associated with exactly one output. This concept is fundamental in describing how variables interact and is a building block for analyzing more complex mathematical relationships.
- Functions can be transformed through translations, reflections, stretches, and compressions. These transformations affect the graph of the function and help in understanding how changes in the function's formula impact its graph.
- Functions can be combined through addition, subtraction, multiplication, and division, and can be composed with each other. Understanding these operations is essential for building more complex functions and analyzing their behavior.
- An inverse function essentially reverses the effect of the original function. Understanding how to find and use inverse functions is important for solving equations and understanding the relationship between functions and their inverses.
- Functions are used to model and solve real-world problems in various fields such as physics, economics, biology, and engineering. Applying functions to practical situations helps in interpreting data and making predictions.

Essential Questions:

- What is a function and what are the different ways they can be represented?
- How are functions used in the real world?
- What are the key characteristics of a graph and why are they important?
- How does composing functions affect the domain?
- What is the relationship between a function and its inverse?

Students will know…	Students will be able to...
• A function is a specific type of relation in which each element in the domain is paired with a unique element of the range. Functions can be represented verbally, numerically, graphically, and algebraically. • The terms domain, input and independent can be used interchangeably. • The terms range, output and dependent variable can be used interchangeably. • An independent variable is the controlling factor of the function. • The dependent variable changes based on the value of the independent variable. • Functions can model many important phenomena • Every function creates a unique graph. Graphs are used to illustrate solutions and solve problems. • Key characteristics such as zeros, extrema, increasing, decreasing, symmetry, and end behavior are important because they describe the behavior of the outputs. • Having an understanding of the basic characteristics of a variety of functions facilitates the solving of complex problems. • Arithmetic operations (addition, subtraction, multiplication, composition) can be performed on functions. • When composing functions the restrictions on the domain of each original function must be considered in determining the domain of the composite function. • Every function has an inverse and that inverse is not necessarily a function, which is graphically represented by its reflection over the y = x line. • An inverse relation interchanges the domain and range of the original function, and therefore “undoes” the original function.	• Determine if a relationship is a function. • Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. • For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. • Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. • Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. • Graph linear and quadratic functions and show intercepts, maxima, and minima. • Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. • Write a function that describes a relationship between two quantities. • Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative) and find the value of k given the graphs. • Combine standard function types using arithmetic operations. • Compose functions. • Find inverse functions. • Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. • Verify by composition that one function is the inverse of another. • Read values of an inverse function from a graph or a table, given that the function has an inverse. • Produce an invertible function from a non-invertible function by restricting the domain.
Key Vocabulary:	

Function Input Output Independent variable Dependent variable Domain Range Function notation: f(x)	Parent function Transformation Translations Vertical stretch Vertical compression Vertical reflection Horizontal reflection	Piecewise function Vertical line test One-to-one function Inverse function Domain Restricted domain Composition

Assessment Evidence

Performance Tasks:

Summative Unit Assessment: TI1(9-12); AA1(9-12) Topics will include:

- Determine if a relationship is a function
- Identify and perform transformations of functions
- Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes
- Graph quadratic, cubic, square root, cube root, and piecewise-defined functions, including step functions*(honors only)* and absolute value functions.
- Compose functions
- Verify by composition that one functions is the inverse of another
- Evaluating inverse functions

Real world applications of functions include:

- Physics - motion and trajectory of a projectile
- Financing - calculate interest, investments, loan payments, maximizing profit for business
- Engineering - to model systems to predict outcomes in relationships, materials, or electrical circuits
- Also used in computer science, economics, and statistics

Other Evidence:

Interim: Characteristics of graphs of functions and domain.**P1(9-12)**
Interim: Transformations of graphs of functions, operations and inverse functions.**P1(9-12)**
Interim: Function operations and composition and domain and on inverses.**P1(9-12)**
Interim: Piecewise project- students create their own piecewise function with a partner. **CCE3(9-12)**

Learning Plan

Precalculus continues the learning of the study of functions. Students will use prior knowledge of functions from Algebra 2. They will use this knowledge to create connections throughout the year including transformations of trigonometric functions.

- Students will explore how transformations (translations, reflections, stretches and compressions) affect all functions through discovery methods including desmos and noticing patterns through graphs. **TI3(9-12)**
- Students will create a graphic organizer when revisiting and adding new functions to their learning. **CCE4(9-12)**
- Students will practice identifying the transformations of a given function and create a graph that represents the function. They will be able to discuss the domain and range for each. **TCC1(9-12)**

Teacher Resources: Blitzer Precalculus Textbook, TI-84 graphing calculator, Desmos calculator, mini white boards, teacher created worksheet

Unit 3 - Polynomials

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (* indicates a modeling standard; ★ indicates greater emphasis in the Honors sections)

- **N.CN.A 1** Know there is a complex number i such that i^2 = −1, and every complex number has the form a + bi with a and b real.
- **N.CN.A 2** Use the relation i^2 = –1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
- **N.CN.3** Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.
- **N.CN.C 7** Solve quadratic equations with real coefficients that have complex solutions.
- **N.CN.C 8** Extend polynomial identities to the complex numbers. For example, rewrite x^2 + 4 as (x + 2i)(x – 2i). (Honors)
- **N.CN.C 9** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
- **A.SSE.A.1** Interpret expressions that represent a quantity in terms of its context.*
- **A.SSE.B.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
- **A.APR.A.1** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials.
- **A.APR.B.2** Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
- **A.APR.B.3** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
- **F.IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
    - **F.IF.C.7.C** Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.*
- **F.IF.C.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
- **F.IF.C.9** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.

Standards for Mathematical Practice

- MP1: Make sense of problems and persevere in solving them.
- MP2: Reason abstractly and quantitatively.
- MP3: Construct viable arguments and critique the reasoning of others.
- MP4: Model with mathematics.
- MP5: Use appropriate tools strategically.

- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- **TCC1(9-12**): I can ask purposeful, insightful questions to find a variety of innovative solutions.
- **TCC3(9-12)**: I can integrate relevant information to produce multiple valid solutions.
- **P2(9-12):** I can strengthen my weaknesses by identifying, initiating, and practicing appropriate strategies to become confident in my abilities to overcome my challenges.
- **P3(9-12):** I can accept constructive feedback and use setbacks to adjust my learning journey in order to reach my goals.
- **TI3(9-12)**: I can formulate and investigate probing questions to further my learning.
- **CCE2(9-12)**: I can give and receive actionable and relevant feedback with openness to be able to determine meaningful revisions for success.

Understandings: Students will understand that…

- Polynomial functions can be classified by their degree, which determines their general shape and the maximum number of roots they can have.
- The degree and leading coefficient determine the end behavior of a polynomial function, which helps in sketching and analyzing graphs.
- The zeros/roots of a polynomial function correspond to the x-intercepts of its graph. The multiplicity of a zero affects the behavior of the graph at that intercept, such as whether it touches or crosses the x-axis.
- Polynomial functions model various real-world phenomena, from projectile motion to economics.
- The complex number set is composed and the real numbers and the imaginary numbers.
- Complex solutions will not be x-intercepts.
- The intermediate value theorem can be used to determine the zeros of a function. ★
- Recognizing symmetry in polynomial functions (even, odd, or neither) helps in graphing and analyzing these functions.
- The remainder theorem tells us that if a polynomial f(x) is divided by x-c, then the remainder is f(c).
- The factor theorem tells us that if f(c)=0, then x - c is a factor of

Essential Questions:

- What are the defining characteristics of polynomial functions and how do their degrees and coefficients affect their behavior?
- Why are there different methods of finding the zeros of a polynomial and how do we decide which method is appropriate?
- How can polynomial functions represent real-life situations?
- What is the Fundamental Theorem of Algebra, and how does it relate to the existence of complex roots?
- How can you use the Factor Theorem and synthetic division to factorize polynomials?
- How do different types of variation apply to real-world problems, and how can understanding these relationships help solve those problems? ★

f(x). • Graphs of rational functions have asymptotes and have many forms ★ • Many real-life quantities can be modeled by types of variation including: direct, inverse, and joint. ★	
Students will know… • When performing operations with complex numbers to combine the real numbers and combine the imaginary numbers • The quantity i squared must be replaced with negative 1. • To rationalize the denominator of a complex fraction, multiply the numerator and denominator by the complex conjugate • Dividing polynomials can produce the zeros of a function. • Synthetic division is a preferred method of long division of polynomials, but may only be used when the divisor is of the form x ± k • The Rational Zero Theorem (rational root theorem) helps to find a zero/root of a polynomial. • The remainder theorem, along with synthetic division, provides a handy shortcut for evaluating polynomials. • Key characteristics such as continuity, zeros, multiplicity of zeros, and end behavior are important because they describe the behavior of the outputs. These characteristics can be used to sketch a graph of the polynomial. • Polynomials can represent real-life situations, including simple financial modeling and wooden roller coasters. • The intermediate value theorem determines if there is a real zero between two x values. ★ • The graph of the parent function of 1/x and 1/x^2 and how to use transformations with them. ★ • Vertical asymptotes occur at x values that make a denominator zero. ★ • Horizontal asymptotes are found by comparing the largest exponents in the numerator and denominator. ★ • Slant asymptotes are found by using long division. ★ • Keywords that indicate direct, inverse, or joint variation in a real-life situation. ★	**Students will be able to…** • Simplify expressions with complex numbers including add, subtract, multiply and divide. • Rationalize the denominator of a complex fraction • Divide polynomial expressions using both long division and synthetic division. • Apply the Fundamental Theorem of Algebra to write the simplest polynomial with the given roots. • Use the rational zero theorem and the factor theorem to find all the zeros of a polynomial function. • Sketch the graph of a polynomial using the end behavior, zeros, multiplicities, y-intercept, and turning points. • Solve application problems that involve representing a real life situation with a polynomial function. • Create a polynomial model and make predictions for real-world phenomena. • Use the intermediate value theorem. ★ • Graph the functions 1/x and 1/x^2 using transformations. ★ • Graph a rational function by finding horizontal and vertical (or slant) asymptotes, intercepts, symmetry, and holes. ★ • Write variation equations that model real-life situations and solve problems related to that equation. ★

Key Vocabulary:

Complex Number	Intermediate Value Theorem	Vertical Asymptote
Complex Conjugate	Turning Points	Horizontal Asymptote
Polynomial Function	Synthetic Division	Slant Asymptote
Leading coefficient test	Remainder Theorem	Direct Variation
End Behavior	Factor Theorem	Inverse Variation
Multiplicities	Rational Zero Theorem	Joint Variation

Assessment Evidence

Performance Tasks:

Summative Unit Assessment: TCC3(9-12): Topics include:

- Add, subtract, multiply and divide complex numbers
- Find all the zeros of a polynomial function
- Sketch the graph of a polynomial function
- Model a real-life situation with a polynomial function
- Use the intermediate value theorem ★
- Graph rational functions with transformations and without. ★
- Solve variation problems ★

Other Evidence:

Interim: P3(9-12):

- Quiz on complex numbers and graphing polynomial functions
- Quiz on finding all zeros of a polynomial (including complex)

Common Formative Assessments: P3(9-12)

- Graphing rational functions- stations activity ★
- Variation practice ★

Learning Plan

- Students will complete textbook practice problems working in groups during class and individually outside of class **TCC1(9-12**)
- Class discourse on correct solutions to textbook problems - compare strategies and ask questions **P2(9-12):**
- Extra practice on graphing rational functions- work in groups-class discourse on correct solutions- compare strategies and ask questions **CCE2(9-12)★**
- Individual/group in class practice on concepts that will be measured on the summative assessment**TCC3(9-12)**

Teacher Resources: Blitzer Precalculus textbook, teacher created notes pages and extra practice, graphing calculators

Unit 4 - Rational Functions (CP only)

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (* indicates a modeling standard)

- **A.APR.D.7** (+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
- **A.SSE.B.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
    - **A.SSE.B.3.A** Factor a quadratic expression to reveal the zeros of the function it defines.
- **A.REI.A.2** Solve simple rational equations in one variable, and give examples showing how extraneous solutions may arise.
- **F.IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases*
    - **F.IF.C.7.D** (+)Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior

Standards for Mathematical Practice

- MP1: Make sense of problems and persevere in solving them.
- MP2: Reason abstractly and quantitatively.
- MP3: Construct viable arguments and critique the reasoning of others.
- MP4: Model with mathematics.
- MP5: Use appropriate tools strategically.
- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... (“I can” statements to be demonstrated)

- **TCC3(9-12)**: I can integrate relevant information to produce multiple valid solutions.
- **TCC4(9-12)**: I can integrate my learning to adapt to experiences in the classroom, career and life.
- **TCC1(9-12)**: I can ask purposeful, insightful questions to find a variety of innovative solutions.
- **CCE3(9-12):** I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.
- **TI3(9-12)**: I can formulate and investigate probing questions to further my learning.

- **P2(9-12)**: I can strengthen my weaknesses by identifying, initiating, and practicing appropriate strategies to become confident in my ability to overcome my challenges.
- **P3(9-12)**: I can accept constructive feedback and use setbacks to adjust my learning journey in order to reach my goals.

Understandings: Students will understand that…	**Essential Questions:**
• Equations, verbal descriptions, graphs and tables provide insight into the relationship between quantities • Students will learn what rational functions are, including their structure as the ratio of two polynomials. They will understand how to identify and work with these functions in various mathematical contexts.	• How can the relationship between quantities best be represented? • How do the properties of rational functions affect their graphs and behaviors • What are the strategies for simplifying and solving equations involving rational functions? • How can you use rational functions to model and solve real-world problems? • In what ways do the characteristics of rational functions differ from those of polynomial functions and how do these differences influence their application? • How do vertical and horizontal asymptotes of rational functions help in predicting the long-term behavior of the function?
Students will know…	**Students will be able to…**
• The domain restrictions and the end behavior define the asymptotes in the graph. • Holes occur when the numerator and denominator have a common linear factor. • The asymptotes, intercepts, and domains are used to create the graph of the rational function. • How to apply simplifying and solving skills to solve rational function applications • Students will develop skills to graph rational functions by considering asymptotic behavior, intercepts, and any discontinuities. They will practice sketching accurate graphs and understanding the function's overall shape and behavior.	• Define rational function as the ratio of two polynomials • State the end behavior of a rational function when looking at the graph of the function • Find y-intercept of a rational function • Find the x-intercepts of a rational function • Determine if a rational function has a horizontal asymptote and find the asymptote • Determine domain and range of a rational function • Determine if a rational function has a hole • Sketch a graph of a rational function based on its domain, x-intercepts, y-intercept, and end behavior • Use technology to graph rational functions and to find precise values for the x-intercepts and the maximums and minimum • Simplify, multiply,divide rational functions • Add and subtract rational functions • Solve rational functions

Key Vocabulary:

Rational Equation	Vertical Asymptote	End Behavior
X-Intercept	Horizontal Asymptote	Factor
Y-Intercept	Hole	Polynomial

Assessment Evidence

Performance Tasks:
Summative:

- **Test:** Rational Expressions and Equations **TCC3(9-12)**
  - Simplify, add, subtract, multiply divide rational expressions
  - Solve rational equations
- **Graphing Rational Equations Project**: Students are randomly assigned a rational function. Students find all necessary information and graph the function. Projects will be assessed using department rubric. **CCE3(9-12)**

Other Evidence:
Class Activities:

- Error Analysis: Multiplying and dividing Rational Expressions activity
- Partner Multiplying/dividing activity **CCE3(9-12)**
- Google Slides Adding and subtracting Rational Expressions digital activity
- Solving rational equations partner activity **CCE3(9-12)**
- Pass the Graph Activity **CCE3(9-12)**
- Graphing Rational Functions Flip book activity**P2(9-12)**

Interims:

- Quiz: Simplify, Multiply, Divide Rational Expressions **P2(9-12)**
- Quiz Adding and Subtracting Rational Expressions **P3(9-12)**

Learning Plan

The unit will emphasize the importance of simplifying rational functions by factoring both the numerator and the denominator, and canceling common factors. This process helps in identifying features such as holes in the graph.

- Students will complete textbook practice problems working in groups during class and individually outside of class **TI3(9-12), TCC3(9-12)**
- Class discourse on correct solutions to textbook problems - compare strategies and ask questions**TCC1(9-12)**
- Extra practice on simplifying radicals- white board practice-class discourse on correct solutions- compare strategies and ask questions **CCE3(9-12)**
- Individual/group in class practice on concepts that will be measured on the summative assessment**TCC3(9-12), TCC3(9-12)**

Teacher Resources: Blitzer Precalculus Textbook, TI-84 graphing calculator, Desmos calculator, mini white boards, teacher created worksheets

Unit 4 - Exponential Functions/Logs (*Honors only)

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (* indicates a modeling standard; ★ indicates greater emphasis in the Honors sections)

- **A.SSE.A.1** Interpret expressions that represent a quantity in terms of its context.*
  - **A.SSE.A.1.B** Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)^n as the product of P and a factor not depending on P.
- **A.SSE.B.3** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
  - **A.SSE.B.3.C** Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as [1.15^(1/12)]^(12t) ≈ 1.012^(12t) to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.
- **A.CED.A.2** Create equations that describe numbers or relationships. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
- **A.REI.D.11** Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
- **F.IF.C.7** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
  - **F.IF.C.7.E** Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.*
- **F.IF.C.8** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
  - **F.IF.C.8.B** Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as y = (1.02)^t, y = (0.97)^t, y = (1.01)^(12t), y = (1.2)^(t/10), and classify them as representing exponential growth and decay.
- **F.BF.A.1** Write a function that describes a relationship between two quantities.*
- **F.BF.B.5 (+)** Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.
- **F.LE.A.1** Distinguish between situations that can be modeled with linear functions and with exponential functions.
  - **F.LE.A.1.C** Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
- **F.LE.A.2** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
- **F.LE.A.3** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly,

quadratically, or (more generally) as a polynomial function.

- **F.LE.A.4** For exponential models, express as a logarithm the solution to ab^(ct) = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
- **F.LE.B.5** Interpret the parameters in a linear or exponential function in terms of a context.
- **S.ID.B.6** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
    - **S.ID.B.6.A** Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear, quadratic, and exponential models.

Standards for Mathematical Practice

- MP1: Make sense of problems and persevere in solving them.
- MP2: Reason abstractly and quantitatively.
- MP3: Construct viable arguments and critique the reasoning of others.
- MP4: Model with mathematics.
- MP5: Use appropriate tools strategically.
- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... (“I can” statements to be demonstrated)

- **TCC4(9-12):** I can integrate my learning to adapt to experiences in the classroom, career and life.
- **CCE3(9-12):** I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.
- **AA1(9-12):** I can evaluate different approaches and justify the best pathway to success.
- **TI3(9-12):** I can formulate and investigate probing questions to further my learning.

Understandings: Students will understand that…

- Exponential functions model situations which grow or decline at a constant percent rate.
- Graphing exponential functions and logarithmic functions will result in asymptotic behavior. Transformations (shifts, stretches, reflections) will affect the graphs.
- Logarithms can be used to solve exponential functions; and conversely, exponents are used to solve logarithmic equations.

Essential Questions:

- How can exponential functions be used to model growth and decay?
- What is the purpose of logarithms and exponential functions and how are they related to each other?
- How do you graph exponential and logarithmic functions by analyzing intercepts and end behavior?
- How are exponential and logarithmic functions used to model real-world phenomena?

• Logarithms can be used to solve equations for which no other algebraic method exists. • Exponential functions model situations involving growth or decay, such as population growth, radioactive decay, and interest calculations, while logarithmic functions are used to model phenomena like the Richter scale for earthquakes and pH in chemistry. • Exponential and logarithmic functions are inverse functions.	
Students will know… • The graph of exponential growth and decay functions has a horizontal asymptote. • Exponential functions to model real-world situations, such as student loan interest, car depreciation, and • The constant *e* is an irrational number that appears in many different areas of mathematics, including continuous interest. • Expressions can be translated between exponential and logarithmic forms. • The common log has base 10 and the natural log has base *e*. • Exponential and logarithmic functions have restrictions on the value of the base *b*. This then influences the range or domain. • The graph of a logarithmic function has a vertical asymptote. • Condensing and expanding logarithmic expressions are techniques used when solving. • Solving exponential and logarithmic equations involves applying properties of exponents and logarithms, such as the power rule, product rule, and change of base formula, to isolate the variable and find solutions. • Techniques for solving equations involving exponential and logarithmic functions include using properties of logarithms, isolating the exponential or logarithmic expression, and applying logarithms to both sides of the equation. • The exponential function $y=b^x$ is one-to-one, so its inverse $x=b^y$ is a function. To express “y as a function of x” for the inverse you write $y=\log_b x$. • Logistic growth is a model used when there is an upper limit on the size of a population, called the carrying capacity. ★	**Students will be able to...** • Graph exponential growth and decay functions. • Identify the asymptote and transformation and solve for the intercepts of the graph. • Apply transformations with exponential and logarithmic functions to graph • Interpret graphs of exponential and logarithmic functions. • Model exponential functions for both growth (e.g. population growth and compound interest).and decay (e.g. depreciation and half-life). • Use compound interest to discover the number *e*. • Evaluate logarithms • Expand and condense logarithms • Solve exponential and logarithmic equations. • Solve real-world problems involving the common base *e*. • Model practical uses of logarithms (e.g., measuring sound intensity, pH scale). • Create and apply exponential and logarithmic functions to model and make predictions. • Solve problems using Newton’s Law of Cooling. ★ • Model functions using logistic growth and solve application problems. ★

Key Vocabulary:

Exponential Growth	Asymptote	Common Logarithm
Exponential Decay	Base	Natural Logarithm
Exponential Function	Natural base e	Power Function
Exponential Regression	Compounded continuously	Logarithmic Equation
Growth Factor	Logarithm	Extraneous Solution
Decay Factor	Logarithmic Regression	Inverse functions
Compounded monthly, quarterly, etc.		

Assessment Evidence

Performance Tasks:

Summative: Students will research colleges in state and out of state specifically focused on tuition prices over the years. Through the use of graphing calculators they will use the data to create an exponential regression model to make predictions for the future. Students will work collaboratively in small groups and will present their findings to the class. **TCC4(9-12)**, **CCE3(9-12)**

Summative Unit Assessment TCC4(9-12) Topics will include:

- Graphing exponential and logarithmic functions.
- Identifying key features of the graph.
- Simplify logarithms
- Expand and condense logarithmic expressions
- Solve exponential and logarithmic equations.
- Model real-world applications.
- Inverse functions

Other Evidence:

- **Activity - Which will grow faster?** Introduction to exponential growth. If you double a penny every day for 30 days what will happen? **AA1(9-12), TI3(9-12)**
- **Activity - Discovery of the number e:** Students will explore the number e through a discovery based lesson to attempt to see what happens when you increase the compounding interest. They will discuss practical applications of e in various fields like finance, population growth, and physics. **AA1(9-12), TI3(9-12)**
- **Interim:** Graphing exponential functions, comprehension of key features such as growth/decay factor, growth/decay rate, asymptote and transformations.
- **Interim:** Applications involving compound growth/decay including continuously. Simplifying with the number e.
- **Interim:** Evaluating logarithms, comprehension of natural logs, common logs and expanding and condensing. Using logarithms to solve exponential equations, which may have extraneous solutions. **AA1(9-12)**

Learning Plan

In this unit, students will extend their knowledge of exponential functions to model growth and decay in numerous applications.Students will explore how transformations (translations, reflections, stretches and compressions) affect exponential and logarithmic functions. Students will learn and apply the fundamental properties of logarithms including the production, quotient, and power rules. Students will solve complex logarithmic and exponential equations and apply them to real world problems and make predictions.

- Students will extend their knowledge of transformations of functions by graphing exponential and logarithmic functions. Discussion around similarities and differences both between these two functions graphically and algebraically to discover they are inverse functions of each other. **CCE3(9-12)**
- Students will evaluate different approaches when solving exponential and logarithmic equations. This may include needing to use the properties of expanding and condensing, as well as utilizing their previous skills for solving and checking for extraneous solutions. **AA1(9-12)**
- Students will model exponential growth and decay models involving applications with simple interest, compound interest, depreciation. As they explore which model grows faster and complete the discovery activity for the number *e*, students will work on collaborating and adjusting to meet their learning needs. **TI3(9-12)** Then students will complete the College Tuition Project (described above) **TCC4(9-12); CCE3(9-12)**

Teacher Resources: Blitzer Precalculus Textbook, TI-84 graphing calculator, Desmos calculator, mini white boards, teacher created worksheets

Unit 5 - Trigonometric Functions

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (* indicates a modeling standard; ★ indicates greater emphasis in the Honors sections)

- **F.TF.A.1** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
- **F.TF.A.2** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
- **F.TF.A.3 (+)** Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for π - x, π + x, and 2π - x in terms of their values for x, where x is any real number.
- **F.TF.A.4 (+)** Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.
- **F.TF.B.6 (+)** Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.★
- **G.SRT.C.6** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
- **G.SRT.C.7** Explain and use the relationship between the sine and cosine of complementary angles.
- **G.SRT.C.8** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Standards for Mathematical Practice

- MP1: Make sense of problems and persevere in solving them.
- MP2: Reason abstractly and quantitatively.
- MP3: Construct viable arguments and critique the reasoning of others.
- MP4: Model with mathematics.
- MP5: Use appropriate tools strategically.
- MP6: Attend to precision.
- MP7: Look for and make use of structure.
- MP8: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... (“I can” statements to be demonstrated)

- **TCC3(9-12):** I can integrate relevant information to produce multiple valid solutions.
- **P2(9-12):** I can strengthen my weaknesses by identifying, initiating, and practicing appropriate strategies to become confident in my abilities to overcome my challenges.
- **P3(9-12):** I can accept constructive feedback and use setbacks to adjust my learning journey in order to reach my goals.

- **TI4(9-12):** I can apply my strengths and anticipate challenges to reach my current and future goals
- **CCE3(9-12):** I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.

Understandings: Students will understand that…

- The trigonometric functions (sine, cosine, tangent, and their reciprocals) represent specific relationships between the angles and side lengths of a right triangle. Understanding these functions as ratios helps students connect geometric intuition with algebraic representation.
- The unit circle is a foundational concept in trigonometry that connects the angle measures in radians with the coordinates of points on the circle. This understanding leads to the concept of periodicity in trigonometric functions, where patterns repeat over regular intervals.
- The graphs of sine, cosine, and tangent functions exhibit unique characteristics such as amplitude, period, phase shift, and vertical shift. Recognizing these properties allows students to model real-world phenomena and understand oscillatory behavior.
- Inverse trigonometric functions, such as arcsine, arccosine, and arctangent, are essential for finding angles when given a trigonometric ratio. Understanding their domains, ranges, and interpretations is crucial for solving trigonometric equations and modeling. ★
- Trigonometric functions can be represented analytically (using equations) and graphically (using graphs). Understanding both forms and how they interrelate is important for solving problems and interpreting data.
- Understanding different units of angle measurement, such as degrees and radians, and how to convert between them is crucial for working with trigonometric functions and applying them in various contexts.

Essential Questions:

- How are sine, cosine, and tangent defined using the sides of a right triangle?
- What are the reciprocal trigonometric ratios (cosecant, secant, cotangent)?
- How do the coordinates of points on the unit circle relate to trigonometric functions?
- What are the radian and degree measures, and how do they relate to each other?
- How do amplitude, period, phase shift, and vertical shift affect the graphs of trigonometric functions?
- What are the key features of the graphs of sine, cosine, tangent, cosecant, secant, and cotangent functions?
- What are the definitions and domains of the inverse sine, cosine, and tangent functions? ★
- How are the inverse functions used to find angles given a trigonometric ratio? ★

Students will know…

- A positive angle is formed by rotating a ray about its endpoint in

Students will be able to…

- Draw angles in standard position

a counterclockwise direction • 180 degrees is equivalent to pi radians. • That coterminal angles are formed by adding or subtracting 2 pi radians or 360 degrees/ • All six exact trigonometric values of π, π/2, π/4, π/3, π/6 and their reference angles. • The domain of sine and cosine is all real numbers and the range is from -1 to 1, inclusive. • How to use the calculator to find the values of the six trig functions. • How to graph the sine and cosine functions with transformations including changes in amplitude, period, and phase shift. • How to graph the parent function of tangent, cosecant, secant, and cotangent functions ★ • How to evaluate inverse sine, cosine, and tangent functions using the unit circle values including composition functions. ★ • How to write a sine or cosine function given a graph of the function ★	• Covert angles between radian and degree measures • Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle • Find coterminal angles • Use the unit circle to find the exact values of the six trigonometric functions • Recognize the domain and range of sine and cosine • Evaluate trigonometric functions with a calculator • Graph the sine and cosine functions using transformations • Graph the tangent, cosecant, secant, and cotangent functions ★ • Understand and use the inverse sine, cosine, and tangent functions ★ • Find the exact values of the composite functions with inverse trigonometric functions ★ • Write a sine or cosine equation that represents a given graph ★
Key Vocabulary: Radian, Degree, Coterminal Angles, Complement, Supplement, Sine, Cosine, Tangent, Secant, Cosecant, Cotangent, Reference Angle, Unit Circle, Amplitude, period	
Assessment Evidence	
Performance Tasks: **Summative Assessment:** Unit 5 Test **(TCC3(9-12), TI4(9-12))** Topics to include: • Sketch angles in standard position • Find coterminal angles • Convert between degrees and radians • Find points on the unit circle given and angle in radians • Find exact values of the 6 trig functions • Graph sine and cosine functions (with transformations) • Graph tangent, cotangent, secant, cosecant parent functions (Honors) • Graph inverse sine, cosine, tangent (Honors) • Evaluate inverse trig function values including composition and	**Other Evidence:** **Interim assessments: P3(9-12)** • Unit circle quiz • Graphing sine and cosine quiz **Common Formative Assessments: P3(9-12)** • Unit circle practice • Graphing sine and cosine practice • Inverse trig function practice ★

algebraic quantities (Honors) Real World applications are most prevalent when: ● Calculating sound waves ● Projectile motion	

Learning Plan

- Students will complete textbook practice problems working in groups during class and individually outside of class **TCC3(9-12)**
- Class discourse on correct solutions to textbook problems - compare strategies and ask questions **CCE3(9-12), P2(9-12)**
- Extra practice on unit circle, graphing functions, and inverse trig functions(★)- white board practice-class discourse, on correct solutions-compare strategies and ask questions **P2(9-12), CCE3(9-12)**
- Individual/group in class practice on concepts that will be measured on the summative assessment**TCC3(9-12)**

Teacher Resources: Blitzer Precalculus Textbook, TI-84 graphing calculator, Desmos calculator, mini white boards, teacher created worksheets

Unit 6 - Trigonometric Identities

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (* indicates a modeling standard; ★ indicates greater emphasis in the Honors sections)

- **F.TF.C.8** Prove the Pythagorean identity $sin^2(\theta) + cos^2(\theta) = 1$ and use it to find $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ given $sin(\theta)$, $cos(\theta)$, or $tan(\theta)$ and the quadrant of the angle.
- **F.TF.C.9** (+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems
- **A.SSE.A.2** Use the structure of an expression to identify ways to rewrite it.

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8**: Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... ("I can" statements to be demonstrated)

- **TCC3(9-12)**: I can integrate relevant information to produce multiple valid solutions.
- **CCE2(9-12)**: I can give and receive actionable and relevant feedback with openness to be able to determine meaningful revisions for success.
- **CCE3(9-12)**: I can show initiative in prompting group discourse and fostering collaboration among others, providing actionable feedback, and working with others to solve problems and/or design products.
- **CCE4(9-12)**: I can communicate and express my understanding in an authentic, respectful and relevant way, using the most effective mode of expression.
- **DE1(9-12)**: I can acknowledge, process, and respond appropriately to others' comments and perspectives.
- **TI3(9-12)**: I can formulate and investigate probing questions to further my learning.
- **P1(9-12)**: I can self-reflect and seek feedback to build upon my strengths, apply those strengths to overcome obstacles, and share my strategies with others.
- **P3(9-12)**: I can accept constructive feedback and use setbacks to adjust my learning journey in order to reach my goals.
- **AA1(9-12)**: I can evaluate different approaches and justify the best pathway to success.
- **AA2(9-12)**: I can assess my past successes and mistakes to change my approach.

Understandings: Students will understand that… • Trigonometric functions are useful for modeling periodic phenomena. • Expressions can be written in multiple ways using the rules of Trigonometry and Algebra and used to solve trigonometric functions.	**Essential Questions:** • Why do we structure trig functions in different ways? • What is the difference between solving an equation and verifying a trigonometric identity? • How do you decide which strategies to use when verifying a trigonometric identity? • How are trigonometric identities used in the process of solving trig equations?
Students will know… • How to use right triangles to prove the sum of angles identities for cosine and sine • How to convert the sum of angles identities into the difference of angles identities • How to apply the sum and difference formulas for sine and cosine to prove the sum and difference formulas for tangent • Double and half angle math lib activity • How to recognize angles that can be written using sums of $\Pi/6,\ \Pi/4,\ and\ \Pi/3$. • How to use the sum and difference formulas for sine, cosine, and tangent to solve for exact values of the trigonometric functions of other angles	**Students will be able to…** • Conduct Trigonometric proofs using the Pythagorean Identities • Conduct Trigonometric proofs using Reciprocal identities • Conduct Trigonometric proofs using Quotient identities • Conduct Trigonometric proofs using Angle sum and difference identities for sine, cosine, and tangent. • Conduct Trigonometric proofs using the Double-Angle Identities for sine, cosine, and tangent.★ • Conduct Trigonometric proofs using the Half-angle Identities for sine, cosine, and tangent★ • Solve trigonometric equations using identities • Solve using Half-Angle and Double-Angle Identities★

Key Vocabulary:

Sine	Cotangent	Pythagorean identity
Cosine	Sum Formula	Pythagorean theorem
Tangen	Difference Formula	Unit circle
Secant	Double Angle Formula	Quotient identity
Cosecant	Half Angle Formulas	

Assessment Evidence	
Performance Tasks:	**Other Evidence:**

Summative Assessment: Trigonometric Identities Test **TCC3(9-12), AA1(9-12), AA1(9-12)** Topics to include:

- Verify trig identities using
  - Pythagorean Identities
  - reciprocal identities
  - Quotient identities
  - Angle sum and difference Formulas★
- Find all solutions of all six trig functions given a domain interval
- Use Double-Angle and Half-Angle Identities to solve ★
- Use right triangles to solve trig functions ★

Real World Examples include:

- Solving real-word problems involving periodicity,angles and wave-like behavior
- Engineering and construction: calculating forces, stresses, and angles in bridges, buildings, and other structures
- Physics and Engineering (Waves and Oscillations) such as sound and light waves.
- Computer graphic and animations
- Medical imaging.

Interim assessments:

- Identities Quiz **AA1(9-12), P3(9-12), AA1(9-12, AA2(9-12)**
  - Verify trig identities using reciprocal, quotient, and pythagorean identities
  - Use rationalizing and factoring strategies to verify trig identities.★
- Quiz 2: Solving Trig Functions on a restricted interval using: **TCC3(9-12), P3(9-12), AA1(9-12, AA2(9-12)**
  - Reciprocal Identities
  - Quotient Identities
  - Pythagorean Identities
  - Sum and Difference Formulas

Learning Plan

Precalculus continues the learning of the study of functions. Students will learn strategies to solve trig functions on a restricted domain in order to choose trigonometric functions to model periodic phenomena.

- Students will complete textbook practice problems working in groups during class and individually outside of class
- Class discourse on correct solutions to textbook problems - compare strategies and ask questions
- Extra practice on trig identities- white board practice-class discourse on correct solutions- compare strategies and ask questions
- Verifying Trig Identities Google slides digital activity with immediate feedback **AA1(9-12)**
- Trigonometric Identities Triangle puzzle group project activity **CCE2(9-12), CCE3(9-12), CCE4(9-12)**
- Check your mate partner activity for solving Trig functions **CCE2(9-12), DE1(9-12)**
- Graphic organizer for referencing All trig identities
- Individual/group in class practice on concepts that will be measured on the summative assessment

Teacher Resources: Blitzer Precalculus Textbook, TI-84 graphing calculator, Desmos calculator, mini white boards, teacher created worksheets, Google Slides, chrome book

Unit 7 - Modeling with Law of Sines and Cosines/ Sinusoids

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (* indicates a modeling standard; ★ indicates greater emphasis in the Honors sections)

- **G.SRT.D.9 (+)** *Apply trigonometry to general triangles.* Derive the formula A = (1/2)ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
- **G.SRT.D.10 (+)** *Apply trigonometry to general triangles.* Prove the Laws of Sines and Cosines and use them to solve problems.
- **G.SRT.D.11 (+)** *Apply trigonometry to general triangles.* Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, resultant forces).

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes:

- **TCC3(9-12)**: I can integrate relevant information to produce multiple valid solutions.
- **TCC2(9-12)**: I can evaluate evidence from multiple perspectives, and recognize their limitations and implications, in order to justify new conclusions.
- **DE1(9-12)**: I can acknowledge, process, and respond appropriately to others' comments and perspectives.
- **P1(9-12)**: I can self-reflect and seek feedback to build upon my strengths, apply those strengths to overcome obstacles, and share my strategies with others.
- **P3(9-12)**: I can accept constructive feedback and use setbacks to adjust my learning journey in order to reach my goals.
- **AA2(9-12)**: I can assess my past successes and mistakes to change my approach.

Understandings: Students will understand that…	**Essential Questions:**

• The law of sines and cosines are fundamental trigonometry laws that can be used to solve problems involving the sides and angles of non-right triangles. • In a non-right triangle the given information determines which law is most appropriate to solve a problem. • When you know two sides and a non-included angle (SSA), there can be zero, one, or two possible triangles, known as the "ambiguous case." • If the altitude of a non-right triangle is not given, it can be expressed using trigonometry and used to calculate its area.	• How are trigonometric ratios used in solving real-world problems? • How do you determine whether the law of sines or law of cosines is most appropriate for solving a particular problem? • In ambiguous cases, how many possible triangles can the Law of Sines produce? • Is it possible to solve for the area of a triangle if the altitude is not known? • What kind of real-life situations can be modeled by sinusoidal equations? • How can sinusoidal equations be used to predict future values? • How can the area of a triangle be found using the law of sines and cosines?
Students will know... • Law of Sines is most effective for angle-side pairs, especially when dealing with two angles and one side or two sides and a non-included angle. • Law of Cosines is more versatile for side-side-angle scenarios or when working with three sides of the triangle, as it can handle cases where the triangle is not right-angled. • Trigonometry can be used to calculate the area of a non-right triangle when you have the lengths of two sides and the included angle or only the length of the 3 sides • Many real-life situations are cyclical in nature and can be represented with sinusoidal equations.	**Students will be able to…** • Prove the Laws of Sines and Cosines and use them to solve problems. • Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side. • Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems, bearing problems) • Model a real-life situation with a sinusoidal equation and use the graphing calculator to predict other values. • Use the law of sines area formula or Heron's formula to find the area of a triangle.
Key Vocabulary: Law of Sines Law of Cosines Angle of Elevation Angle of Depression Bearings Heron's Formula Ambiguous Case Included Angle Solving Triangles Acute Triangle Right Triangle Obtuse Triangle Oblique Triangle	
Assessment Evidence	

Performance Tasks:	**Other Evidence:**
Performance Task 1: Create one creative application problem that would be solved by the law of sines/law of cosines. Students will follow a rubric to help guide them to clearly express their problem as well as get peer feedback. **TCC3(9-12), DE1(9-12), P1(9-12)** **Summative Unit Assessment: TCC2(9-12), AA2(9-12)** Topics will include: ● Law of Sines ● Law of Cosines ● Solving Triangles ● Recognizing and solving ambiguous case situations ● Area of a triangle using trigonometry ● Modeling with the Law of Sines/Cosine ● Create sinusoidal equations that represent real-life situations ● Enter sinusoids in the calculator and make predictions	● **Interim:** Solving non-right triangles for missing sides and angles with the Law of Sines. Students will also show their knowledge of the ambiguous case and if two triangles exist they will be asked to produce both possible solutions. **TCC3(9-12), P3(9-12)** ● **Interim**: Solving a triangle for missing sides and angles and calculating the area of a triangle A=1/2bh with the Law of Cosines. **TCC2(9-12)** ● **Interim:** Solving application problems involving the Law of Sines and the Law of Cosines. Application problems may involve bearings as well as angles of elevation or depression. **P3(9-12)**

Learning Plan

In this unit, students will extend their knowledge of solving right triangles with trigonometry to non-right triangles with trigonometry. They will also create a formula for finding the area of a triangle with trigonometry. Students will apply this learning to real world problems.

- Students will extend their knowledge of right triangle trigonometry through discovery lessons of the Law of Sines, Law of Cosines and area formulas through an investigative approach. **TCC2(9-12)**
- Students will evaluate different scenarios when solving with the Law of Sines and Law of Cosines and decipher which is more appropriate to utilize in solving for the missing parts of a non-right triangle. **TCC3(9-12)**
- Students will solve real world application problems involving different scenarios including bearings and angles of elevation and depression. **P1(9-12), P3(9-12)**

Teacher Resources: Blitzer Precalculus Textbook, TI-84 graphing calculator, Desmos calculator, mini white boards, teacher created worksheets

Unit 8 - Matrices

Desired Results - Goals, Transfer, Meaning, Acquisition

Established Goals:

Common Core State Standards (* indicates a modeling standard; ★ indicates greater emphasis in the Honors sections)

- **N.VM.C.6** (+) Use matrices to represent and manipulate data, e.g., to represent payoffs or incidence relationships in a network.
- **N.VM.C.7** (+) Multiply matrices by scalars to produce new matrices, e.g., as when all of the payoffs in a game are doubled.
- **N.VM.C.8** (+) Add, subtract, and multiply matrices of appropriate dimensions.
- **N.VM.C.9** (+) Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties.
- **N.VM.C.10** (+) Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.

Standards for Mathematical Practice

- **MP1:** Make sense of problems and persevere in solving them.
- **MP2:** Reason abstractly and quantitatively.
- **MP3:** Construct viable arguments and critique the reasoning of others.
- **MP4:** Model with mathematics.
- **MP5:** Use appropriate tools strategically.
- **MP6:** Attend to precision.
- **MP7:** Look for and make use of structure.
- **MP8:** Look for and express regularity in repeated reasoning.

Vision of A Learner Attributes: Students will be able to independently use their learning to... (“I can” statements to be demonstrated)

- **TCC3(9-12):** I can integrate relevant information to produce multiple valid solutions.
- **P2(9-12):** I can strengthen my weaknesses by identifying, initiating, and practicing appropriate strategies to become confident in my abilities to overcome my challenges.
- **P3(9-12):** I can accept constructive feedback and use setbacks to adjust my learning journey in order to reach my goals.
- **TI4(9-12):** I can apply my strengths and anticipate challenges to reach my current and future goals.

Understandings: Students will understand that…

- A matrix is a rectangular array of numbers arranged in rows and columns, representing a set of data or a system of linear equations.
- The size or dimension of a matrix (rows × columns) is a

Essential Questions:

- What is a matrix, and how does it represent data or systems of equations in a structured way?
- How can matrices be used to solve systems of linear equations,

fundamental concept that affects how matrices can be manipulated and used in calculations. ● Students should understand that matrices of the same dimensions can be added or subtracted element-wise, with each element in the result being the sum or difference of corresponding elements. ● Multiplying a matrix by a scalar (a single number) scales each element of the matrix, maintaining its structure while altering its magnitude. ● Matrix multiplication is not commutative, meaning the order of multiplication matters. Students should understand the conditions under which matrix multiplication is defined and how to perform it. ● Matrices provide a powerful way to represent and solve systems of linear equations, with each system corresponding to a matrix equation. ● Matrices can be used to model and solve problems in fields such as economics, engineering, computer graphics, and more. ● The determinant of a square matrix is a special number that can provide insights into the matrix's properties, such as whether it is invertible. ● A matrix that has an inverse (i.e., it is invertible) can be used to solve systems of linear equations. Students should understand the conditions under which a matrix has an inverse and how to compute it. ● The identity matrix acts as the multiplicative identity in matrix multiplication, analogous to the number 1 in regular multiplication. ● As matrices grow in size, operations involving them can become computationally intensive. Understanding the complexity of these operations is important for practical applications. ● While manual computation with matrices is important for learning, recognizing the role of technology in handling large and complex matrices is also essential.	and why is this method sometimes more efficient than traditional methods? ● In what ways do matrices model real-world phenomena, such as geometric transformations, computer graphics, and economic systems? ● Under what conditions does a matrix have an inverse, and why is the inverse of a matrix important in solving systems of equations and other applications?
Students will know…	**Students will be able to…**

• The order of a matrix is represented by the number of rows by the number of columns. • Only matrices of the same dimensions can be added and subtracted. • It is only possible to multiply two matrices when the number of columns in the first matrix is the same as the number of rows in the second matrix. • Matrix operations can be performed on the graphing calculator. • A determinant is a real number associated with a square matrix and can be used to find the inverse of a matrix. • Matrices can be used to solve a system of equations by multiplying the inverse of the coefficient matrix by the constant matrix. • How to represent a real-life situation with matrices.	• State the order of a matrix • Add, subtract, and multiply matrices by hand and on the graphing calculator • Find the determinant of a 2 X 2 matrix and of a 3 X 3 matrix on the calculator and by hand. • Find the inverse of a 2X2 matrix by hand and of a 3X3 matrix on the calculator • Translate a system of equations into a matrix equation • Use matrices to solve a system of linear equations: a 2X2 system by hand and a 3X3 system on the calculator • Solve a real-life application problem using matrices

Key Vocabulary:

Matrix	Order of a matrix	Inverse Matrix
Matrices	Square Matrix	Coefficient Matrix
Row	Row Matrix	Constant Matrix
Column	Column Matrix	Determinant

Assessment Evidence

Performance Tasks: **Summative Unit Assessment TCC3(9-12)** Topics include: • Add, subtract and multiply matrices by hand • Find the determinant of a matrix • Find the inverse of a matrix • Solve a system of equations using matrices by hand and on the calculator • Solve application problems using matrices	**Other Evidence:** Note: these will be given if time permits at the end of the year: **Interim: P3(9-12)** • Quiz on adding, subtracting, and multiplying matrices by hand • Quiz on solving a system by hand using matrices

Learning Plan

- Students will complete textbook practice problems working in groups during class and individually outside of class **TCC3(9-12)**
- Class discourse on correct solutions to textbook problems - compare strategies and ask questions **P2(9-12) P3(9-12)**
- Strategies on the graphing calculator will be stressed in this unit

• Individual/group in class practice on concepts that will be measured on the summative assessment **TCC3(9-12)**
Teacher Resources: Blitzer Precalculus Textbook, TI-84 graphing calculator