Marietta City Schools

2024–2025 District Unit Planner

Advanced Algebra: Concepts & Connections

Unit title	Unit 4: Modeling Polynomial Functions	**Unit duration (hours)**	*22.5*

Mastering Content and Skills through INQUIRY (Establishing the purpose of the Unit): ***What will students learn?***

GA DoE Standards

Standards

AA.FGR.5: Extend exploration of quadratic solutions to include real and non-real numbers and explore how these numbers behave under familiar operations and within real-world situations; create polynomial expressions, solve polynomial equations, graph polynomial functions, and model real-world phenomena.

AA.FGR.5.1 Graph and analyze quadratic functions in contextual situations and include analysis of data sets with regressions.

Fundamentals

• As an introduction to polynomial functions, students should be able to use quadratic functions in standard, factored, and vertex forms to graph and identify key features in context in order to answer questions about real-life phenomena.

• Key features of quadratic functions should include x and y-intercepts, roots, zeros, and solutions; domain, range, and intervals where the function is increasing, decreasing, positive, and/or negative (using inequality and interval notations); vertex, extreme value, and axis of symmetry; end behavior, using technology where appropriate.

• Students should be able to calculate the slope of average rate of change for a given interval, including the estimated rate of change.

• Through contextual exploration, students should recognize that there are data sets for which a quadratic function is not the best model, and therefore, explore other types of polynomial regression.

• Analysis of data sets with regressions should be done informally with verbal descriptions and with the use of technology.

AA.FGR.5.2 Define complex numbers i such that $i^2 = -1$ and show that every complex number has the form a + bi where a and b are real numbers and that the complex conjugate is a - bi.

Fundamentals

• Students should be able to identify the real part of a complex number and the imaginary part.

• Students should convert any power of the imaginary unit, i, to an equivalent form and identify the pattern that emerges.

• Students should have opportunities to identify the complex conjugate of any complex number and recognize that complex numbers always occur as pairs when they represent solutions to a polynomial function.

AA.FGR.5.3 Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Fundamentals

• Students should be provided opportunities to solve real-life problems that require the addition, subtraction, or multiplication of complex numbers.
• Division of complex numbers is beyond the scope of Advanced Algebra

AA.FGR.5.4 Use the structure of an expression to factor quadratics.

Relevance and Application • Expressions should include special-case quadratics such as perfect-square trinomials and the difference of two perfect squares.

AA.FGR.5.5 Write and solve quadratic equations and inequalities with real coefficients and use the solution to explain a mathematical, applicable situation.

Relevance and Application • Equations and inequalities presented in real-life, mathematical problems should include quadratics with complex solutions.

Fundamentals

• In previous grades, students had opportunities to create and solve quadratic equations. Given a real life scenario, students should be able to model the scenario using quadratic equations and inequalities in one variable.
• Given a quadratic equation or inequality in one variable (model), students should be able to create a real-life scenario that matches the model.
• Students should be able to connect the solutions of quadratic equations and inequalities to the graph of the corresponding quadratic function and use these to solve real-life problems that can be modeled by quadratic equations and inequalities.
• Students should be able to model real-life occurrences with quadratic equations or inequalities and use these to solve problems.

Strategies and Methods

• Students should be able to solve quadratic equations and inequalities fluently (flexibly, accurately, efficiently) by inspection, taking square roots, factoring, completing the square, and applying the quadratic formula, as appropriate to the initial form of the equation.
• Students should be provided opportunities to explore a variety of real-life problems modeled by quadratic equations and inequalities.

Example • Students can create equations involving areas using unknown dimensions

AA.FGR.5.6 Solve systems of quadratic and linear functions to determine points of intersection.

Fundamentals • Students should be able to solve real-life problems modeled by systems of quadratic and linear functions using algebraic techniques (by hand) or using technology to identify the intersections of a parabola and a line.

AA.FGR.5.7 Create and analyze quadratic equations to represent relationships between quantities as a model for contextual situations.

Fundamentals

• Students should be able to solve real-life problems modeled by quadratic equations and inequalities in two or more variables.
• Given a real-life scenario, students should be able to model the scenario using quadratic equations and inequalities in two or more variables.
• Given a quadratic equation or inequality in two or more variables (model), students should be able to create a real-life scenario that matches the model.

Example • Students can create equations involving projectile motion.

AA.FGR.5.8 Identify the number of zeros that exist for any polynomial based upon the greatest degree of the polynomial and the end behavior of the polynomial by observing the sign of the leading coefficient.

Fundamentals

• Given a polynomial function, students should be able to apply the Fundamental Theorem of Algebra to describe the maximum number of times the function may cross the x-axis.
• Given a polynomial function, students should be able to tell if the left and right sides are increasing as x approaches negative and positive infinity based upon the sign of the leading coefficient and whether the greatest exponent is even or odd.
• Students should verify, using technology, if their predictions are correct for the number of zeros a polynomial has. When there are fewer zeros than the highest exponent led them to expect, students should understand that this is because there are complex solutions.
• Students should understand that complex solutions always occur in pairs.

Example • Given the polynomial function, $f(x)=-2x^5 - 4x^4 + x^3 -6x^2 +9x + 4$ students should be able to say that there are five or fewer real zeros, and that as x approaches negative infinity, f(x) approaches infinity, and as x approaches infinity, f(x) approaches negative infinity. Using technology, students should be able to verify the number of zeroes and/or complex solutions of this polynomial.

AA.FGR.5.9 Identify zeros of polynomial functions using technology or pre-factored polynomials and use the zeros to construct a graph of the function defined by the polynomial function. Analyze identify key features of these polynomial functions.

Fundamentals

• Students should be able to graph and identify key features of polynomial functions to include x and y-intercepts, roots of multiplicity, zeros, and solutions; domain, range, and intervals where the function is increasing, decreasing, positive, and/or negative (using inequality and interval notations); vertex, extreme value, and axis of symmetry; end behavior, using technology where appropriate.

• Students are not expected to graph polynomial functions presented in standard form, by hand.

• Students should be able to identify key features of a polynomial equation to create a rough sketch of a graph, by hand.

• When presented with a polynomial function in standard form, students should be able to use technology to graph the function and identify key features, including the zeros of the function.

AA.FGR.5.10 Use the structure of an expression to factor polynomials, including the sum of cubes, the difference of cubes, and higher-order polynomials that may be expressed as a quadratic within a quadratic.

Fundamentals • Students should be able to rewrite polynomial expressions in various equivalent forms, based on the context of the problem.

Example • $x^4 - y^4 = (x^2)^2 - (y^2)^2 = (x^2 + y^2)(x^2 - y^2) = (x^2 + y^2)(x + y)(x - y)$.

AA.FGR.5.11 Using all the zeros of a polynomial function, list all the factors and multiply to write a multiple of the polynomial function in standard form.

Fundamentals

• Students should be able to analyze a graph of a polynomial to identify where multiplicity exists due to a local maximum or minimum value that is situated on the x-axis and recognize that repeating that factor may be necessary.

• Students should note that multiplying factors to write a polynomial generates one of many possible polynomials with the same zeros, because the leading coefficient of a polynomial is not evident from factor alone.

Example • Given the solutions of -2, 4, and 5 with a multiplicity of 2, students would be asked to write an equation that represents a multiple of the polynomial, f(x), in standard form. The number of possible equations is endless, but answers could include:

o $(x+2)(x-4)(x-5)(x-5) = x^4 - 12x^3 + 29x^2 + 30x - 200$, and

o $2(x+2)(x-4)(x-5)(x-5) = 2x^4 - 24x^3 + 58x^2 + 60x - 4$

AA.MM.1: Apply mathematics to real-life situations; model real-life phenomena using mathematics.

AA.MM.1.1 Explain applicable, mathematical problems using a mathematical model.

Fundamentals

• Students should be provided with opportunities to learn mathematics in the context of culturally relevant problems.

• Mathematically applicable problems are problems presented in context where the context makes sense, realistically and mathematically, and allows for students to make decisions about how to solve the problem (i.e., model with mathematics).

AA.MM.1.2 Create mathematical models to explain phenomena that exist in the natural sciences, social sciences, liberal arts, fine and performing arts, and/or humanities contexts.

Fundamentals

• Mathematically proficient students should be able to use the content learned in this course to create a mathematical model to explain real-life phenomena.

AA.MM.1.3 Using abstract and quantitative reasoning, make decisions about information and data from a mathematical, applicable situation.

Fundamentals

• Students should be able to:

to analyze functions, graphs, tables, and equations and make decisions about the real-life situations they describe based upon their understanding of mathematical functions.

to analyze statistical results to decide the best course of action or approach to a problem.

Example

• Given a rectangle with length = (x – 2) and width = (2x + 3), a student could discover and articulate that the area = (x – 2)(2x + 3) = 2x2 – x – 6. From the student's understanding of parabolas, a student would know that the parabola that represents all possible areas of this rectangle opens upwards and that there is no maximum area possible for this rectangle.

AA.MM.1.4 Use various mathematical representations and structures to represent and solve real-life problems.

Fundamentals

• Students should be able to generate models, graphs, charts, and equations, to represent real-world phenomena in order to solve problems.

• Students should be provided opportunities to generate representations of real-world phenomena utilizing technology to show these phenomena and to solve problems.

Concepts/Skills to support mastery of standards

Vocabulary

Axis	Coefficient	Complex Number	Concave Down	Concave Up	Decreasing
Degree	End Behavior	Exponent	Extrema	Factor	Features
Fundamental Theorem of Algebra	Higher Order Polynomials	i (the number "i")	Imaginary Number	Increasing	Intersection Point
Leading Coefficient	Limit	Maximum	Minimum	Multiple	Quadrant
Quartic	Polynomial	Root	Real Number	Regression	Zero

Notation

Standard Form: f(x) = ax^2 + bx + c
Vertex Form: f(x) = a(x - h)^2 + k
Factored Form: f(x) = a(x - r1)(x - r2)
Quadratic Formula: x = (-b ± √(b^2 - 4ac)) / (2a)
Discriminant Notation: b^2 - 4ac
Difference of Squares: a^2 - b^2 = (a + b)(a - b)
Sum of Square: a^2 + b^2 = (a + bi)(a - bi)
Sum or Difference of Cubes:
a^3 + b^3 = (a + b)(a^2 - ab + b^2)
a^3 - b^3 = (a - b)(a^2 + ab + b^2)

Interval or Inequality for Graphing Characteristics

Essential Questions

What are polynomial functions and how are they different from other types of functions?

How do we identify the degree and leading coefficient of a polynomial function?

What are the different forms of polynomial functions and how are they interconnected?

How do we use polynomial functions to model real-world situations or problems?

What role do the roots, zeros, and factors play in understanding polynomial functions?

How do we analyze and interpret the behavior of polynomial functions, including end behavior and turning points?

What strategies and techniques can be employed to graph polynomial functions accurately?

How do we apply transformations to basic polynomial functions to create more complex ones?

What methods exist for solving polynomial equations, and how do we determine their solutions?

Assessment Tasks

List of common formative and summative assessments.

Formative Assessment(s):

Unit Quiz

Summative Assessment(s):

Unit Test Part A

Unit Test Part B

Learning Experiences

Add additional rows below as needed.

Objective or Content	Learning Experiences	Personalized Learning and Differentiation All information included by PLC in the differentiation box is the responsibility and ownership of the local school to review and approve per Board Policy IKB.
AA.FGR.5 **AA.FGR.5.1** **AA.FGR.5.2** **AA.FGR.5.3** **AA.MM.1** **AA.MM.1.1** **AA.MM.1.4**	**iPatterns** In this learning plan, students are introduced to the concept of the imaginary number i and learn about complex numbers. Students will define complex numbers i such that i^2=–1, and be able to convert any power of i to an equivalent form. **Learning Goal(s):** ● I can convert any power of i to an equivalent form. ● I can write numbers in the form $a+bi$.	Students will be able to work at their own pace in collaborative groups where additional scaffolding is available as needed.

Content Resources

Textbook Correlation: enVision A|G|A - Algebra 2

AA.FGR.5.1 - Lessons 2-1, 2-2, 3-3, 3-6
AA.FGR.5.2 - Lesson 2-4
AA.FGR.5.3 - Lessons 2-4
AA.FGR.5.4 - Lessons 2-3, 2-4
AA.FGR.5.5 - Lesson 1-5, 2-3, 2-4, 2-5, 2-6, Topic 2-Mathematical Modeling in 3 Acts
AA.FGR.5.6 - Lessons 2-7
AA.FGR.5.7 - Lessons 2-1, 2-2, Topic 2-Mathematical Modeling in 3 Acts
AA.FGR.5.8 - Lessons 3-5, 3-6, Topic 3-Mathematical Modeling in 3 Acts
AA.FGR.5.9 - Lessons 3-1, 3-5, Topic 3-Mathematical Modeling in 3 Acts
AA.FGR.5.10 - Lesson 3-3
AA.FGR.5.11 - Lesson 3-6

Exploring Polynomials in Geometric Contents (G.PAR.2.2, 2.3)