Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

AP CALCULUS BC

034010

Updated: July 2023

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit Title: Limits and Continuity	
In this unit students develop an understanding of limits as the foundational building blocks for both derivatives and integration. It is essential for discovering and developing important ideas, definitions, formulas and theorems in calculus. Students will solve limit problems graphically, algebraically, and conceptually. They will generate and work with tables, sketch and analyze various graphs, and apply numerous algebraic techniques to find limits of indeterminate forms. Students must have a solid, intuitive understanding of limits and be able to compute various limits, such as, one-sided limits, limits at infinity, infinite limits, and trigonometric limits. In addition, they will communicate both orally and in written form effectively what their answers mean in the context of the problems they are given. Finally, students will understand how limits are used to determine continuity, which is a fundamental property of functions, and apply the Intermediate Value Theorem.	
Essential Questions	**Learning Targets / Objectives**
1. What is a limit and how can you determine the limit of a function as x approaches c? 2. What algebraic techniques can you use to evaluate a limit? 3. What is continuity and how does it apply to the Intermediate Value Theorem? 4. What is an infinite limit?	Students will be able to: • Represent limits analytically using correct notation • Interpret limits expressed in analytic notation • Estimate limits of functions • Determine the limits of functions using limit theorems • Determine the limits of functions using equivalent expressions for the function or the squeeze theorem. • Justify conclusions about continuity at a point using the definition • Determine intervals over which a function is continuous • Determine values of x or solve for parameters that make discontinuous functions continuous, if possible • Interpret the behavior of functions using limits involving infinity. • Explain the behavior of a function on an interval using IVT

Tier 2 Vocabulary *High-frequency words used throughout the unit*		**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
1. Function 2. Continuity 3. Infinite	4. Asymptote 5. Removable Discontinuity	1. Limit

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. A-APR.D.7	1. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
2. F-IF.B.6	2. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Climate Change Example: Students may calculate the average rate of change of a function c(m) presented symbolically or as a table, where c(m) represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).
NJSLS	**Interdisciplinary Connections**
1. RI.CR.11–12.1	1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
2. W.IW.11–12.2	2. Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
3. SL.PE.11–12.1	3. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
4. HS-PS2-1	4. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.

5. HS-PS2-2	5. Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.
6. HS-PS3-5	6. Develop and use a model of two objects interacting through electric or magnetic fields to illustrate the forces between objects and the changes in energy of the objects due to the interaction.
2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills	
1. 9.1.12.FI.3	1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g.,college, retirement).
2. 9.1.12.CFR.6	2. Identify and explain the consequences of breaking federal and/or state employment or financial laws.
3. 9.3.12.AG-PST.1	3. Apply physical science principles and engineering applications to solve problems and improve performance in AFNR power, structural and technical systems.
4. 9.4.12.CT.2	4. Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
5. 9.4.12.CI.1	5. Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a).
2020 New Jersey Student Learning Standards for Computer Science and Design Thinking	
1. 8.1.12.AP.2	1. Create generalized computational solutions using collections instead of repeatedly using simple variables.
2. 8.1.12.DA.1	2. Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
3. 8.2.12.ETW.3	3. Identify a complex, global environmental or climate change issue, develop a systematic plan of investigation, and propose an innovative sustainable solution.
4. 8.2.12.ED.6	4. Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Defi nitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures

- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately

- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook

Calculus for AP 2nd Edition: Larson and Battaglia

Online Resources

- AP Central
- CalcChat
- Desmos Activities
- Pear Assessment
- Quizizz
- EdPuzzle
- Canva
- Khan Academy
- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning

- New Jersey Climate Education Hub

Videos

- CalcView - Video Solutions of selected problems in the textbook
- Khan Academy

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

1.2 - Finding Limits Graphically and Numerically		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Represent limits analytically using correct notation • Interpret limits expressed ns in analytic notation • Estimate limits of functions	Determine the Vertical Asymptotes for the given functions	Larson Text Section: 1.2 Pg: 72 - 75

1.3 - Evaluating Limits Analytically		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Determine the limits of functions using limit theorems • Determine the limits of functions using equivalent expressions for the function or the squeeze theorem.	Interpret the given limit and estimate its value	Larson Text Section: 1.3 Pg: 84 - 87

1.4 - Continuity and One-Sided Limits		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Justify conclusions about continuity at a point using the definition • Determine intervals over which a function is continuous • Determine values of x or solve for parameters that make discontinuous functions continuous, if possible • Explain the behavior of a function on an interval using the IVT	Determine the value of the given limit and verify graphically using calculator	Larson Text Section: 1.4 Pg: 96 - 99

1.5 - Infinite Limits		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
Interpret the behavior of functions using limits involving infinity	Define asymptote. What is a function's value at an asymptote? Explain.	Larson Text Section: 1.5 Pg: 105 - 107

1.6 - Limits at Infinity		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
Interpret the behavior of functions using limits involving infinity	In your own words, describe the meaning of an infinite limit. Is infinity a real number?	Larson Text Section: 1.6 Pg: 115 - 117

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. ● Diagnostic Pre-Test ● Chapter Tests ● Projects ● End-Of –Course Assessment	The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.	The following assessments require students to utilize various strands of mathematics. ● Projects ● Practice AP Exam Questions ● Homework ● Classwork
List of Accommodations and Modifications ● Special Education ● 504 Students ● At Risk Students ● ELL ● Gifted and Talented		

State Mandates and Resources
● New Jersey Student Learning Standards ● Career Readiness, Life Literacies, and Key Skills ● LGBT and Disabilities Law ● Asian and Pacific Islander

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PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit Title: Differentiation	
In this unit students will get into the heart of calculus. Derivatives are a major portion of the course, so they will spend a significant amount of time in this unit. Derivatives are used to describe the rate of change of one variable with respect to another to understand change in a variety of contexts. At first students build the derivative using the concept of limits and use it to progamily compute the instantaneous rate of change of a function. Students should be able to use different definitions of derivatives, estimate derivatives from tables and graphs, and apply various derivative rules and properties. As they progress through this unity they will spend some time on the relationship between position, velocity, and acceleration on problems involving projectile motion and rectilinear motion.	
Essential Questions	**Learning Targets / Objectives**
1. What is a derivative and what is the relationship of continuity? 2. How do you find the derivatives of basic algebraic functions, trigonometric functions, and exponential functions? 3. How do you find the derivatives of functions involving products and quotients? 4. How do you find the derivatives of composite functions, natural logarithmic functions, and exponential functions with bases other than e? 5. How do you find the derivative of implicitly defined functions? 6. How do you find the derivatives of inverse functions, including inverse trigonometric functions? 7. What is a related rate and how do you find it?	1. Determine average rates of change using difference of quotients 2. Represent the derivative as a limit of a difference quotient 3. Calculate and interpret equations of tangent lines to a curve 4. Calculate derivatives and higher order derivative for familiar functions and composite functions 5. Calculate and interpret derivatives of implicitly defined functions 6. Calculate and interpret related rates in applied contexts 7. Determine limits of functions utilizing L'Hopital's Rule.

Tier 2 Vocabulary *High-frequency words used throughout the unit*		Tier 3 Vocabulary *Discipline-specific words used throughout the unit*
1. Limit 2. Continuity 3. Tangent	4. Secant 5. Linear Equation 6. Function	1. Derivative 2. Difference Quotient 3. Related Rate

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- EdPuzzle
- Canva
- Khan Academy
- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub

Videos

- CalcView - Video Solutions of selected problems in the textbook
- Khan Academy

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

2.1 - The Derivative and The Tangent Line Problem		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Find the slope of the tangent line to a curve at a point • Use the limit definition to find the derivative of a function • Understand the relationship between differentiability & continuity • Find the derivative of a function given by a table or a graph	Find the slope of the given secant line. Can you find slope with only 1 point?	Larson Text Section: 2.1 Pg: 132-134

2.2 - Basic Differentiation Rules and Rates of Change		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Find the derivative of a function using various Differentiation techniques • Find the derivative of trigonometric functions • Find the derivative of exponential functions • Use derivatives to find rates of change	Solve the following using first principles.	Larson Text Section: 2.2 Pg: 144-147

2.3 - Product & Quotient, Trigonometric, and Higher-Oder Derivatives		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Find the derivative of a function using the Product Rule	Find the equation of the tangent line	Larson Text Section: 2.3 Pg: 155-158

• Find the derivative of a function using the QUotient Rule • Find the derivative of a trigonometric function • Find a higher-order derivative of a function		

2.4 - The Chain Rule and Exponential Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Find the derivative of a composite function using the Chain Rule • Find the derivative of a function using the General Power Rule • Simplify the derivative of a function using algebra • Find the derivative of a transcendental function using the Chain Rule • Find the derivative of a function involving the natural logarithmic function • Define and differentiate exponential functions that have bases other than e.	Determine f'(x) and dy/dx of the given functions	Larson Text Section: 2.4 Pg: 169-173

2.5 - Implicit Differentiation and Logarithmic Differentiation		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Distinguish between functions written in implicit form and explicit form	Determine f'(x) of the given functions	Larson Text Section: 2.5 Pg: 180-182

• Use implicit differentiation to find the derivative of a function • Find derivatives of functions using logarithmic differentiation		

2.6 - Derivatives of Inverse Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Find the derivative of an inverse function • Differentiate an inverse trigonometric function	Solve for dy/dx. (use implicit differentiation)	Larson Text Section: 2.6 Pg: 187-189

2.7 - Related Rates		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Find a Related Rate • Use related rates to solve real-life problems	Determine if A, B, C, or D is the solution to the given problems (MC AP Practice)	Larson Text Section: 2.7 Pg: 195-198

7.7 - Indeterminate Forms and L'Hopital's Rule		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Recognize Limits that produce indeterminate forms • Apply L'Hopital's Rule to evaluate a limit	Solve the given limit using an algebraic approach	Larson Text Section: 7.7 Pg: 513-516

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
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List of Accommodations and Modifications ● Special Education ● 504 Students ● At Risk Students ● ELL ● Gifted and Talented		

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● New Jersey Student Learning Standards ● Career Readiness, Life Literacies, and Key Skills ● LGBT and Disabilities Law ● Asian and Pacific Islander

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PART I: UNIT RATIONALE

<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit Title: Applications of Differentiation	
In this unit, students will go on to understand and apply the Mean Value Theorem and will have the opportunity to see how the average rate of change can be used to justify instantaneous speed. They will also spend a significant amount of time developing a comprehensive analysis of functions using not only their graphs but their derivatives as well. Students should be familiar with a variety of real-world applications, including related rates, optimization, linear approximation, and growth and decay models. This is most likely the first time students will be asked to think deeply on a conceptual level, so they may struggle to make connections at first. Students will also learn how far to simplify solutions and provide meaningful simplifications to clarify solutions.	
Essential Questions	**Learning Targets / Objectives**
1. What are extrema and how can you find them on open and closed intervals? 2. What is the Mean Value Theorem and how is it used? 3. How can you determine the intervals on which a function is increasing or decreasing and the location of the function's relative extrema? 4. How do you determine the concavity of a function and find its inflection points? 5. How do you analyze a function and sketch its graph? 6. How do you maximize or minimize quantities? 7. How are differentials used to explain the tangent line approximation?	1. Justify conclusions about functions by applying the Extreme Value Theorem 2. Justify conclusions about the behavior of a function based on the behavior of its derivatives 3. Determine critical points of implicit relations 4. Justify conclusions about the behavior of a function based on the behavior of its derivatives 5. Calculate minimum and maximum values in applied contexts or analysis of functions 6. Interpret minimum and maximum values calculated in applied contexts 7. Approximate a value on a curve using the equation of a tangent line

Tier 2 Vocabulary *High-frequency words used throughout the unit*		Tier 3 Vocabulary *Discipline-specific words used throughout the unit*	
1. Limit 2. Continuity 3. Derivative	4. Interval 5. Increase/Decrease	1. Concavity 2. MVT 3. Rolle's Theorem	4. Extrema 5. Inflection Point

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. F-IF.C.7.c	1. .Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
2. F-IF.B.4	2. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity
3. F-IF.B.6	3. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Climate Change Example: Students may calculate the average rate of change of a function c(m) presented symbolically or as a table, where c(m) represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).
4. F-LE.B.5	4. Interpret the parameters in a linear or exponential function in terms of a context.
5. G-MG.A.3	5. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). Climate Change Example: Students may apply geometric methods to solve design problems such as increasing access to green spaces in cities given physical and cost constraints.

NJSLS	Interdisciplinary Connections
1. RI.CR.11–12.1	1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
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1. 9.1.12.FI.3	1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g.,college, retirement).
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The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

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- Reason Abstractly

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- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
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Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

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- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units

- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

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- Repeat Calculations
- Find General Methods
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- Evaluate Results

Resources

Textbook

Calculus for AP 2nd Edition: Larson and Battaglia

Online Resources

- AP Central
- CalcChat
- Desmos Activities
- Pear Assessment
- IXL
- Quizizz
- EdPuzzle
- Canva
- Khan Academy

- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub

Videos

- CalcView - Video Solutions of selected problems in the textbook
- Khan Academy

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

<u>How</u> will students uncover content and build skills?

3.1 - Extrema on an Interval		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Understand the definition of extrema of a function on an interval • Understand the definition of relative extrema of a function on an open interval • Find extrema on a closed interval	Find the coordinate of the vertices for the given function	Larson Text Section: 3.1 Pg: 217-219

3.2 - Rolle's Theorem and the Mean Value Theorem		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Understand and use Rolle's Theorem • Understand and use the Mean Value Theorem	Determine the max or min value of the function and determine if its a local or extrema	Larson Text Section: 3.2 Pg: 224-226

3.3 - Increasing and Decreasing Functions and the First Derivative Test		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Determine intervals on which a function is increasing or decreasing • Apply the First Derivative Test to find relative extrema of a function	Use the MVT to verify that an extrema point exists on the given interval	Larson Text Section: 3.3 Pg: 233-236

3.4 - Concavity and the Second Derivative Test		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Determine intervals on which a function is concave upward or concave downward. • Find any points of inflection of the graph of a function • Apply the Second Derivative Test to find relative extrema of a function	Determine if the function is increasing or decreasing on the given interval. Verify by graphical analysis.	Larson Text Section: 3.4 Pg: 242-244

3.5 - A Summary of Curve Sketching		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Analyze and Sketch the graph of a function	What is concavity? Determine the intervals of concavity for the given graph.	Larson Text Section: 3.5 Pg: 253-256

3.6 - Optimization Problems		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Solve applied minimum and maximum problems.	Determine the intervals of increase and decrease and concavity given the following family of curve sketches.	Larson Text Section: 3.6 Pg: 262-265

3.7 - Linear Approximation and Differentials		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Understand the concept of tangent line approximation • Compare the value of the differential, dy, with the actual change in y, delta y • Estimate a propagated error using a differential • Find the differential of a function using differentiation formulas.	Various optimization problems.	Larson Text Section: 3.7 Pg: 272-275

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. • Diagnostic Pre-Test • Chapter Tests • Projects • End-Of –Course Assessment	The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.	The following assessments require students to utilize various strands of mathematics. • Projects • Practice AP Exam Questions • Homework • Classwork

List of Accommodations and Modifications

- Special Education
- 504 Students
- At Risk Students
- ELL
- Gifted and Talented

State Mandates and Resources

- New Jersey Student Learning Standards
- Career Readiness, Life Literacies, and Key Skills
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Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

AP CALCULUS BC
034010

Updated: July 2023

PART I: UNIT RATIONALE

<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 4: Integration

In this unit, students will establish the relationship between differentiation and integration. Students begin by exploring the contextual meaning of areas of certain regions bounded by rate functions. Integration determines the accumulation of change over an interval, just as differentiation determines instantaneous rate of change at a point. Students will understand that integration is a limiting case of a sum of products (areas) in the same way that differentiation is a limiting case of a quotient of differences (slopes).

Essential Questions	Learning Targets / Objectives
1. What are antiderivatives and how are they used? 2. How can you approximate the area of a plane region? 3. How are Reimann sums similar to the Trapezoidal Rule and how are they different? 4. How do you integrate composite functions? 5. How do you integrate rational functions and trigonometric functions other than sine and cosine? 6. What is integration by parts and when is it used? 7. How do you evaluate trigonometric integrals involving powers? 8. How can trigonometric substitution be used to solve an integral? 9. How do you integrate a complex rational function?	1. Definite integrals allow us to solve problems involving the accumulation of change over an interval. 2. Definite integrals can be approximated using geometric and numerical methods. 3. Recognizing opportunities to apply knowledge of geometry and mathematical rules can simplify integration. 4. The use of limits allows us to show that the areas of unbounded regions may be finite.

Tier 2 Vocabulary *High-frequency words used throughout the unit*			Tier 3 Vocabulary *Discipline-specific words used throughout the unit*		
Derivative	Limits/Continuity	Area	Antiderivatives Riemann Sum	Integration Sigma Notation	Partial Fractions Integration by Parts

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. F-BF.A.2	1. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
2. F-IFB.6	2. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Climate Change Example: Students may calculate the average rate of change of a function c(m) presented symbolically or as a table, where c(m) represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).
3. F-LE.B.5	3. Interpret the parameters in a linear or exponential function in terms of a context
NJSLS	**Interdisciplinary Connections**
1. RI.CR.11–12.1	1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
2. W.IW.11–12.2	2. Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately

	through the effective selection, organization, and analysis of content.
3. SL.PE.11–12.1	3. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
4. HS-PS2-1	4. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.
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  - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE

<u>How</u> will students uncover content and build skills?

4.1 Antiderivatives and Indefinite Integration		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Write the general solution of a differential equation and use indefinite integral notation for antiderivatives. -Use basic integration rules to find antiderivatives. -Find a particular solution of a differential equation.	Determine the dy/dx for 2-3 functions. Describe the process using words.	Larsen Text Section: 4.1 Pg: 287-289

4.2 Area		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
-Use sigma notation to write and evaluate a sum. -Understand the concept of area. -Approximate the area of a plane region. -Find the area of a plane region using limits.	Find the sum of the numbers from 1 to 100.	Larsen Text Section: 4.2 Pg: 299-301

4.3 Riemann Sums and Definite Integrals		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
-Understand the definition of a Riemann sum. -Evaluate a definite integral using limits and geometric formulas. -Evaluate a definite integral using properties of definite integrals. -Approximate a definite integral using the Trapezoidal Rule.	Given a basic definite integral, have students explain what it represents.	Larsen Text Section: 4.3 Pg: 312-315

4.6 Integration by Substitution

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply
-Use pattern recognition to find a definite integral. -Use a change of variable to find an indefinite integral. -Use the General Power Rule for Integration to find an indefinite integral. -Use a change of variables to evaluate a definite integral. -Evaluate a definite integral involving an even or odd function.	Solve an integral with trigonometric functions.	Larsen Text Section: 4.6 Pg: 343-346

4.7 Natural Log Function Integration		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
-Use the Log Rule for Integration to integrate a rational function. -Integrate trigonometric functions.	Error analysis problem in Larsen text on pg. 344 #82.	Larsen Text Section: 4.7 Pg: 353-355

7.2 Integration by Parts		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
-Find an antiderivative using integration by parts.	Write down the integration by parts formula. Write down at least two things you notice. Talk with a partner about what you notice.	Larsen Text Section: 7.2 Pg: 469-472

7.3 Trigonometric Integrals		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
-Solve trigonometric integrals involving powers of sine, cosine, secant and tangent. -Solve trigonometric integrals involving sines and	Write down the Pythagorean trigonometric identities.	Larsen Text Section: 7.3 Pg: 479-481

cosines of different angles.		

7.4 Trigonometric Substitutions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
-Use trigonometric substitution to find an integral. -Use integrals to model and solve real-life applications.	Have students solve the Pythagorean trigonometric identities for sin, cosine and tangent.	Larsen Text Section: 47.4 Pg: 488-490

7.5 Partial Fractions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
-Understand the concept of partial fraction decomposition. -Use partial fractions decomposition with linear factors to integrate rational functions. -Use partial fraction decomposition with quadratic factors to integrate rational functions.	Ask students: How can we decompose a function? Give students a rational function with 1/basic quadratic. Allow them to factor the denominator and then see if they can write it as the sum of two fractions.	Larsen Text Section: 7.5 Pg: 498-499

PART IV: EVIDENCE OF LEARNING

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Assessments		
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The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the	The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments,	The following assessments require students to utilize various strands of mathematics. ● Projects ● Practice AP Exam Questions ● Homework

Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. ● Diagnostic Pre-Test ● Chapter Tests ● Projects ● End-Of –Course Assessment	learning/response logs, discussions and practice presentations.	● Classwork
List of Accommodations and Modifications ● Special Education ● 504 Students ● At Risk Students ● ELL ● Gifted and Talented		

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Updated: July 2023

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Applications of Integration

Students will use integration to find the area between two curves. They will also use rotation and other techniques to find the volume of a variety of 3-D solids as well as of solids with known cross-sectional areas. The First Fundamental Theorem of Calculus will play a major role in the development in understanding of both area and volume, and students will have to rely on a variety of integration techniques to help them progress through the chapter including: substitution, transcendental and trigonometric techniques.

Essential Questions	Learning Targets / Objectives
1. What is the Fundamental Theorem of Calculus? 2. How do you find the area of a region between two curves? 3. How can you use integrals to find the volume of a solid? 4. How can you use definite integrals to find the arc length of a smooth curve and the area of a surface of revolution?	1. The Fundamental Theorem of Calculus connects differentiation and integration. 2. Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval. 3. Definite integrals allow us to solve problems involving the accumulation of change in length over an interval.
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
Definite Integrals closed interval revolution arc length	Fundamental Theorem of Calculus Disc Method Washer Method

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. N-Q.A.1	1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. Climate Change Example: Students may use units to guide the solution of multi-step problems about how variations in the flow of energy into and out of the Earth's systems result in climate change. Note: Changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.
2. F-IF.C.7c	2. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
3. G-GMD.A.1	3. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
4. G-GMD.A.2	4. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
5. G-GMD.A.3	5. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
6. G-GMD.B.4	6. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
NJSLS	**Interdisciplinary Connections**
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- View as Components
- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

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Textbook

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- CalcView - Video Solutions of selected problems in the textbook
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Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

4.4 The Fundamental Theorem of Calculus		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
Determine the average value of a function using definite integrals. Interpret the meaning of a definite integral in accumulation problems. Evaluate a definite integral using the Fundamental Theorem of Calculus. Calculate a definite integral using areas and properties of definite integrals.	Determine the area under the curve given an interval.	Larson Text Section 4.4 Pg: 326-328

6.1 Area of a Region Between Two Curves		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
Find the area of a region between two curves using integration. Find the area of a region between intersecting curves using integration. Describe integration as an accumulation process.	Given a picture of a shaded area and the equations bounding it, have students find the area.	Larson Text Section 6.1 Pg: 416-419

6.2 Volume: The Disk and Washer Method		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
Find the volume of a solid of revolution using the disk method. Find the volume of a solid of revolution using the washer method. Find the volume of a solid with known cross sections.	Students can explain the difference between a disk and a washer. How would their difference affect the volume of a rotating object?	Larson Text Section 6.2 Pg: 427-430

6.4 Arc Length and Surfaces of Revolution		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
Find the arc length of a smooth curve. Find the area of a surface of revolution.	Think of an example of when you would need to find the arc length that is not a circle.	Larson Text Section 6.4 Pg: 446-449

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. ● Diagnostic Pre-Test ● Chapter Tests ● Projects ● End-Of –Course Assessment	The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.	The following assessments require students to utilize various strands of mathematics. ● Projects ● Practice AP Exam Questions ● Homework ● Classwork
List of Accommodations and Modifications ● Special Education ● 504 Students ● At Risk Students ● ELL ● Gifted and Talented		

State Mandates and Resources
● New Jersey Student Learning Standards ● Career Readiness, Life Literacies, and Key Skills ● LGBT and Disabilities Law ● Asian and Pacific Islander

Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

AP CALCULUS BC
034010

Updated: July 2023

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 6: Differential Equations

In this unit, students will learn to set up and solve separable differential equations. Slope fields can be used to represent solution curves to a differential equation and build understanding that there are infinitely many general solutions to a differential equation, varying only by a constant of integration. Students can locate a unique solution relevant to a particular situation, provided they can locate a point on the solution curve. By writing and solving differential equations leading to models for exponential growth and decay and logistic growth, students build understanding of topics introduced in Algebra II and other courses.

Essential Questions	Learning Targets / Objectives
1. How do you approximate the particular solution of a differential equation? 2. How are differential equations used in application problems, such as the exponential growth and decay models? 3. How do you solve separable differential equations? 4. How do you solve logistic differential equations?	1. Solving differential equations allows us to determine functions and develop models.
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
Exponential Growth and Decay Differential Equation	Slope Field Euler's Method Separable Particular Solution General Solution Order

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. F-LE.B.5	1. Interpret the parameters in a linear or exponential function in terms of a context.
2. F-BF.A.1.b	2. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
3. F-BF.A.1.c	3. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
4. F-IF.B.6	4. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Climate Change Example: Students may calculate the average rate of change of a function c(m) presented symbolically or as a table, where c(m) represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).
NJSLS	**Interdisciplinary Connections**
1. RI.CR.11–12.1	1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
2. W.IW.11–12.2	2. Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
3. SL.PE.11–12.1	3. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
4. HS-PS2-1	4. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship

	among the net force on a macroscopic object, its mass, and its acceleration.
2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills	
1. 9.1.12.FI.3	1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g.,college, retirement).
2. 9.3.12.AG-PST.1	2. Apply physical science principles and engineering applications to solve problems and improve performance in AFNR power, structural and technical systems.
3. 9.4.12.CT.2	3. Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
4. 9.4.12.CI.1	4. Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a).
2020 New Jersey Student Learning Standards for Computer Science and Design Thinking	
1. 8.1.12.AP.2	1. Create generalized computational solutions using collections instead of repeatedly using simple variables.
2. 8.1.12.DA.1	2. Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
3. 8.2.12.ETW.3	3. Identify a complex, global environmental or climate change issue, develop a systematic plan of investigation, and propose an innovative sustainable solution.
4. 8.2.12.ED.6	4. Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
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- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

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- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions

- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components

- Look for Patterns
- Look for Structure

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PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

<u>How</u> will students uncover content and build skills?

5.1 Slope Fields and Euler's Method		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
Use initial conditions to find particular solutions of differential equations. Use slope fields to approximate solutions of differential equations. Use Euler's Method to approximate solutions of differential equations.	Students will sketch a line with the following slopes, -3, -1, 0, 1, 5.	Larson Text Section: 5.1 Pg: 375-378

5.2 Growth and Decay		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
Use separation of variables to solve a simple differential equation. Use exponential functions to model growth and decay in applied problems.	Students will sketch the graph of $y = \frac{1}{2}e^{2x}$ How can you make 1 change to make the graph represent an exponential decay function?	Larson Text Section: 5.2 Pg: 384-386

5.3 Separation of Variables		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
Recognize and solve differential equations that can be solved by separation of variables. Use differential equations to model and solve applied problems.	Solve the following: $\int_0^1 \frac{24x}{(4x^2+4)^2}$	Larson Text Section: 5.3 Pg: 393-396

5.4 The Logistic Equation		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
Solve and analyze logistic differential equations. Use logistic differential equations to model and solve applied problems.	Match the slope field for 3 differential equations.	Larson Text Section: 5.4 Pg: 402-403

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for	The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning	The following assessments require students to utilize various strands of mathematics. ● Projects ● Practice AP Exam Questions

Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. ● Diagnostic Pre-Test ● Chapter Tests ● Projects ● End-Of –Course Assessment	strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.	● Homework ● Classwork
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AP CALCULUS AB AND BC

Mathematical Practices

Practice 1

Implementing Mathematical Processes 1

Determine expressions and values using mathematical procedures and rules.

Practice 2

Connecting Representations 2

Translate mathematical information from a single representation or across multiple representations.

Practice 3

Justification 3

Justify reasoning and solutions.

Practice 4

Communication and Notation 4

Use correct notation, language, and mathematical conventions to communicate results or solutions.

SKILLS

1.A Identify the question to be answered or problem to be solved *(not assessed)*.

1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*.

1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*).

1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems.

1.E Apply appropriate mathematical rules or procedures, with and without technology.

1.F Explain how an approximated value relates to the actual value.

2.A Identify common underlying structures in problems involving different contextual situations.

2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

2.C Identify a re-expression of mathematical information presented in a given representation.

2.D Identify how mathematical characteristics or properties of functions are related in different representations.

2.E Describe the relationships among different representations of functions and their derivatives.

3.A Apply technology to develop claims and conjectures *(not assessed)*.

3.B Identify an appropriate mathematical definition, theorem, or test to apply.

3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.

3.D Apply an appropriate mathematical definition, theorem, or test.

3.E Provide reasons or rationales for solutions and conclusions.

3.F Explain the meaning of mathematical solutions in context.

3.G Confirm that solutions are accurate and appropriate.

4.A Use precise mathematical language.

4.B Use appropriate units of measure.

4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, y', *and* $\frac{dy}{dx}$).

4.D Use appropriate graphing techniques.

4.E Apply appropriate rounding procedures.

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Updated: July 2023

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 7: Parametric Equations, Polar Coordinates, & Vectors

In this unit, students will build on their understanding of straight-line motion to solve problems in which particles are moving along curves in the plane. Students will define parametric equations and vector-valued functions to describe planar motion and apply calculus to solve motion problems. Students will learn that polar equations are a special case of parametric equations and will apply calculus to analyze graphs and determine lengths and areas. This unit should be treated as an opportunity to reinforce past learning and transfer knowledge and skills to new situations, rather than as a new list of facts or strategies to memorize.

Essential Questions	Learning Targets / Objectives
1. What are parametric equations and how do you find a set of parametric equations to represent a curve? 2. How can you find the slope of a tangent line and the arc length of a curve using a set of parametric equations? 3. What is the polar coordinate system and the properties of curves written in polar form? 4. How do you integrate and find arc length of curves in polar coordinates? 5. What are vectors and how do you perform operations using vectors? 6. What are vector-valued functions and how do you apply Calculus to them? 7. How do you describe velocity and acceleration associated with vector-valued functions?	1. Derivatives allow us to solve real-world problems involving rates of change. 2. Definite integrals allow us to solve problems involving the accumulation of change in length over and interval. 3. Recognizing opportunities to apply derivative rules can simplify differentiation 4. Solving an initial value problem allows us to determine an expression for the position of a particle moving in the plane. 5. Derivatives can be used to determine velocity, speed, and acceleration for a particle moving along a curve in the plane defined using parametric or vector-valued functions.

Tier 2 Vocabulary *High-frequency words used throughout the unit*			Tier 3 Vocabulary *Discipline-specific words used throughout the unit*	
Planar Curve	Smooth	Derivative	Parameter	Parametric Equation
Tangent	Area	Arc Length	Polar Coordinates	Polar Graph
Vectors	Scalar	Limits / Continuity	Vector-Valued Function	

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. F-IF.A.2	1. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
2. F-IF.B.4	2. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
3. F-IF.B.5	3. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Climate Change Example: Students may relate the domain of a function c(m) representing the amount of carbon dioxide produced by burning m molecules of ethane (gasoline), to its graph in order to determine the appropriate domain for c(m).
4. F-BF.A.1.b	4. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
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4. HS-PS2-1	4. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.
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- Justify Conclusions
- Compare Arguments
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- Use Logic
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Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

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PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

9.2 - Plane Curves and Parametric Equations		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Sketch the graph of a curve given by a set of parametric equations - Eliminate the parameter in a set of parametric equations. - Find a set of parametric equations to represent a curve	Explain how to write any ordered pair on the unit circle using trig functions and the angle θ formed by the positive x-axis and the point on the circle.	Larson Text Section: 9.2 Pg: 652-654

9.4 - Polar Coordinates and Polar Graphs		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Understand the polar coordinate system. - Rewrite rectangular coordinates and equations in polar form and vice versa. - Sketch the graph of an equation given in polar form. - Find the slope of a tangent line to a polar graph. - Identify several types of special polar graphs.	Evaluate 3-4 trigonometric values of angle measurements	Larson Text Section: 9.4 Pg: 670 - 672

9.6 - Vectors in the Plane		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Write the component form of a vector. - Perform vector operations and interpret the results geometrically.	- Define the terms 'magnitude' and 'direction' in your own words in relation to mathematics. - Try and recall how to add two vectors geometrically using an xy coordinate plane.	Larson Text Section: 9.6 Pg: 686 - 688

9.3 - Parametric Equations and Calculus		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Find the slope of a tangent line to a curve given by a set of parametric equations. - Find the arc length of a curve given by a set of parametric equations.	- Eliminate the parameter and write the corresponding equation in rectangular coordinates. - Create a set of parametric equations using trig functions that describe the path of a particle moving on the unit circle from (1,0) to (0,1) in a counterclockwise motion.	Larson Text Section: 9.3 Pg: 659-62

9.5 - Area and Arc Length in Polar Coordinates		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- FInd the area of a region bounded by a polar graph. - Find the points of intersection of two polar graphs. - Find the arc length of a polar graph.	- Converting polar coordinates to rectangular and vice versa (3-4 examples) - If integration in rectangular coordinates uses sums of rectangles, what shape do you think we will be adding up if our	Larson Text Section: 9.5 Pg: 687 - 680

	representation is in polar form? Justify your conjecture.	

9.7 - Vector-Valued Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Analyze and sketch a plane curve given by a vector-valued function. - Extend the concepts of limits and continuity to vector-valued functions. - Differentiate a vector-valued function. - Integrate a vector-valued function.	- Find the following vector magnitudes (3 examples) - What are the three criteria for a function to be continuous at a point in the real-valued function case?	Larson Text Section: 9.7 Pg: 695 - 697

9.8 - Velocity and Acceleration		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Describe the velocity and acceleration associated with a vectored-value function.	- Find derivatives for the following vector-valued functions.	Larson Text Section: 9.8 Pg: 702-703

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

<table>
<tr><th colspan="3">Assessments</th></tr>
<tr><th>Summative</th><th>Formative</th><th>Performance</th></tr>
<tr>
<td>The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.
Diagnostic Pre-Test
Chapter Tests
Projects
End-Of –Course Assessment</td>
<td>The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.</td>
<td>The following assessments require students to utilize various strands of mathematics.
Projects
Practice AP Exam Questions
Homework
Classwork</td>
</tr>
<tr>
<td colspan="3">List of Accommodations and Modifications
Special Education
504 Students
At Risk Students
ELL
Gifted and Talented</td>
</tr>
</table>

State Mandates and Resources
• New Jersey Student Learning Standards • Career Readiness, Life Literacies, and Key Skills

- LGBT and Disabilities Law
- Asian and Pacific Islander

AP CALCULUS AB AND BC

Mathematical Practices

Practice 1

Implementing Mathematical Processes 1

Determine expressions and values using mathematical procedures and rules.

Practice 2

Connecting Representations 2

Translate mathematical information from a single representation or across multiple representations.

Practice 3

Justification 3

Justify reasoning and solutions.

Practice 4

Communication and Notation 4

Use correct notation, language, and mathematical conventions to communicate results or solutions.

SKILLS

1.A Identify the question to be answered or problem to be solved *(not assessed)*.

1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*.

1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*).

1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems.

1.E Apply appropriate mathematical rules or procedures, with and without technology.

1.F Explain how an approximated value relates to the actual value.

2.A Identify common underlying structures in problems involving different contextual situations.

2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

2.C Identify a re-expression of mathematical information presented in a given representation.

2.D Identify how mathematical characteristics or properties of functions are related in different representations.

2.E Describe the relationships among different representations of functions and their derivatives.

3.A Apply technology to develop claims and conjectures *(not assessed)*.

3.B Identify an appropriate mathematical definition, theorem, or test to apply.

3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.

3.D Apply an appropriate mathematical definition, theorem, or test.

3.E Provide reasons or rationales for solutions and conclusions.

3.F Explain the meaning of mathematical solutions in context.

3.G Confirm that solutions are accurate and appropriate.

4.A Use precise mathematical language.

4.B Use appropriate units of measure.

4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, y', *and* $\frac{dy}{dx}$).

4.D Use appropriate graphing techniques.

4.E Apply appropriate rounding procedures.

Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

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034010

Updated: July 2023

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 8: Infinite Series

In this unit, students will build on their understanding of straight-line motion to solve problems in which particles are moving along curves in the plane. Students will define parametric equations and vector-valued functions to describe planar motion and apply calculus to solve motion problems. Students will learn that polar equations are a special case of parametric equations and will apply calculus to analyze graphs and determine lengths and areas. This unit should be treated as an opportunity to reinforce past learning and transfer knowledge and skills to new situations, rather than as a new list of facts or strategies to memorize.

Essential Questions	Learning Targets / Objectives
1. How do you write terms of a sequence and determine whether the sequence converges or diverges? 2. What is a convergent infinite series? 3. How do you use the Integral Test or properties of special series (p and Harmonic) to determine whether or not an infinite series converges or diverges? 4. How do you use comparison tests to determine the convergence of infinite series? 5. What is an alternating series and how do you determine its convergence? 6. What are the root and ratio tests to determine convergence of series? 7. How do you find polynomial approximations (such as Taylor and Maclaurin polynomials) of elementary functions? 8. What is a power series and how do you find the radius and intervals	1. Applying limits may allow us to determine the finite sum of infinitely many terms. 2. Power series allow us to represent associated functions on an appropriate interval

of convergence of them? 9. How do you construct power series using operations? 10. How do you find Taylor or Maclaurin series for a function?	
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
Sequence, Series, Limit Geometric Series, Harmonic Series, Polynomial Ratio, Root, Derivative	Convergence, Divergence, Monotonic Bounded, Partial Sums, Alternating Series Power Series, Radius of Convergence Interval of Convergence

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. F-IF.A.3	1. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.
2. F-IF.C.8.a	2. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
3. F-IF.C.9	3. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
4. F-BF.A.2	4. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
NJSLS	**Interdisciplinary Connections**
1. RI.CR.11–12.1	1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.

2. W.IW.11–12.2	2. Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
3. SL.PE.11–12.1	3. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
4. HS-PS2-1	4. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.
2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills	
1. 9.1.12.FI.3	1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g.,college, retirement).
2. 9.3.12.AG-PST.1	2. Apply physical science principles and engineering applications to solve problems and improve performance in AFNR power, structural and technical systems.
3. 9.4.12.CT.2	3. Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
4. 9.4.12.CI.1	4. Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a).
2020 New Jersey Student Learning Standards for Computer Science and Design Thinking	
1. 8.1.12.AP.2	1. Create generalized computational solutions using collections instead of repeatedly using simple variables.
2. 8.1.12.DA.1	2. Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
3. 8.2.12.ETW.3	3. Identify a complex, global environmental or climate change issue, develop a systematic plan of investigation, and propose an innovative sustainable solution.
4. 8.2.12.ED.6	4. Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Defi nitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures

- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately

- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook

Calculus for AP 2nd Edition: Larson and Battaglia

Online Resources

- AP Central
- CalcChat
- Desmos Activities
- Pear Assessment
- IXL
- Quizizz
- EdPuzzle
- Canva
- Khan Academy
- Inside Mathematics
- NJDOE Digital Item Library

- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub

Videos

- CalcView - Video Solutions of selected problems in the textbook
- Khan Academy

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

8.1 - Sequences		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**

- Write the terms of a sequence. - Determine whether a sequence converges or diverges. - Write a formula for the *n*th term of a sequence. - Use properties of monotonic sequences and bounded sequences.	Anything Fibonacci!	Larson Text Section: 8.1 Pg: 542 - 544

8.2 - Series and Convergence		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Understand the definition of a convergent infinite series. - Use properties of infinite geometric series. - Use the *n*th-Term Test for Divergence of an infinite series.	Use recursive or explicit formulas to write out the first 5 terms of given sequences.	Larson Text Section: 8.2 Pg: 552 - 554

8.3 - The Integral Test and *p*-Series		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Use the Integral Test to determine whether an infinite series converges or diverges. - Use properties of *p*-series and harmonic series.	Determine the first 5 partial sums of a set of series. Determine convergence or divergence of given series.	Larson Text Section: 8.3 Pg: 559 - 561

8.4 - Comparisons of Series		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Use the Direct Comparison test to	Error analysis of the use of the Integral Test to	Larson Text Section: 8.4 Pg: 566 - 568

determine whether a series converges or diverges. - Use the Limit Comparison Test to determine whether a series converges or diverges.	determine convergence for 2 given series. State the conditions on p-series convergence.	

8.5 - Alternating Series		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Use the Alternating Series Test to determine whether an infinite series converges. - Use the Alternating Series Remainder to approximate the sum of an alt. series. - Classify a convergent series as absolutely or conditionally convergent. - Rearrange an infinite series to obtain a different sum	Use comparison tests to determine the convergence or divergence of given series.	Larson Text Section: 8.5 Pg: 575 - 576

8.6 - The Ratio and Root Tests		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Use the Ratio Test to determine whether a series converges or diverges. - Use the Root Test to determine whether a series converges or diverges. - Review all convergence/divergence tests.	List all convergence tests for series that we have learned thus far. Which is your favorite and why?	Larson Text Section: 8.6 Pg: 583 - 585

8.7 - Taylor Polynomials and Approximations		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**

- Find polynomial approximations of elementary functions and compare them with elementary functions. - Find Taylor and Maclaurin polynomial approximations of elementary functions. - Use the remainder of a Taylor polynomial.	Find equations of tangent lines for given functions through given points. Open question. Can you think of any other way to approximate a function with other curves?	Larson Text Section: 8.7 Pg: 594 - 596

8.8 - Power Series		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Understand the definition of a power series. - Find the radius and interval of convergence of a power series. - Differentiate and integrate a power series.	Determine the Maclaurin polynomials of degree 3 or 4 for a set of elementary functions	Larson Text Section: 8.8 Pg: 604 - 606

8.9 - Representation of Functions by Power Series		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Find a geometric power series that represents a function. - Construct a power series using series operations.	Find the function representation of a set of geometric series with known common ratios and starting terms.	Larson Text Section: 8.9 Pg: 612 - 613

8.10 - Taylor and Maclaurin Series		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
- Find a Taylor or Maclaurin series for a function. - Find a binomial series. - Use a basic list of Taylor Series to find	Use desmos.com to graph the partial sums (Maclaurin polynomial) of the exponential function. What do you notice about the approximation as you add more and more terms?	Larson Text Section: 8.9 Pg: 623 - 625

other Taylor Series.		

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. ● Diagnostic Pre-Test ● Chapter Tests ● Projects ● End-Of –Course Assessment	The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.	The following assessments require students to utilize various strands of mathematics. ● Projects ● Practice AP Exam Questions ● Homework ● Classwork
List of Accommodations and Modifications ● Special Education ● 504 Students ● At Risk Students ● ELL		

- Gifted and Talented

State Mandates and Resources

- New Jersey Student Learning Standards
- Career Readiness, Life Literacies, and Key Skills
- LGBT and Disabilities Law
- Asian and Pacific Islander

AP CALCULUS AB AND BC

Mathematical Practices

Practice 1

Implementing Mathematical Processes 1

Determine expressions and values using mathematical procedures and rules.

Practice 2

Connecting Representations 2

Translate mathematical information from a single representation or across multiple representations.

Practice 3

Justification 3

Justify reasoning and solutions.

Practice 4

Communication and Notation 4

Use correct notation, language, and mathematical conventions to communicate results or solutions.

SKILLS

1.A Identify the question to be answered or problem to be solved *(not assessed)*.

1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*.

1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*).

1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems.

1.E Apply appropriate mathematical rules or procedures, with and without technology.

1.F Explain how an approximated value relates to the actual value.

2.A Identify common underlying structures in problems involving different contextual situations.

2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

2.C Identify a re-expression of mathematical information presented in a given representation.

2.D Identify how mathematical characteristics or properties of functions are related in different representations.

2.E Describe the relationships among different representations of functions and their derivatives.

3.A Apply technology to develop claims and conjectures *(not assessed)*.

3.B Identify an appropriate mathematical definition, theorem, or test to apply.

3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.

3.D Apply an appropriate mathematical definition, theorem, or test.

3.E Provide reasons or rationales for solutions and conclusions.

3.F Explain the meaning of mathematical solutions in context.

3.G Confirm that solutions are accurate and appropriate.

4.A Use precise mathematical language.

4.B Use appropriate units of measure.

4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, y', *and* $\frac{dy}{dx}$).

4.D Use appropriate graphing techniques.

4.E Apply appropriate rounding procedures.

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AP CALCULUS BC
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Updated: July 2023

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 9: AP Test Review

In this unit, students will build on their understanding of straight-line motion to solve problems in which particles are moving along curves in the plane. Students will define parametric equations and vector-valued functions to describe planar motion and apply calculus to solve motion problems. Students will learn that polar equations are a special case of parametric equations and will apply calculus to analyze graphs and determine lengths and areas. This unit should be treated as an opportunity to reinforce past learning and transfer knowledge and skills to new situations, rather than as a new list of facts or strategies to memorize.

Essential Questions	Learning Targets / Objectives
1. 1. How do you dissect and apply appropriate calculus techniques to FRQ AP Test questions? 2. 2. How do you dissect and apply appropriate calculus techniques to MCQ AP Test questions?	1. Reasoning with definitions, theorems and properties can be used to justify claims about FRQ style questions. 2. Reasoning with definitions, theorems and properties can be used to justify claims about MCQ style questions.
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
1. Limit 2. Derivative 3. Integral 4. Differential 5. Volume	1. FRQ 2. MCQ

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. A-SSE.A.1b	1. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P.
2. A-APR.D.7	2. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
3. F-IF.C.7c	3. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
4. F-IF.C.8.a	4. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
5. F-IF.C.9	5. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
6. F-IF.B.6	6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.
7. F-LE.B.5	7. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.
8. F-IF.B.4	8. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
9. F-IF.A.3	9. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1.

10. F-BF.A.2	10. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
11. F-BF.A.1.b,	11. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
12. F-BF.A.1.c	12. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
13. G-GMD.A.2	13. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.
14. G-GMD.A.3	14. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.
15. G-GMD.B.4	15. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
16. N-Q.A.1	16. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
NJSLS	**Interdisciplinary Connections**
1. RI.CR.11–12.1	1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
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4. HS-PS2-1	4. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship

	among the net force on a macroscopic object, its mass, and its acceleration.
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3. 9.4.12.CT.2	3. Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
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The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Defi nitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions

- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components

- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook

Calculus for AP 2nd Edition: Larson and Battaglia

Online Resources

- AP Central
- CalcChat
- Desmos Activities
- Pear Assessment
- IXL
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- New Jersey Climate Education Hub

Videos

- CalcView - Video Solutions of selected problems in the textbook
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Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
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PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

AP Test Review		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Identify, calculate, and analyze various FRQ and MCQ problems	Describe how the three main components of calculus (Limits, Derivatives, and Integrals) are connected.	AP Central FRQ & MCQ

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. • Diagnostic Pre-Test • Chapter Tests • Projects • End-Of –Course Assessment	The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.	The following assessments require students to utilize various strands of mathematics. • Projects • Practice AP Exam Questions • Homework • Classwork
List of Accommodations and Modifications • Special Education • 504 Students • At Risk Students • ELL • Gifted and Talented		

State Mandates and Resources
• New Jersey Student Learning Standards • Career Readiness, Life Literacies, and Key Skills • LGBT and Disabilities Law • Asian and Pacific Islander

AP CALCULUS AB AND BC

Mathematical Practices

Practice 1

Implementing Mathematical Processes 1

Determine expressions and values using mathematical procedures and rules.

Practice 2

Connecting Representations 2

Translate mathematical information from a single representation or across multiple representations.

Practice 3

Justification 3

Justify reasoning and solutions.

Practice 4

Communication and Notation 4

Use correct notation, language, and mathematical conventions to communicate results or solutions.

SKILLS

1.A Identify the question to be answered or problem to be solved *(not assessed)*.

1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*.

1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*).

1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems.

1.E Apply appropriate mathematical rules or procedures, with and without technology.

1.F Explain how an approximated value relates to the actual value.

2.A Identify common underlying structures in problems involving different contextual situations.

2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

2.C Identify a re-expression of mathematical information presented in a given representation.

2.D Identify how mathematical characteristics or properties of functions are related in different representations.

2.E Describe the relationships among different representations of functions and their derivatives.

3.A Apply technology to develop claims and conjectures *(not assessed)*.

3.B Identify an appropriate mathematical definition, theorem, or test to apply.

3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.

3.D Apply an appropriate mathematical definition, theorem, or test.

3.E Provide reasons or rationales for solutions and conclusions.

3.F Explain the meaning of mathematical solutions in context.

3.G Confirm that solutions are accurate and appropriate.

4.A Use precise mathematical language.

4.B Use appropriate units of measure.

4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, y', *and* $\frac{dy}{dx}$).

4.D Use appropriate graphing techniques.

4.E Apply appropriate rounding procedures.

Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

AP CALCULUS BC
034010

Updated: July 2023

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 10: Additional Topics

In this unit, students will build on their understanding of straight-line motion to solve problems in which particles are moving along curves in the plane. Students will define parametric equations and vector-valued functions to describe planar motion and apply calculus to solve motion problems. Students will learn that polar equations are a special case of parametric equations and will apply calculus to analyze graphs and determine lengths and areas. This unit should be treated as an opportunity to reinforce past learning and transfer knowledge and skills to new situations, rather than as a new list of facts or strategies to memorize.

Essential Questions	Learning Targets / Objectives
1. 1. What is the Net Change Theorem? 2. 2. How do you determine the volume of a solid using the Shell Method	1. Reasoning with theorems of calculus to justify values of the rate of change function 2. Definite integrals allow us to solve problems involving the accumulation of change in area or volume over an interval
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
1. Washer 2. Disc 3. Axis of Rotation 4. Cylinder 5. Integral	1. Shell

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. F-IF.C.7.C	1. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior .
2. F-IF.B.6	2. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph
3. F-IF.B.4	3. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity
4. F-LE.B.5	4. Interpret the parameters in a linear or exponential function in terms of a context.
5. G-MG.A.3	5. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). Climate Change Example: Students may apply geometric methods to solve design problems such as increasing access to green spaces in cities given physical and cost constraints.
6. N-Q.A.1,	6. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. Climate Change Example: Students may use units to guide the solution of multi-step problems about how variations in the flow of energy into and out of the Earth's systems result in climate change. Note: Changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.
7. G-GMD.A.1	7. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
8. G-GMD.A.2	8. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures .
9. G-GMD.A.3	9. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.

10. G-GMD.B.4	10. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.
NJSLS	**Interdisciplinary Connections**
1. RI.CR.11–12.1	1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
2. W.IW.11–12.2	2. Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
3. SL.PE.11–12.1	3. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively.
4. HS-PS2-1	4. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.
2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills	
1. 9.1.12.FI.3	1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g.,college, retirement).
2. 9.3.12.AG-PST.1	2. Apply physical science principles and engineering applications to solve problems and improve performance in AFNR power, structural and technical systems.
3. 9.4.12.CT.2	3. Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
4. 9.4.12.CI.1	4. Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a).
2020 New Jersey Student Learning Standards for Computer Science and Design Thinking	
1. 8.1.12.AP.2	1. Create generalized computational solutions using collections instead of repeatedly using simple variables.
2. 8.1.12.DA.1	2. Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
3. 8.2.12.ETW.3	3. Identify a complex, global environmental or climate change issue, develop a systematic plan of investigation, and propose an innovative sustainable solution.

4. 8.2.12.ED.6	4. Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Defi nitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

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- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

4.5 - The Net Change Theorem		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• Understand and use the Net Change Theorem	Describe, in words, the fundamental theorem of calculus	Larson Text Section: 4.5 Pg: 333 - 335

Section number and Title:		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
• How do you determine the volume of a solid using the Shell Method	Determine the volume of a shape rotated on a given Axis of Rotation (Disc or Washer)	Larson Text Section: 6.3 Pg: 436 - 439

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. ● Diagnostic Pre-Test ● Chapter Tests ● Projects ● End-Of –Course Assessment	The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.	The following assessments require students to utilize various strands of mathematics. ● Projects ● Practice AP Exam Questions ● Homework ● Classwork
List of Accommodations and Modifications ● Special Education ● 504 Students ● At Risk Students ● ELL ● Gifted and Talented		

State Mandates and Resources
● New Jersey Student Learning Standards ● Career Readiness, Life Literacies, and Key Skills ● LGBT and Disabilities Law ● Asian and Pacific Islander

AP CALCULUS AB AND BC

Mathematical Practices

Practice 1

Implementing Mathematical Processes 1

Determine expressions and values using mathematical procedures and rules.

Practice 2

Connecting Representations 2

Translate mathematical information from a single representation or across multiple representations.

Practice 3

Justification 3

Justify reasoning and solutions.

Practice 4

Communication and Notation 4

Use correct notation, language, and mathematical conventions to communicate results or solutions.

SKILLS

1.A Identify the question to be answered or problem to be solved *(not assessed)*.

1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*.

1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*).

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1.E Apply appropriate mathematical rules or procedures, with and without technology.

1.F Explain how an approximated value relates to the actual value.

2.A Identify common underlying structures in problems involving different contextual situations.

2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

2.C Identify a re-expression of mathematical information presented in a given representation.

2.D Identify how mathematical characteristics or properties of functions are related in different representations.

2.E Describe the relationships among different representations of functions and their derivatives.

3.A Apply technology to develop claims and conjectures *(not assessed)*.

3.B Identify an appropriate mathematical definition, theorem, or test to apply.

3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.

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3.E Provide reasons or rationales for solutions and conclusions.

3.F Explain the meaning of mathematical solutions in context.

3.G Confirm that solutions are accurate and appropriate.

4.A Use precise mathematical language.

4.B Use appropriate units of measure.

4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, y', *and* $\frac{dy}{dx}$).

4.D Use appropriate graphing techniques.

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Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

AP CALCULUS BC
034010

Updated: July 2023

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 11: Marginal Analysis Project

In this unit, students will build on their understanding of straight-line motion to solve problems in which particles are moving along curves in the plane. Students will define parametric equations and vector-valued functions to describe planar motion and apply calculus to solve motion problems. Students will learn that polar equations are a special case of parametric equations and will apply calculus to analyze graphs and determine lengths and areas. This unit should be treated as an opportunity to reinforce past learning and transfer knowledge and skills to new situations, rather than as a new list of facts or strategies to memorize.

Essential Questions	Learning Targets / Objectives
1. 1. What is Marginal Revenue? 2. 2. What is Marginal Profit? 3. 3. What is Marginal Cost? 4. 4. How do you use calculus to determine Marginal Revenue, Profit, and/or Cost?	1. Marginal analysis is utilized as a decision making tool to maximize potential profits for businesses.
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
1. Derivative 2. Function 3. Revenue 4. Cost 5. Profit	1. Marginal

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. F-BF-A.1.B	1. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.
2. F-IF.B.5	2. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Climate Change Example: Students may relate the domain of a function c(m) representing the amount of carbon dioxide produced by burning m molecules of ethane (gasoline), to its graph in order to determine the appropriate domain for c(m).
NJSLS	**Interdisciplinary Connections**
1. RI.CR.11–12.1	1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
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- Problem Solve

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PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section number and Title:		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply**
● Define, Calculate, and Analyze Marginal Profits, Cost, and Revenues for Real-World Problem	Derive the given function	Larson Text Section: 2.5 Pg: 180-182

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

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Chapter Tests
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AP CALCULUS AB AND BC

Mathematical Practices

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Use correct notation, language, and mathematical conventions to communicate results or solutions.

SKILLS

1.A Identify the question to be answered or problem to be solved *(not assessed)*.

1.B Identify key and relevant information to answer a question or solve a problem *(not assessed)*.

1.C Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*).

1.D Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems.

1.E Apply appropriate mathematical rules or procedures, with and without technology.

1.F Explain how an approximated value relates to the actual value.

2.A Identify common underlying structures in problems involving different contextual situations.

2.B Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

2.C Identify a re-expression of mathematical information presented in a given representation.

2.D Identify how mathematical characteristics or properties of functions are related in different representations.

2.E Describe the relationships among different representations of functions and their derivatives.

3.A Apply technology to develop claims and conjectures *(not assessed)*.

3.B Identify an appropriate mathematical definition, theorem, or test to apply.

3.C Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.

3.D Apply an appropriate mathematical definition, theorem, or test.

3.E Provide reasons or rationales for solutions and conclusions.

3.F Explain the meaning of mathematical solutions in context.

3.G Confirm that solutions are accurate and appropriate.

4.A Use precise mathematical language.

4.B Use appropriate units of measure.

4.C Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, y', *and* $\frac{dy}{dx}$).

4.D Use appropriate graphing techniques.

4.E Apply appropriate rounding procedures.