# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## Course Name: AP Calculus AB

## Course Number: 034000

**Updated:** June 2024

### PART I: UNIT RATIONALE

**<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| **Unit Title: Limits and Their Properties** | |
|---|---|
| *In this unit students develop an understanding of limits as the foundational building blocks for both derivatives and integration. It is essential for discovering and developing important ideas, definitions, formulas and theorems in calculus. Students will solve limit problems graphically, algebraically, and conceptually. They will generate and work with tables, sketch and analyze various graphs, and apply numerous algebraic techniques to find limits of indeterminate forms. Students must have a solid, intuitive understanding of limits and be able to compute various limits, such as, one-sided limits, limits at infinity, infinite limits, and trigonometric limits. In addition, they will communicate both orally and in written form effectively what their answers mean in the context of the problems they are given. Finally, students will understand how limits are used to determine continuity, which is a fundamental property of functions, and apply the Intermediate Value Theorem.* | |
| **Essential Questions** | **Learning Targets/Objectives** |
| 1. What is a limit and how can you determine the limit of a function as x approaches c?<br>2. What algebraic techniques can you use to evaluate a limit?<br>3. What is continuity and how does it apply to the Intermediate Value Theorem?<br>4. What is an infinite limit? | Students will be able to:<br>• Represent limits analytically using correct notation<br>• Interpret limits expressed in analytic notation<br>• Estimate limits of functions<br>• Determine the limits of functions using limit theorems<br>• Determine the limits of functions using equivalent expressions for the function or the squeeze theorem.<br>• Justify conclusions about continuity at a point using the definition<br>• Determine intervals over which a function is continuous<br>• Determine values of x or solve for parameters that make discontinuous functions continuous, if possible |

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|---|---|
| | • Interpret the behavior of functions using limits involving infinity.<br>• Explain the behavior of a function on an interval using Intermediate Value Theorem |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| Piecewise function, direct substitution, polynomial functions, rational functions, radical function, composite function, domain, one-sided limit, vertical asymptote, horizontal asymptote, secant line, tangent line, continuity, discontinuity, removable discontinuity, nonremovable discontinuity, | limit, right handed behavior, left handed behavior, unbounded behavior, oscillating behavior, transcendental functions, indeterminate form, rationalizing technique, the Squeeze Theorem, Intermediate Value Theorem, infinite limit, limits at infinity, logistic function |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| **New Jersey Student Learning Standards That Support Learning Targets** | |
|---|---|
| **2023 New Jersey Student Learning Standards for Mathematics** | |
| 1. A-APR.D.7 | 1. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. |
| 2. F-IF.B.6 | 2. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.<br>Climate Change Example: Students may calculate the average rate of change of a function c(m) presented symbolically or as a table, where c(m) represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline). |
| **NJSLS** | **Interdisciplinary Connections** |

| | |
|---|---|
| 1. RI.CR.11–12.1 | 1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text. |
| 2. W.IW.11–12.2 | 2. Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content. |
| 3. SL.PE.11–12.1 | 3. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively. |
| 4. HS-PS2-1 | 4. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration. |
| 5. HS-PS2-2 | 5. Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system. |
| 6. HS-PS3-5 | 6. Develop and use a model of two objects interacting through electric or magnetic fields to illustrate the forces between objects and the changes in energy of the objects due to the interaction. |
| **2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills** | |
| 1. 9.1.12.FI.3 | 1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g.,college, retirement). |
| 2. 9.1.12.CFR.6 | 2. Identify and explain the consequences of breaking federal and/or state employment or financial laws. |
| 3. 9.3.12.AG-PST.1 | 3. Apply physical science principles and engineering applications to solve problems and improve performance in AFNR power, structural and technical systems. |
| 4. 9.4.12.CT.2 | 4. Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a). |
| 5. 9.4.12.CI.1 | 5. Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a). |
| **2020 New Jersey Student Learning Standards for Computer Science and Design Thinking** | |
| 1. 8.1.12.AP.2 | 1. Create generalized computational solutions using collections instead of repeatedly using simple variables. |
| 2. 8.1.12.DA.1 | 2. Create interactive data visualizations using software tools to help others better understand real world phenomena, |

| | |
|---|---|
| | including climate change. |
| 3. 8.2.12.ETW.3 | 3. Identify a complex, global environmental or climate change issue, develop a systematic plan of investigation, and propose an innovative sustainable solution. |
| 4. 8.2.12.ED.6 | 4. Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor). |

## The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

**Make sense of problems and persevere in solving them**: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

**Reason abstractly and quantitatively**: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities

- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

**Construct viable arguments and critique the reasoning of others:** Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Defi nitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

**Model with mathematics**: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

**Use appropriate tools strategically:** Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

**Attend to precision:** Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

**Look for and make use of structure:** Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

**Look for and express regularity in repeated reasoning:** Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

## Resources

**Textbook**

Calculus for AP 2nd Edition:  Larson and Battaglia

## Online Resources

- AP Central
- CalcChat
- Desmos Activities
- Pear Assessment
- Quizizz
- EdPuzzle
- Canva
- Khan Academy
- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub

## Videos

- CalcView - Video Solutions of selected problems in the textbook
- Khan Academy

## Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
    - Chromebooks
    - Texas Instrument TI-84 Plus Graphing Calculator

## ML Resources

- Multi-Language Glossary

## Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| Section 1.2: Finding Limits Graphically and Numerically | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Represent limits analytically using correct notation<br>• Interpret limits expressed ns in analytic notation<br>• Estimate limits of functions | Suggestions include but not limited to: Create a list of numbers that get closer and closer to a number without reaching it. Also refer to Lesson Motivator in textbook | Section 1.2, p. 72-75 |

| Section 1.3: Evaluating Limits Analytically | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Determine the limits of functions using limit theorems<br>• Determine the limits of functions using equivalent expressions for the function or the squeeze theorem. | Suggestions include but not limited to: Have students calculate the limit of a polynomial f(x) numerically around x=0 and compare to f(0).<br>Also refer to Lesson Motivator in textbook | Section 1.3, P. 84-86 |

| Section 1.4: Continuity and One-Sided Limits | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Justify conclusions about continuity at a point using the definition<br>• Determine intervals over which a function is continuous<br>• Determine values of x or solve for parameters that make discontinuous functions continuous, if possible<br>• Explain the behavior of a function on an interval using the IVT | Suggestions include but not limited to: Exploration on pg. 87.<br>Also refer to Lesson Motivator in textbook | Section 1.4, p. 96-99 |

| Section 1.5: Infinite Limits | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Interpret the behavior of functions using limits involving infinity | Suggestions include but not limited to: Small Socratic Seminar about the nature of infinity. Also refer to Lesson Motivator in textbook | Section 1.5, p. 105-107 |

| Section 1.6: Limits at Infinity | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Interpret the behavior of functions using limits involving infinity | Suggestions include but not limited to: Review methods of discovering end behavior for polynomials and rational function from precalculus. Also refer to Lesson Motivator in textbook | Section 1.6, p. 115-117 |

## PART IV: EVIDENCE OF LEARNING

### IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

| Assessments | | |
|---|---|---|
| **Summative** | **Formative** | **Performance** |
| The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<br><br>● Diagnostic Pre-Test<br>● Chapter Tests<br>● Cumulative Semester Assessments | The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to:<br><br>● Teacher observations<br>● Self-Assessments<br>● Student record-keeping<br>● Quizzes<br>● Warm-ups<br>● Exit Tickets<br>● Participation in class discussions<br>● Independent practice | The following assessments require students to utilize various strands of mathematics.<br><br>● Projects<br>● Performance Tasks<br>● Homework<br>● Classwork |
| **List of Accommodations and Modifications**<br>● Special Education<br>● 504 Students<br>● At Risk Students<br>● MLL<br>● Gifted and Talented | | |

| State Mandates and Resources |
|---|
| ● New Jersey Student Learning Standards<br>● Standards for Mathematical Practices |

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## Course Number: 034000

**Updated:** June 2024

## PART I: UNIT RATIONALE

**WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| Unit Title: Differentiation | |
|---|---|
| *In this unit students will get into the heart of calculus. Derivatives are a major portion of the course, so they will spend a significant amount of time in this unit. Derivatives are used to describe the rate of change of one variable with respect to another variable to understand change in a variety of contexts. At first students build the derivative using the concept of limits and use it primarily to compute the instantaneous rate of change of a function. Students should be able to use different definitions of the derivative, estimate derivatives from tables and graphs, and apply various derivative rules and properties. As they progress through this unit they will spend some time on the relationship between position, velocity and acceleration on problems involving projectile motion and rectilinear motion.* | |
| **Essential Questions** | **Learning Targets/Objectives** |
| 1. What is a derivative and what is the relationship of continuity?<br>2. How do you find the derivatives of basic algebraic functions, trigonometric functions, and exponential functions?<br>3. How do you find the derivatives of functions involving products and quotients?<br>4. How do you find the derivatives of composite functions, natural logarithmic functions, and exponential functions with bases other than e?<br>5. How do you find the derivative of implicitly defined functions?<br>6. How do you find the derivatives of inverse functions, including inverse trigonometric functions? | Students will be able to:<br>• Determine average rates of change using difference quotients<br>• Represent the derivative of a function as the limit of a difference quotient<br>• Determine the equation of a line tangent to a curve at a given point<br>• Estimate derivatives<br>• Interpret meaning of a derivative in context<br>• Explain relationship between differentiability and continuity<br>• Calculate derivatives of familiar functions<br>• Calculate rates of change in applied contexts<br>• Calculate derivatives of products and quotients of differentiable |

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| 7. What is a related rate and how do you find it? | functions<br>• Determine higher-order derivatives of a function<br>• Calculate derivatives of compositions of differentiable functions<br>• Calculate derivatives of implicitly defined functions<br>• Justify conclusions about the behavior of an implicitly defined function based on evidence from its derivative<br>• Calculate derivatives of inverse and inverse trigonometric functions<br>• Calculate related rates in applied contexts<br>• Interpret related rates in applied contexts. |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| Secant line, slope, change in x, change in y, tangent line, point-slope formula, position function, velocity, speed, acceleration, vertical asymptote, trigonometric function, logarithmic function, exponential function, natural base, inverse function, area, volume, angle of elevation, angle of depression, surface area | Difference quotient, differentiation, derivative, vertical tangent line, horizontal tangent line, instantaneous rate of change, average rate of change, open interval, closed interval, differentiability, continuity, cusp, power rule, constant multiple rule, sum and difference rule, product rule, quotient rule, second derivative, third derivative, higher-order derivative, chain rule, transcendental function, implicit differentiation, explicit form, related rate, Newton's Method |

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| 3. F-LE.B.5 | 3. Interpret the parameters in a linear or exponential function in terms of a context. |
| **NJSLS** | **Interdisciplinary Connections** |
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## Integrated Technology

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- WebAssign
- Devices:
    - Chromebooks
    - Texas Instrument TI-84 Plus Graphing Calculator

## ML Resources

| |
|---|
| • Multi-Language Glossary |
| **Gifted & Talented Resources** |
| • Leveled Assessments<br>• Enrichment worksheets |

**PART III: TRANSFER OF KNOWLEDGE AND SKILLS**

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| Section 2.1: The Derivative and the Tangent Line Problem | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Determine average rates of change using difference quotients<br>• Represent the derivative of a function as the limit of a difference quotient<br>• Determine the equation of a line tangent to a curve at a given point<br>• Estimate derivatives<br>• Interpret meaning of a derivative in context | Suggestions include but not limited to: Have students work with the difference quotient on opening problems to refresh.<br>Refer to Lesson Motivator in textbook | Section: 2.1 Pg: 132-134 |

| | | |
|---|---|---|
| • Explain relationship between differentiability and continuity<br>• Calculate derivatives of familiar functions | | |

| Section 2.2: Basic Differentiation Rules and Rates of Change | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Calculate rates of change in applied contexts<br>• Interpret rates of change in applied contexts | Suggestions include but not limited to: Complete Exploration-“Writing a Conjecture”, pg. 135 Also refer to Lesson Motivator in textbook | Section 2.2, p. 144-147 |

| Section 2.3: Product and Quotient Rules and Higher-Order Derivatives | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Calculate derivatives of products and quotients of differentiable functions<br>• Determine higher-order derivatives of a function | Suggestions include but not limited to: Algebra review, pages A38 and A39 Also refer to Lesson Motivator in textbook | Section 2.3, p. 155-158 |

| Section 2.4: The Chain Rule |
|---|

| Specific Learning Objective | Warm-Up/Starting Options | Practice & Apply Exercises |
|---|---|---|
| • Calculate derivatives of compositions of differentiable functions | Suggestions include but not limited to: Review composite function algebra<br>Also refer to Lesson Motivator in textbook | Section 2.4, p. 169-173, |

| Section 2.5: Implicit Differentiation | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Calculate derivatives of implicitly defined functions<br>• Justify conclusions about the behavior of an implicitly defined function based on evidence from its derivative | Suggestions include but not limited to: Discussion on how to write the equation of a conic section as a combination of functions. Also refer to Lesson Motivator in textbook | Section 2.5, p. 180-182 |

| Section 2.6: Derivatives of Inverse Function | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Calculate derivatives of inverse and inverse trigonometric functions. | Suggestions include but not limited to: Solve for dy/dx. (use implicit differentiation) | Section 2.6. P. 187-189 |

| Section 2.7: Related Rates | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |

| | | |
|---|---|---|
| • Calculate related rates in applied contexts<br>• Interpret related rates in applied contexts | Suggestions include but not limited to:<br>Determine if A, B, C, or D is the solution to the given problems (MC AP Practice) | Section 2.7, P. 195-198 |

## PART IV: EVIDENCE OF LEARNING

**IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR**

**UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.**

| Assessments | | |
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| **Summative** | **Formative** | **Performance** |
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| **List of Accommodations and Modifications**<br>● Special Education<br>● 504 Students<br>● At Risk Students<br>● MLL<br>● Gifted and Talented | | |

| State Mandates and Resources |
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| ● New Jersey Student Learning Standards<br>● Standards for Mathematical Practices |

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## Course Name: AP Calculus AB
## Course Number: 034000

**Updated:** June 2024

## PART I: UNIT RATIONALE

**WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| Unit Title: Applications of Differentiation | |
|---|---|
| *In this unit students will go on to understand and apply the Mean Value Theorem and will have the opportunity to see how the average rate of change can be used to justify instantaneous speed. They will also spend a significant amount of time developing a comprehensive analysis of functions (for example, determining whether a function is increasing or decreasing and finding concavity and extreme values), using not only their graphs but their derivatives as well. Students should be able to solve separable differential equations, and be familiar with a variety of real-world applications, including related rates, optimization, linear approximation and growth and decay models. This is most likely the first time students will be asked to think deeply on a conceptual level, so they may struggle to make connections at first. Students often ask how far they should simplify their answers and it should be stressed that they should look to simplify only if it provides meaningful progress, such as a much shorter or cleaner answer.* | |
| **Essential Questions** | **Learning Targets/Objectives** |
| 1. What are extrema and how can you find them on open and closed intervals?<br>2. What is the Mean Value Theorem and how is it used?<br>3. How can you determine the intervals on which a function is increasing or decreasing and the location of the function's relative extrema?<br>4. How do you determine the concavity of a function and find its inflection points?<br>5. How do you analyze a function and sketch its graph? | Students will be able to:<br>• Justify conclusions about functions by applying the Extreme Value Theorem<br>• Justify conclusions about the behavior of a function based on the behavior of its derivatives<br>• Determine critical points of implicit relations<br>• Justify conclusions about the behavior of a function based on the behavior of its derivatives<br>• Calculate minimum and maximum values in applied contexts or |

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| 6. How do you maximize or minimize quantities?<br>7. How are differentials used to explain the tangent line approximation? | analysis of functions<br>• Interpret minimum and maximum values calculated in applied contexts<br>• Approximate a value on a curve using the equation of a tangent line |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| Minimum, maximum, increasing, decreasing, constant, open interval, closed interval, derivative, average rate of change, instantaneous rate of change, horizontal tangent, secant line, x-intercept, y-intercept, symmetry, domain, range, continuity, vertical asymptote, horizontal asymptote, differentiability, cusp, first derivative, second derivative | Absolute minimum, absolute maximum, global minimum, global maximum, relative minimum, relative maximum, extreme value, critical number, Rolle's Theorem, Mean Value Theorem, First Derivative Test, concavity, concave up, concave down, Second Derivative Test, inflection point, optimization problems, primary equation, secondary equation, tangent line approximation (linear approximation), differential form, L'Hopital's Rule, indeterminate form |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| New Jersey Student Learning Standards That Support Learning Targets | |
|---|---|
| **2023 New Jersey Student Learning Standards for Mathematics** | |
| 1. F-IF.C.7c | 1. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. |
| 2. F-IF.B.4 | 2. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity |
| 3. F-IFB.5 | 3. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Climate Change Example: Students may relate the domain of a function c(m) representing the amount of carbon dioxide produced by burning m molecules of ethane (gasoline), to its graph in order to determine the appropriate domain for c(m). |
| 4. F-LE.B.5 | 4. Interpret the parameters in a linear or exponential function in terms of a context. |
| 5. G-MG.A.3 | 5. Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios). Climate Change Example: Students may apply geometric methods to solve design problems such as increasing access to green spaces in cities given physical and cost constraints. |
| **NJSLS** | **Interdisciplinary Connections** |
| 1. RI.CR.11–12.1 | 1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text. |
| 2. W.IW.11–12.2 | 2. Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content. |
| 3. SL.PE.11–12.1 | 3. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) |

| | |
|---|---|
| | with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively. |
| 4. HS-PS2-1 | 4. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration. |
| **2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills** | |
| 1. 9.1.12.FI.3 | 1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g.,college, retirement). |
| 2. 9.3.12.AG-PST.1 | 2. Apply physical science principles and engineering applications to solve problems and improve performance in AFNR power, structural and technical systems. |
| 3. 9.4.12.CT.2 | 3. Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a). |
| 4. 9.4.12.CI.1 | 4. Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a). |
| **2020 New Jersey Student Learning Standards for Computer Science and Design Thinking** | |
| 1. 8.1.12.AP.2 | 1. Create generalized computational solutions using collections instead of repeatedly using simple variables. |
| 2. 8.1.12.DA.1 | 2. Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change. |
| 3. 8.2.12.ETW.3 | 3. Identify a complex, global environmental or climate change issue, develop a systematic plan of investigation, and propose an innovative sustainable solution. |
| 4. 8.2.12.ED.6 | 4. Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor). |

## The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

**Make sense of problems and persevere in solving them**: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

**Reason abstractly and quantitatively**: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

**Construct viable arguments and critique the reasoning of others:** Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions

- Use Defi nitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

**Model with mathematics**: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

**Use appropriate tools strategically:** Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

**Attend to precision:** Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely

- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

**Look for and make use of structure:** Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

**Look for and express regularity in repeated reasoning:** Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

| Resources |
| --- |
| **Textbook** |
| Calculus for AP 2nd Edition: Larson and Battaglia |
| **Online Resources** |
| • AP Central<br>• CalcChat<br>• Desmos Activities<br>• Pear Assessment<br>• IXL<br>• Quizizz |

- EdPuzzle
- Canva
- Khan Academy
- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub

**Videos**

- CalcView - Video Solutions of selected problems in the textbook
- Khan Academy

**Integrated Technology**

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

**ML Resources**

- Multi-Language Glossary

**Gifted & Talented Resources**

- Leveled Assessments
- Enrichment worksheets

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| Section 3.1: Extrema on an Interval | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Justify conclusions about functions by applying the Extreme Value Theorem<br>• Justify conclusions about the behavior of a function based on the behavior of its derivatives<br>• Determine critical points of implicit relations | Suggestions include but not limited to: Have students identify locations of a maximum using the feature on the calculator. Refer to Lesson Motivator in textbook | Section 3.1, p.217-219 |

| Section 3.2: Rolle's Theorem and the Mean Value Theorem | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Justify conclusions about functions by applying the Mean Value Theorem over an interval | Suggestions include but not limited to: Complete Exploration-Extreme Values in a Closed Interval on p.220. Also refer to Lesson Motivator in textbook | Section 3.2, p. 224-226 |

| Section 3.3: Product and Quotient Rules and Higher-Order Derivatives | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Justify conclusions about the behavior of a function based on the behavior of its derivatives | Suggestions include but not limited to: Have students identify intervals of increase and decrease by viewing a graph. Also refer to Lesson Motivator in textbook | Section 3.3, P 233-236 |

| 3.4: Concavity and Second Derivative Test | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Justify conclusions about the behavior of a function based on the behavior of its derivatives | Suggestions include but not limited to: Have students find the first two derivatives of a rational function. Also refer to Lesson Motivator in textbook | Section 3.4, p. 242-244 |

| Section 3.5: A Summary of Curve Sketching | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Justify conclusions about the behavior of a function based on the behavior of its derivatives | Suggestions include but not limited to: Have students list all information needed to sketch a polynomial function precisely without a calculator. Also refer to Lesson Motivator in textbook | Section 3.5, P253-256 |

| Section 3.6: Optimization Problems | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Solve applied maximum and minimum problems | Suggestions include but not limited to: Have students list possible dimensions for a rectangular yard given the area Also refer to Lesson Motivator in textbook | Section 3.6<br>P 262-266 # 2, 5, 9-23 odd, 29, 31, 37, 41, 48, 54-56 |

| Section 3.7 Linear Approximation and Differentials | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Calculate minimum and maximum values in applied contexts or analysis of functions<br>• Interpret minimum and maximum values calculated in applied contexts | Suggestions include but not limited to: Have students complete the exploration on p267. Also refer to Lesson Motivator in textbook | Section 3.7, P 272-275 |

## PART IV: EVIDENCE OF LEARNING

**IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.**

| Assessments | | |
|---|---|---|
| **Summative** | **Formative** | **Performance** |
| The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<br><br>● Diagnostic Pre-Test<br>● Chapter Tests<br>● Cumulative Semester Assessments | The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to:<br><br>● Teacher observations<br>● Self-Assessments<br>● Student record-keeping<br>● Quizzes<br>● Warm-ups<br>● Exit Tickets<br>● Participation in class discussions<br>● Independent practice | The following assessments require students to utilize various strands of mathematics.<br><br>● Projects<br>● Performance Tasks<br>● Homework<br>● Classwork |
| **List of Accommodations and Modifications**<br>● Special Education<br>● 504 Students<br>● At Risk Students<br>● MLL<br>● Gifted and Talented | | |

| State Mandates and Resources |
|---|
| ● New Jersey Student Learning Standards |

- Standards for Mathematical Practices

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## Course Name: AP Calculus AB
## Course Number: 034000

**Updated:** June 2024

## PART I: UNIT RATIONALE

**<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| **Unit Title: Integration** | |
|---|---|
| *In this unit students will discover the relationship between differentiation and integration as inverse operations. Students will learn how to integrate functions using definite integrals and indefinite integrals through the use of Riemann sums. Students will use integration to solve many real-world applications. Students will also learn the importance of the Fundamental Theorem of Calculus and its many applications. Students will revisit differentiation when they learn to integrate using transcendental functions. This will instill the importance of the chain rule which they learned in the Derivative unit.* | |
| **Essential Questions** | **Learning Targets/Objectives** |
| 1. What are antiderivatives and how are they used?<br>2. How can you approximate the area of a plane region?<br>3. How are Riemann sums similar to the Trapezoidal Rule and how are they different?<br>4. What is the Fundamental Theorem of Calculus?<br>5. How do you integrate composite functions?<br>6. How do you integrate rational functions and trigonometric functions other than sine or cosine?<br>7. How can you recognize when an integral results in an inverse trigonometric function? | Students will be able to:<br>• Evaluate definite integrals analytical using the Fundamental Theorem of Calculus<br>• Determine antiderivatives of functions and indefinite integrals, using knowledge of derivatives.<br>• Interpret verbal statements of problems as differential equations involving a derivative expression<br>• Determine general solutions to differential equations<br>• Determine particular solutions to differential equations<br>• Interpret the meaning of areas associated with the graph of a rate of change in context.<br>• Approximate a definite integral using geometric and numerical |

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| | methods<br>• Interpret the limiting case of the Riemann sum as a definite integral<br>• Represent the limiting case of the Riemann sum as a definite integral<br>• Calculate a definite integral using areas and properties of definite integrals<br>• Determine the average value of a function using definite integrals<br>• Interpret the meaning of a definite integral in accumulation problems.<br>• Calculate a definite integral using areas and properties of definite integrals<br>• Represent accumulation functions using definite integrals<br>• For integrands requiring substitution or rearrangements into equivalent forms:<br>a) Determine indefinite integrals<br>b) Evaluate definite integrals |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| Differential equation, initial condition, position, velocity, acceleration, area of a rectangle, area of a trapezoid, continuity, u-substitution, even function, odd function | Antiderivative, integration, indefinite integral, definite integral, particular solution, sigma notation, upper bound, lower bound, area under the curve, inscribed rectangle, circumscribe rectangle, lower sum, upper sum, Riemann sum, general partition, Trapezoidal Rule, Fundamental Theorem of Calculus, Mean Value Theorem for Integrals, average value, Second Fundamental Theorem of Calculus, change of variables, Power Rule for Integration, Log Rule for Integration, area between two curves |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| New Jersey Student Learning Standards That Support Learning Targets | |
|---|---|
| **2023 New Jersey Student Learning Standards for Mathematics** | |
| 1. F-BF.A.2 | 1. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. |
| 2. F-IFB.6 | 2. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.<br>Climate Change Example: Students may calculate the average rate of change of a function c(m) presented symbolically or as a table, where c(m) represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline). |
| 3. F-LE.B.5 | 3. Interpret the parameters in a linear or exponential function in terms of a context. |
| **NJSLS** | **Interdisciplinary Connections** |
| 1. RI.CR.11–12.1 | 1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text. |
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| 4. HS-PS2-1 | 4. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration. |
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| | |
|---|---|
| 1. 9.1.12.FI.3 | 1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g.,college, retirement). |
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| 3. 9.4.12.CT.2 | 3. Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a). |
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| 1. 8.1.12.AP.2 | 1. Create generalized computational solutions using collections instead of repeatedly using simple variables. |
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| 3. 8.2.12.ETW.3 | 3. Identify a complex, global environmental or climate change issue, develop a systematic plan of investigation, and propose an innovative sustainable solution. |
| 4. 8.2.12.ED.6 | 4. Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor). |

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**Make sense of problems and persevere in solving them**: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
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- Check Progress
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- Problem Solve

**Reason abstractly and quantitatively**: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

**Construct viable arguments and critique the reasoning of others:** Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Defi nitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions

- Critique Reasoning
- Use Logic
- Error Analysis

**Model with mathematics**: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

**Use appropriate tools strategically:** Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
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**Attend to precision:** Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

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- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
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- Understand Mathematical Terms

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- Look for Patterns
- Look for Structure

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- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

## Resources

### Textbook

Calculus for AP 2nd Edition: Larson and Battaglia

### Online Resources

- AP Central
- CalcChat
- Desmos Activities
- Pear Assessment
- IXL
- Quizizz
- EdPuzzle
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- Khan Academy
- Inside Mathematics
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### Videos

- CalcView - Video Solutions of selected problems in the textbook
- Khan Academy

**Integrated Technology**

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

**ML Resources**

- Multi-Language Glossary

**Gifted & Talented Resources**

- Leveled Assessments
- Enrichment worksheets

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| Section 4.1: Antiderivatives | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Evaluate definite integrals analytical using the Fundamental Theorem of Calculus<br>• Determine antiderivatives of functions and indefinite integrals, using knowledge of derivatives.<br>• Interpret verbal statements of problems as differential equations involving a derivative expression<br>• Determine general solutions to differential equations<br>• Determine particular solutions to differential equations | Suggestions include but not limited to: Approximate Antiderivatives from Derivatives.<br>Refer to Lesson Motivator in textbook | Section 4.1, p. 287-289 |

| Section 4.2: Area | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Interpret the meaning of areas associated with the graph of a rate of change in context. | Suggestions include but not limited to: Discuss Archimedes method of exhaustion for finding area. Also refer to Lesson Motivator in textbook | Section 4.2, p.299-301 |

| Section 4.3 Riemann Sums and Definite Integrals | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Approximate a definite integral using geometric and numerical methods<br>• Interpret the limiting case of the Riemann sum as a definite integral<br>• Represent the limiting case of the Riemann sum as a definite integral<br>• Calculate a definite integral using areas and properties of definite integrals | Suggestions include but not limited to:<br>Model Riemann sum by drawing smaller rectangles and<br>discuss trapezoid rule to approximate area under curve.<br>Also refer to Lesson Motivator in textbook | Section 4.3, p. 312-316 |

| Section 4.4 The Fundamental Theorem of Calculus | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Determine the average value of a function using definite integrals<br>• Interpret the meaning of a definite integral in accumulation problems.<br>• Calculate a definite integral using areas and properties of definite integrals<br>• Represent accumulation functions using definite integrals | Suggestions include but not limited to:<br>Discuss notation difference between the definite and Indefinite Integral. Also refer to Lesson Motivator in textbook | Section 4.4, p.326-328 |

| Section 4.6 Integration | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |

| | | |
|---|---|---|
| • For integrands requiring substitution or rearrangements into equivalent forms:<br>a) Determine indefinite integrals<br>b) Evaluate definite integrals | Suggestions include but not limited to:<br>Show how the need for a new technique arises with the introduction of composite functions.<br>Also refer to Lesson Motivator in textbook | Section 4.6, p.343-346 |

| Section 4.7 The Natural Logarithmic Functions: Integration | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • For integrands requiring substitution or rearrangements into equivalent forms:<br>a) Determine indefinite integrals<br>b) Evaluate definite integrals | Suggestions include but not limited to:<br>Give examples of Integrating Rational Functions. Also refer to Lesson Motivator in textbook | Section 4.7<br>p.353-355 #1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 43, 44, 47-49, 53, 55, 58, 68, 70, 71-77 odd, 81, 92, 94, 104-107. |

| Section 4.8: Inverse Trigonometric Functions: Integration | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • For integrands requiring substitution or rearrangements into equivalent forms:<br>a) Determine indefinite integrals<br>b) Evaluate definite integrals | Suggestions include but not limited to:<br>Evaluate the given integral (logarithmic function) | Section 4.8, P. 361-363 |

## PART IV: EVIDENCE OF LEARNING

**IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.**

## Assessments

| Summative | Formative | Performance |
|---|---|---|
| The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<br><br>● Diagnostic Pre-Test<br>● Chapter Tests<br>● Cumulative Semester Assessments | The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to:<br><br>● Teacher observations<br>● Self-Assessments<br>● Student record-keeping<br>● Quizzes<br>● Warm-ups<br>● Exit Tickets<br>● Participation in class discussions<br>● Independent practice | The following assessments require students to utilize various strands of mathematics.<br><br>● Projects<br>● Performance Tasks<br>● Homework<br>● Classwork |

**List of Accommodations and Modifications**

- Special Education
- 504 Students
- At Risk Students
- MLL
- Gifted and Talented

## State Mandates and Resources

- New Jersey Student Learning Standards
- Standards for Mathematical Practices

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## Course Name: AP Calculus AB
## Course Number: 034000

**Updated:** June 2024

## PART I: UNIT RATIONALE

**<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| **Unit Title: Integration** | |
|---|---|
| *Students will see in this unit a substantial amount of real-life applications. It starts with the process of differential equations with both general and particular solutions, with an emphasis on slope fields and how they produce solutions. It is followed up with specific growth and decay models that are tied to everyday situations. Separation of variables follows with more emphasis placed on real-world applications including but not limited to: population growth, financial mathematics, and other scientific principles such as radioactive decay.* | |
| **Essential Questions** | **Learning Targets/Objectives** |
| 1. How do you approximate the particular solution of a differential equation?<br>2. How are differential equations used in application problems, such as the exponential growth and decay model?<br>3. How do you solve separable differential equations? | Students will be able to:<br>• Interpret verbal statements of problems as differential equations involving a derivative expression.<br>• Verify solutions to differential equations.<br>• Estimate solutions to differential equations.<br>• Determine general solutions to differential equations.<br>• Determine particular solutions to differential equations.<br>• Interpret the meaning of a differential equation and its variables in context.<br>• Determine general and particular solutions for problems involving differential equations in context. |

| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
|---|---|
| Differential equation, general solution, singular solution, particular solution, slope, initial conditions, exponential growth, exponential decay, | Solution curves, slope field, Euler's Method, proportionality constant, separable, separation of variables, |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| **New Jersey Student Learning Standards That Support Learning Targets** | |
|---|---|
| **2023 New Jersey Student Learning Standards for Mathematics** | |
| 1. F-LE.B.5 | 1. Interpret the parameters in a linear or exponential function in terms of a context. |
| 2. F-BF.A.1.b | 2. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. |
| 3. F-BF.A.1.c | 3. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. |
| 4. F-IF.B.6 | 4. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph.<br>Climate Change Example: Students may calculate the average rate of change of a function c(m) presented symbolically or as a table, where c(m) represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline). |
| **NJSLS** | **Interdisciplinary Connections** |
| 1. RI.CR.11–12.1 | 1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a |

| | |
|---|---|
| | comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text. |
| 2. W.IW.11–12.2 | 2. Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content. |
| 3. SL.PE.11–12.1 | 3. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others' ideas and expressing their own clearly and persuasively. |
| 4. HS-PS2-1 | 4. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration. |
| **2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills** | |
| 1. 9.1.12.FI.3 | 1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g.,college, retirement). |
| 2. 9.3.12.AG-PST.1 | 2. Apply physical science principles and engineering applications to solve problems and improve performance in AFNR power, structural and technical systems. |
| 3. 9.4.12.CT.2 | 3. Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a). |
| 4. 9.4.12.CI.1 | 4. Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a). |
| **2020 New Jersey Student Learning Standards for Computer Science and Design Thinking** | |
| 1. 8.1.12.AP.2 | 1. Create generalized computational solutions using collections instead of repeatedly using simple variables. |
| 2. 8.1.12.DA.1 | 2. Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change. |
| 3. 8.2.12.ETW.3 | 3. Identify a complex, global environmental or climate change issue, develop a systematic plan of investigation, and propose an innovative sustainable solution. |
| 4. 8.2.12.ED.6 | 4. Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor). |

## The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

**Make sense of problems and persevere in solving them**: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

**Reason abstractly and quantitatively**: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

**Construct viable arguments and critique the reasoning of others:** Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Defi nitions
- Use Prior Results
- Make Conjectures

- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

**Model with mathematics**: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

**Use appropriate tools strategically:** Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

**Attend to precision:** Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units

- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

**Look for and make use of structure:** Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

**Look for and express regularity in repeated reasoning:** Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

## Resources

### Textbook

Calculus for AP 2nd Edition: Larson and Battaglia

### Online Resources

- AP Central
- CalcChat
- Desmos Activities
- Pear Assessment
- IXL
- Quizizz
- EdPuzzle
- Canva
- Khan Academy

- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub

**Videos**

- CalcView - Video Solutions of selected problems in the textbook
- Khan Academy

**Integrated Technology**

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

**ML Resources**

- Multi-Language Glossary

**Gifted & Talented Resources**

- Leveled Assessments
- Enrichment worksheets

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| Section 5.1: Slope Fields | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Interpret verbal statements of problems as differential equations involving a derivative expression.<br>• Verify solutions to differential equations.<br>• Estimate solutions to differential equations. | Suggestions include but not limited to: Discuss the idea of slope and the difference how positive, negative, zero and undefined slopes are represented on a graph. | Section: 5.1 Pg: 375-377 |

| Section 5.2: Growth and Decay | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Determine general and particular solutions for problems involving differential equations in context.<br>• Interpret the meaning of a differential equation and its variables in context. | Suggestions include but not limited to: Discuss situations real world applications in medicine and science which require the modeling of growth and decay. | Section: 5.2 Pg: 384-386 |

| Section 5.3: Separation of Variables | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Determine general solutions to differential equations.<br>• Determine particular solutions to differential equations. | Suggestions include but not limited to: Find the integral of a transcendental function using u-substitution. | Section: 5.3 Pg: 393-396 |

| | | |
|---|---|---|
| | | |

## PART IV: EVIDENCE OF LEARNING

**IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.**

| Assessments | | |
|---|---|---|
| **Summative** | **Formative** | **Performance** |
| The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<br><br>● Diagnostic Pre-Test<br>● Chapter Tests<br>● Cumulative Semester Assessments | The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to:<br><br>● Teacher observations<br>● Self-Assessments<br>● Student record-keeping<br>● Quizzes<br>● Warm-ups<br>● Exit Tickets<br>● Participation in class discussions<br>● Independent practice | The following assessments require students to utilize various strands of mathematics.<br><br>● Projects<br>● Performance Tasks<br>● Homework<br>● Classwork |
| **List of Accommodations and Modifications**<br>● Special Education<br>● 504 Students<br>● At Risk Students<br>● MLL<br>● Gifted and Talented | | |

| State Mandates and Resources |
|---|
| ● New Jersey Student Learning Standards<br>● Standards for Mathematical Practices |

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## Course Name: AP Calculus AB
## Course Number: 034000

**Updated:** June 2024

## PART I: UNIT RATIONALE

**<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| **Unit Title: Integration** | |
|---|---|
| *Students will use integration to find the area between two curves. They will also use rotation and other techniques to find the volume of a variety of 3-D solids as well as of solids with known cross-sectional areas. The First Fundamental Theorem of Calculus will play a major role in the development in understanding of both area and volume, and students will have to rely on a variety of integration techniques to help them progress through the chapter including: substitution, transcendental and trigonometric techniques.* | |
| **Essential Questions** | **Learning Targets/Objectives** |
| 1. How do you find the area of a region between two curves?<br>2. How can you use integrals to find the volume of a solid? | Students will be able to:<br>• Calculate areas in the plan using the definite integral.<br>• Calculate volumes of solids with known cross sections using definite integrals.<br>• Calculate volumes of solids of revolution using definite integrals. |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| Area under a curve, upper bound, lower bound, point of intersection, continuous function, volume, outer radius, inner radius | Area between two curves, disk method, washer method, solid of revolution, axis of revolution, cross section |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| New Jersey Student Learning Standards That Support Learning Targets | |
|---|---|
| **2023 New Jersey Student Learning Standards for Mathematics** | |
| 1. N-Q.A.1 | 1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. Climate Change Example: Students may use units to guide the solution of multi-step problems about how variations in the flow of energy into and out of the Earth's systems result in climate change. Note: Changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution. |
| 2. F-IF.C.7c | 2. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. |
| 3. G-GMD.A.1 | 3. Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. |
| 4. G-GMD.A.2 | 4. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. |
| 5. G-GMD.A.3 | 5. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. |
| 6. G-GMD.B.4 | 6. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. |
| **NJSLS** | **Interdisciplinary Connections** |
| 1. RI.CR.11–12.1 | 1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text. |

| | |
|---|---|
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- Use Defi nitions
- Use Prior Results
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- Build Arguments
- Analyze Conjectures
- Use Counterexamples

- Justify Conclusions
- Compare Arguments
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- Specify Units
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- Calculate Accurately
- Understand Mathematical Terms

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- Find General Methods
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## Resources

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### Online Resources

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- Desmos Activities
- Pear Assessment
- IXL
- Quizizz
- EdPuzzle
- Canva
- Khan Academy
- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning

- New Jersey Climate Education Hub

**Videos**

- CalcView - Video Solutions of selected problems in the textbook
- Khan Academy

**Integrated Technology**

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

**ML Resources**

- Multi-Language Glossary

**Gifted & Talented Resources**

- Leveled Assessments
- Enrichment worksheets

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| Section 6.1: Area of a Region Between Two Curves | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Calculate areas in the plan using the definite integral. | Suggestions include but not limited to: Find the area of a region under one curve. Then find the area of a region under a second curve underneath that. What is the difference in area? | Section: 6.1 Pg: 416-419 |

| Section 6.2: Volume: The Disk and Washer Method | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Calculate volumes of solids with known cross sections using definite integrals.<br>• Calculate volumes of solids of revolution using definite integrals. | Suggestions include but not limited to: Sketch the general concept behind the Disk Method and its relationship to the area of circles. | Section: 6.2 Pg: 427-430 |

## PART IV: EVIDENCE OF LEARNING

**IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.**

| Assessments | | |
|---|---|---|
| **Summative** | **Formative** | **Performance** |

| | | |
|---|---|---|
| The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<br><br>● Diagnostic Pre-Test<br>● Chapter Tests<br>● Cumulative Semester Assessments | The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to:<br><br>● Teacher observations<br>● Self-Assessments<br>● Student record-keeping<br>● Quizzes<br>● Warm-ups<br>● Exit Tickets<br>● Participation in class discussions<br>● Independent practice | The following assessments require students to utilize various strands of mathematics.<br><br>● Projects<br>● Performance Tasks<br>● Homework<br>● Classwork |
| **List of Accommodations and Modifications**<br>● Special Education<br>● 504 Students<br>● At Risk Students<br>● MLL<br>● Gifted and Talented | | |

## State Mandates and Resources

- New Jersey Student Learning Standards
- Standards for Mathematical Practices

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## AP CALCULUS AB
## 034000

**Updated:** October 2024

## PART I: UNIT RATIONALE

**WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

### Unit 9: AP Test Review

In this unit, students will build on their understanding of straight-line motion to solve problems in which particles are moving along curves in the plane. Students will define parametric equations and vector-valued functions to describe planar motion and apply calculus to solve motion problems. Students will learn that polar equations are a special case of parametric equations and will apply calculus to analyze graphs and determine lengths and areas. This unit should be treated as an opportunity to reinforce past learning and transfer knowledge and skills to new situations, rather than as a new list of facts or strategies to memorize.

| Essential Questions | Learning Targets / Objectives |
|---|---|
| 1. 1. How do you dissect and apply appropriate calculus techniques to FRQ AP Test questions?<br>2. 2. How do you dissect and apply appropriate calculus techniques to MCQ AP Test questions? | 1. Reasoning with definitions, theorems and properties can be used to justify claims about FRQ style questions.<br>2. Reasoning with definitions, theorems and properties can be used to justify claims about MCQ style questions. |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| 1. Limit<br>2. Derivative<br>3. Integral<br>4. Differential<br>5. Volume | 1. FRQ<br>2. MCQ |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| New Jersey Student Learning Standards That Support Learning Targets | |
|---|---|
| **2023 New Jersey Student Learning Standards for Mathematics** | |
| 1. A-SSE.A.1b | 1. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1+r)n as the product of P and a factor not depending on P. |
| 2. A-APR.D.7 | 2. Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. |
| 3. F-IF.C.7c | 3. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. |
| 4. F-IF.C.8.a | 4. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. |
| 5. F-IF.C.9 | 5. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. |
| 6. F-IF.B.6 | 6. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. |
| 7. F-LE.B.5 | 7. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. |
| 8. F-IF.B.4 | 8. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. |
| 9. F-IF.A.3 | 9. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n) + f(n-1) for n ≥ 1. |

| | |
|---|---|
| 10. F-BF.A.2 | 10. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. |
| 11. F-BF.A.1.b, | 11. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. |
| 12. F-BF.A.1.c | 12. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time. |
| 13. G-GMD.A.2 | 13. Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. |
| 14. G-GMD.A.3 | 14. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. |
| 15. G-GMD.B.4 | 15. Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. |
| 16. N-Q.A.1 | 16. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. |
| **NJSLS** | **Interdisciplinary Connections** |
| 1. RI.CR.11–12.1 | 1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text. |
| 2. W.IW.11–12.2 | 2. Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content. |
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| 4. HS-PS2-1 | 4. Analyze data to support the claim that Newton's second law of motion describes the mathematical relationship |

| | |
|---|---|
| | among the net force on a macroscopic object, its mass, and its acceleration. |
| **2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills** | |
| 1. 9.1.12.FI.3 | 1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g.,college, retirement). |
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**The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:**

**Make sense of problems and persevere in solving them**: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
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**Look for and make use of structure:** Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

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**Integrated Technology**

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

**ML Resources**

- Multi-Language Glossary

**Gifted & Talented Resources**

- Leveled Assessments
- Enrichment worksheets

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| AP Test Review | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| • Identify, calculate, and analyze various FRQ and MCQ problems | Describe how the three main components of calculus (Limits, Derivatives, and Integrals) are connected. | AP Central FRQ & MCQ |

## PART IV: EVIDENCE OF LEARNING

**IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.**

| Assessments | | |
|---|---|---|
| **Summative** | **Formative** | **Performance** |
| The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<br>• Diagnostic Pre-Test<br>• Chapter Tests<br>• Projects<br>• End-Of –Course Assessment | The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations. | The following assessments require students to utilize various strands of mathematics.<br>• Projects<br>• Practice AP Exam Questions<br>• Homework<br>• Classwork |
| **List of Accommodations and Modifications**<br>• Special Education<br>• 504 Students<br>• At Risk Students<br>• ELL<br>• Gifted and Talented | | |

| State Mandates and Resources |
| --- |
| • New Jersey Student Learning Standards<br>• Career Readiness, Life Literacies, and Key Skills<br>• LGBT and Disabilities Law<br>• Asian and Pacific Islander |

AP CALCULUS AB AND BC

# Mathematical Practices

## Practice 1

### *Implementing Mathematical Processes* 1

Determine expressions and values using mathematical procedures and rules.

## Practice 2

### *Connecting Representations* 2

Translate mathematical information from a single representation or across multiple representations.

## Practice 3

### *Justification* 3

Justify reasoning and solutions.

## Practice 4

### *Communication and Notation* 4

Use correct notation, language, and mathematical conventions to communicate results or solutions.

**SKILLS**

**1.A** Identify the question to be answered or problem to be solved *(not assessed)*.

**1.B** Identify key and relevant information to answer a question or solve a problem *(not assessed)*.

**1.C** Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*).

**1.D** Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems.

**1.E** Apply appropriate mathematical rules or procedures, with and without technology.

**1.F** Explain how an approximated value relates to the actual value.

**2.A** Identify common underlying structures in problems involving different contextual situations.

**2.B** Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

**2.C** Identify a re-expression of mathematical information presented in a given representation.

**2.D** Identify how mathematical characteristics or properties of functions are related in different representations.

**2.E** Describe the relationships among different representations of functions and their derivatives.

**3.A** Apply technology to develop claims and conjectures *(not assessed)*.

**3.B** Identify an appropriate mathematical definition, theorem, or test to apply.

**3.C** Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.

**3.D** Apply an appropriate mathematical definition, theorem, or test.

**3.E** Provide reasons or rationales for solutions and conclusions.

**3.F** Explain the meaning of mathematical solutions in context.

**3.G** Confirm that solutions are accurate and appropriate.

**4.A** Use precise mathematical language.

**4.B** Use appropriate units of measure.

**4.C** Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, $y'$, *and* $\frac{dy}{dx}$).

**4.D** Use appropriate graphing techniques.

**4.E** Apply appropriate rounding procedures.

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## Course Name: AP Calculus AB
## Course Number: 034000

**Updated:** June 2024

## PART I: UNIT RATIONALE

**WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

| **Unit Title: Integration** | |
|---|---|
| *This unit begins with one of the last topics before the AP Exam, L'Hôpital's Rule. It is then followed with a review of all previous topics for the AP Exam itself. Afterwards, if time permits, students will be introduced to advanced Calculus topics that will be discussed in college classes: integration by parts and integration of partial fractions.* | |
| **Essential Questions** | **Learning Targets/Objectives** |
| 1. How do you evaluate a limit when direct substitution produces an indeterminate form?<br>2. How do you integrate a complex rational function?<br>3. What other techniques can be used to evaluate integrals? | Students will be able to:<br>• Determine limits of functions that result in indeterminate forms.<br>• Evaluate integrands requiring integration by parts (indefinite only)<br>• Evaluate an improper integral or determine that the integral diverges |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| U-substitution, differential, particular solution, linear factors, quadratic factors, divide rational expression, factors | Integration by parts, tabular method, partial fraction, decomposing rational expressions, basic equations, distinct linear factor, distinct quadratic factors |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| New Jersey Student Learning Standards That Support Learning Targets | |
|---|---|
| **2023 New Jersey Student Learning Standards for Mathematics** | |
| 1. F-IF.B.4 | 1. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. |
| 2. F-BF.A.2 | 2. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. |
| 3. F-IF.B.6 | 3. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. |
| 4. F-LE.B.5 | 4. Interpret the parameters in a linear or exponential function in terms of a context. |
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| | |
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  - Chromebooks
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## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| Section 7.7: Indeterminate Forms and L'Hôpital's Rule | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Determine limits of functions that result in indeterminate forms. | Suggestions include but not limited to: Find the limit on the board using techniques that have already been used (factor and canceling, trigonometric identities) | Section: 7.7 Pg: 513-516 |

****THESE NEXT TWO SECTIONS SHOULD ONLY BE COVERED AFTER THE AP EXAM IF TIME PERMITS****

| Section 7.2: Integration by Parts | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • For integrands requiring integration by parts: | Suggestions include but not limited to: Write down the integration by parts formula. What do you notice about it? Discussion about how it is set up. | Section: 7.2 Pg: 469-472 |

| Section 7.5: Partial Fractions | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply Exercises** |
| • Evaluate an improper integral or determine that the integral diverges | Suggestions include but not limited to: How do we decompose a function? Go through the process of factoring and finding the basic equation and solving for the variables. | Section: 7.6 Pg: 504-505 |

## PART IV: EVIDENCE OF LEARNING

**IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.**

## Assessments

| Summative | Formative | Performance |
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| The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<br><br>● Diagnostic Pre-Test<br>● Chapter Tests<br>● Cumulative Semester Assessments | The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to:<br><br>● Teacher observations<br>● Self-Assessments<br>● Student record-keeping<br>● Quizzes<br>● Warm-ups<br>● Exit Tickets<br>● Participation in class discussions<br>● Independent practice | The following assessments require students to utilize various strands of mathematics.<br><br>● Projects<br>● Performance Tasks<br>● Homework<br>● Classwork |

**List of Accommodations and Modifications**

- Special Education
- 504 Students
- At Risk Students
- MLL
- Gifted and Talented

## State Mandates and Resources

- New Jersey Student Learning Standards
- Standards for Mathematical Practices

# Black Horse Pike Regional School District

**Where inspiring excellence is our standard, and student achievement is the result.**

## AP CALCULUS AB
## 034000

**Updated:** October 2024

## PART I: UNIT RATIONALE

**WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?**

### Unit 11: Marginal Analysis Project

In this unit, students will build on their understanding of straight-line motion to solve problems in which particles are moving along curves in the plane. Students will define parametric equations and vector-valued functions to describe planar motion and apply calculus to solve motion problems. Students will learn that polar equations are a special case of parametric equations and will apply calculus to analyze graphs and determine lengths and areas. This unit should be treated as an opportunity to reinforce past learning and transfer knowledge and skills to new situations, rather than as a new list of facts or strategies to memorize.

| Essential Questions | Learning Targets / Objectives |
|---|---|
| 1. 1. What is Marginal Revenue?<br>2. 2. What is Marginal Profit?<br>3. 3. What is Marginal Cost?<br>4. 4. How do you use calculus to determine Marginal Revenue, Profit, and/or Cost? | 1. Marginal analysis is utilized as a decision making tool to maximize potential profits for businesses. |
| **Tier 2 Vocabulary**<br>*High-frequency words used throughout the unit* | **Tier 3 Vocabulary**<br>*Discipline-specific words used throughout the unit* |
| 1. Derivative<br>2. Function<br>3. Revenue<br>4. Cost<br>5. Profit | 1. Marginal |

## PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

**DESCRIBE THE LEARNING TARGETS.**

| New Jersey Student Learning Standards That Support Learning Targets | |
|---|---|
| **2023 New Jersey Student Learning Standards for Mathematics** | |
| 1. F-BF-A.1.B | 1. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. |
| 2. F-IF.B.5 | 2. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Climate Change Example: Students may relate the domain of a function c(m) representing the amount of carbon dioxide produced by burning m molecules of ethane (gasoline), to its graph in order to determine the appropriate domain for c(m). |
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- Look for Patterns
- Look for Structure

**Look for and express regularity in repeated reasoning:** Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

## Resources

### Textbook

Calculus for AP 2nd Edition:  Larson and Battaglia

### Online Resources

- AP Central
- CalcChat
- Desmos Activities
- Pear Assessment
- IXL
- Quizizz
- EdPuzzle
- Canva
- Khan Academy
- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub

### Videos

- CalcView - Video Solutions of selected problems in the textbook
- Khan Academy

### Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
    - Chromebooks

| |
|---|
| ○ Texas Instrument TI-84 Plus Graphing Calculator |
| **ML Resources** |
| ● Multi-Language Glossary |
| **Gifted & Talented Resources** |
| ● Leveled Assessments<br>● Enrichment worksheets |

## PART III: TRANSFER OF KNOWLEDGE AND SKILLS

**DESCRIBE THE LEARNING EXPERIENCE.**

**How will students uncover content and build skills?**

| Section number and Title: | | |
|---|---|---|
| **Specific Learning Objective** | **Warm-Up/Starting Options** | **Practice & Apply** |
| ● Define, Calculate, and Analyze Marginal Profits, Cost, and Revenues for Real-World Problem | Derive the given function | Larson Text Section: 2.5 Pg: 180-182 |

## PART IV: EVIDENCE OF LEARNING

### IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

<table>
<tr><th colspan="3">Assessments</th></tr>
<tr><th>Summative</th><th>Formative</th><th>Performance</th></tr>
<tr>
<td>The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.<ul><li>Diagnostic Pre-Test</li><li>Chapter Tests</li><li>Projects</li><li>End-Of –Course Assessment</li></ul></td>
<td>The effectiveness of the instructional program will be based on numerous activities and strategies including the following: teacher observations, students collaborating with peers, questioning strategies, student record-keeping, quizzes, exit/admit assignments, peer/self-assessments, learning/response logs, discussions and practice presentations.</td>
<td>The following assessments require students to utilize various strands of mathematics.<ul><li>Projects</li><li>Practice AP Exam Questions</li><li>Homework</li><li>Classwork</li></ul></td>
</tr>
<tr>
<td colspan="3">List of Accommodations and Modifications<ul><li>Special Education</li><li>504 Students</li><li>At Risk Students</li><li>ELL</li><li>Gifted and Talented</li></ul></td>
</tr>
</table>

| State Mandates and Resources |
| --- |
| • New Jersey Student Learning Standards<br>• Career Readiness, Life Literacies, and Key Skills<br>• LGBT and Disabilities Law<br>• Asian and Pacific Islander |

AP CALCULUS AB AND BC

# Mathematical Practices

## Practice 1

### *Implementing Mathematical Processes* 1

Determine expressions and values using mathematical procedures and rules.

## Practice 2

### *Connecting Representations* 2

Translate mathematical information from a single representation or across multiple representations.

## Practice 3

### *Justification* 3

Justify reasoning and solutions.

## Practice 4

### *Communication and Notation* 4

Use correct notation, language, and mathematical conventions to communicate results or solutions.

**SKILLS**

**1.A** Identify the question to be answered or problem to be solved *(not assessed)*.

**1.B** Identify key and relevant information to answer a question or solve a problem *(not assessed)*.

**1.C** Identify an appropriate mathematical rule or procedure based on the classification of a given expression (*e.g., Use the chain rule to find the derivative of a composite function*).

**1.D** Identify an appropriate mathematical rule or procedure based on the relationship between concepts (*e.g., rate of change and accumulation*) or processes (*e.g., differentiation and its inverse process, anti-differentiation*) to solve problems.

**1.E** Apply appropriate mathematical rules or procedures, with and without technology.

**1.F** Explain how an approximated value relates to the actual value.

**2.A** Identify common underlying structures in problems involving different contextual situations.

**2.B** Identify mathematical information from graphical, numerical, analytical, and/or verbal representations.

**2.C** Identify a re-expression of mathematical information presented in a given representation.

**2.D** Identify how mathematical characteristics or properties of functions are related in different representations.

**2.E** Describe the relationships among different representations of functions and their derivatives.

**3.A** Apply technology to develop claims and conjectures *(not assessed)*.

**3.B** Identify an appropriate mathematical definition, theorem, or test to apply.

**3.C** Confirm whether hypotheses or conditions of a selected definition, theorem, or test have been satisfied.

**3.D** Apply an appropriate mathematical definition, theorem, or test.

**3.E** Provide reasons or rationales for solutions and conclusions.

**3.F** Explain the meaning of mathematical solutions in context.

**3.G** Confirm that solutions are accurate and appropriate.

**4.A** Use precise mathematical language.

**4.B** Use appropriate units of measure.

**4.C** Use appropriate mathematical symbols and notation (*e.g., Represent a derivative using* $f'(x)$, $y'$, *and* $\frac{dy}{dx}$).

**4.D** Use appropriate graphing techniques.

**4.E** Apply appropriate rounding procedures.