

Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

Course Name: Calculus

Course Number: 034100

Updated: June 2024

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit Title: Preparation for Calculus

All of the skills and concepts in this chapter are prerequisite for a successful experience in Calculus 1. You can assign this as summer work but should certainly touch upon the topics for at least a week or two before jumping into the Limits Unit as these topics are essential to any good Calculus student's understanding of the subject of Calculus.

(Ch. P.1, P.2, P.3, P.4, P.5)

Essential Questions

1. How do you identify intercepts and points of intersection, and test for symmetry?
2. How do you find the slope of a line and use the slope to write an equation of the line?
3. How do you determine the domain and range of a function, and classify functions?
4. How do you find the inverse of a function?
5. How are the functions
6. $fx=ex$ and $gx=\ln x$ related, and what properties can you use to simplify exponential and logarithmic expressions?

Learning Targets/Objectives

Students will be able to:

- Test a graph for symmetry with respect to an axis and the origin.
- Find the points of intersection of two graphs.
- Fit a mathematical model to a real-life data set.
- Find the slope of a line passing through two points.
- Write the equation of a line with a given point and slope
- Interpret slope as a ratio or as a rate in a real-life application.
- Sketch the graph of a linear equation in slope-intercept form.
- Write equations of lines that are parallel or perpendicular to a given line.
- Find the domain and range of a function.
- Identify different types of transformations of functions.
- Classify functions and recognize combinations of functions.

	<ul style="list-style-type: none"> • Verify that one function is the inverse function of another function. • Determine whether a function has an inverse function. • Develop properties of the six inverse trigonometric functions. • Develop and use properties of exponential functions. • Understand the definition of the number e. • Understand the definition of the natural logarithm function and develop and use properties of the natural logarithm function.
Tier 2 Vocabulary <i>High-frequency words used throughout the unit</i>	Tier 3 Vocabulary <i>Discipline-specific words used throughout the unit</i>
<p>Solution point, intercepts, x-intercept, y-intercept, point of intersection, slope, point-slope formula, ratio, rate of change, average rate of change, slope-intercept form, general form, parallel, perpendicular, function, independent variable, dependent variable, domain, range, vertical line test, transformations, leading coefficient, constant term, composition, inverse function, natural logarithmic function, natural base</p>	<p>Symmetric with respect to the y-axis, symmetric with respect to the x-axis, symmetric with respect to the origin, mathematical model, implicitly, explicitly, implied domain, transcendental, even function, odd function, horizontal line test, one-to-one</p>

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. N-RN.A.2	1. Rewrite expressions involving radicals and rational exponents using the properties of exponents.
2. A-APR.B.3	2. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
3. A-CED.A.2	3. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
4. A-SSE.A.1a	4. Interpret parts of an expression, such as terms, factors, and coefficients.
5. F-BF.B.4b	5. Verify by composition that one function is the inverse of another.
6. F-IF.A.1	6. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x . The graph of f is the graph of the equation $y = f(x)$.
7. F-IF.A.2	7. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. Climate Change Example: Students may use function notation to determine the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), m , where $c(m)$ is the number of molecules of carbon dioxide.
8. F-IF.B.4	8. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
9. F-IF.C.7a	9. Graph linear and quadratic functions and show intercepts, maxima, and minima.
10. F-IF.C.7b	10. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.

11. F-IF.C.7c	11. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
12. F-IF.C.7e	12. Graph exponential and logarithmic functions, showing intercepts and end behavior.
13. F-IF.C.7f	13. Graph trigonometric functions, showing period, midline, and amplitude.
14. F-IF.C.8a	14. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
15. F-LE.A.4	15. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to $ab^{(ct)} = d$ where a , c , and d are numbers and the base b is 2, 10, or e ; evaluate the logarithm using technology.
16. F-LE.B.5	16. Interpret the parameters in a linear or exponential function in terms of a context.
17. F-TF.A.3	17. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$ and $\frac{\pi}{6}$ and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$ and $2\pi - x$ in terms of their values for x , where x is any real number.
18. G-SRT.C.6	18. Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
19. G-SRT.C.7	19. Explain and use the relationship between the sine and cosine of complementary angles.
20. G-SRT.C.8	20. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
21. G-GPE.B.5	21. Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
NJSLS	Interdisciplinary Connections
1. RI.CR.11–12.1	1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
2. W.IW.11–12.2	2. Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.

3. SL.PE.11–12.1	3. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others’ ideas and expressing their own clearly and persuasively.
4. HS-PS2-2	4. Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.
5. HS-ETS1-3	5. Evaluate a solution to a complex real-world problem based on prioritized criteria and trade-offs that account for a range of constraints, including cost, safety, reliability, and aesthetics, as well as possible social, cultural, and environmental impacts.

2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills

1. 9.1.12.FI.3	1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g.,college, retirement).
2. 9.1.12.CFR.6	2. Identify and explain the consequences of breaking federal and/or state employment or financial laws.
3. 9.4.12.CT.2	3. Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
4. 9.4.12.CI.1	4. Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a).

2020 New Jersey Student Learning Standards for Computer Science and Design Thinking

1. 8.1.12.AP.2	1. Create generalized computational solutions using collections instead of repeatedly using simple variables.
2. 8.1.12.DA.1	2. Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
3. 8.2.12.ETW.3	3. Identify a complex, global environmental or climate change issue, develop a systematic plan of investigation, and propose an innovative sustainable solution.
4. 8.2.12.ED.6	4. Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Definitions
- Use Prior Results
- Make Conjectures
- Build Arguments

- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation.

Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem.

Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes

- Calculate Accurately
- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook

Calculus for AP 2nd Edition: Larson and Battaglia

Online Resources

- [CalcChat](#)
- [Desmos Activities](#)

- [Pear Assessment](#)
- [IXL](#)
- [Quizizz](#)
- [EdPuzzle](#)
- [Canva](#)
- [Khan Academy](#)
- [Inside Mathematics](#)
- [NJDOE Digital Item Library](#)
- [New Jersey Center for Teaching and Learning](#)
- [New Jersey Climate Education Hub](#)

Videos

- [CalcView](#) - Video Solutions of selected problems in the textbook
- [Khan Academy](#)

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
 - Chromebooks
 - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section P.1: Graphs and Models		
Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none">• Sketch the graph of an equation.• Find the intercepts of a graph.• Test a graph for symmetry with respect to an axis and the origin.• Find the points of intersection of two graphs.• Fit a mathematical model to a real-life data set.	Suggestions include but not limited to have students work with the difference quotient on opening problems to refresh. Refer to Lesson Motivator in textbook	Section P.1 p. 10-13 #2-26 even, 27-37 odd, 57, 59, 64

Section P.2: Linear Models and Rates of Change		
Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none">• Find the slope of a line passing through two points.• Write the equation of a line with a given point and slope.• Interpret slope as a ratio or as a rate in real-life applications.• Sketch the graph of a linear equation in slope-intercept form.	Suggestions include but not limited to: Algebraic justification of the order of coordinates used in the linear slope formula. Also refer to Lesson Motivator in textbook	Section P.2 p. 20-22 #2-62 odd, 65, 67, 73,

<ul style="list-style-type: none"> Write equations of lines that are parallel or perpendicular to a given line. 		
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Section P.3: Functions and their Graphs

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Use function notation to represent and evaluate a function. Find the domain and range of a function. Sketch the graph of a function. Identify different types of transformations of functions. Classify functions and recognize combinations of functions. 	Suggestions include but not limited to: Algebra review, pages A34 Also refer to Lesson Motivator in textbook	Section P.3 p. 31-34, #2-34 odd, 47-52 all, 55, 57, 59, 62, 65, 73-76 all, 81, 83, 84

Section P.4: Inverse Functions

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Verify that one function is the inverse of another function. Determine whether a function has an inverse. Develop properties of the six inverse trigonometric functions. 	Suggestions include but not limited to: Complete the exploration on P. 35 "Finding inverse functions". Also refer to Lesson Motivator in textbook	Section P.4 p. 42-45, #1-12 all, 14, 15, 18, 19, 22, 23, 25, 28, 30, 31-45 odd, 48, 67, 71, 73, 75, 78, 79, 81, 83, 86

Section P.5: Exponential And Logarithmic Functions

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> • Develop and use properties of exponential functions. • Understand the definition of the number e. • Understand the definition of the natural logarithm function, and develop and use properties of the natural logarithmic function. 	<p>Suggestions include but not limited to: Recall the rules of exponentials. Also refer to Lesson Motivator in textbook</p>	<p>Section P.5 p. 51-53 # 2-32 even, 39, 40, 51, 57, 60, 69, 71, 86-102 even, 103, 119, 120</p>

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	Formative	Performance
<p>The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey</p>	<p>The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to:</p>	<p>The following assessments require students to utilize various strands of mathematics.</p> <ul style="list-style-type: none"> • Projects

<p>Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.</p> <ul style="list-style-type: none"> ● Diagnostic Pre- Test ● Chapter Tests ● Cumulative Semester Assessments 	<ul style="list-style-type: none"> ● Teacher observations ● Self-Assessments ● Student record-keeping ● Quizzes ● Warm-ups ● Exit Tickets ● Participation in class discussions ● Independent practice 	<ul style="list-style-type: none"> ● Performance Tasks ● Homework ● Classwork
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<p>List of Accommodations and Modifications</p> <ul style="list-style-type: none"> ● Special Education ● 504 Students ● At Risk Students ● MLL ● Gifted and Talented
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<p>State Mandates and Resources</p>
<ul style="list-style-type: none"> ● New Jersey Student Learning Standards ● Standards for Mathematical Practices

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Course Name: Calculus
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PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit Title: Limits and Their Properties

*Students will begin to learn about limits, the most important aspect of the Calculus course. They will begin to understand the basic operations and algebraic rules of limits, discover the algebra of limits and their properties, and dive into what it means for a function to be continuous.
(Ch. 1.1, 1.2, 1.3, 1.4, 1.5, 1.6)*

Essential Questions

1. What is Calculus?
2. What is a limit and how can you determine the limit of a function as x approaches c ?
3. What algebraic techniques can you use to evaluate a limit?
4. What is continuity and how does it apply to the Intermediate Value Theorem?
5. What is an infinite limit?
6. What is a limit at infinity?

Learning Targets/Objectives

- Students will be able to:
- Understand what Calculus is and how it compares to Pre-calculus.
 - Understand that the tangent line problem is basic to Calculus.
 - Understand that the area problem is also basic to Calculus.
 - Estimate a limit using a numerical or graphical approach.
 - Learn different ways that a limit can fail to exist.
 - Study and use a formal definition of a limit.
 - Evaluate a limit using properties of limits.
 - Develop and use a strategy for finding limits.
 - Evaluate a limit using the dividing out technique.
 - Evaluate a limit using the rationalizing technique.
 - Evaluate a limit using the Squeeze Theorem.
 - Determine continuity at a point and continuity on an open interval.
 - Determine one-sided limits and continuity on a closed interval.

	<ul style="list-style-type: none"> • Use properties of continuity. • Understand and use the Intermediate Value Theorem. • Determine infinite limits from the left and from the right. • Find and sketch the vertical asymptotes of the graph of a function. • Determine (finite) limits at infinity. • Determine the horizontal asymptotes, if any, of the graph of a function. • Determine infinite limits at infinity.
Tier 2 Vocabulary <i>High-frequency words used throughout the unit</i>	Tier 3 Vocabulary <i>Discipline-specific words used throughout the unit</i>
Piecewise function, direct substitution, polynomial functions, rational functions, radical function, composite function, domain, one-sided limit, vertical asymptote, horizontal asymptote,	Secant line, tangent line, limit, right handed behavior, left handed behavior, unbounded behavior, oscillating behavior, transcendental functions, dividing out technique, indeterminate form, rationalizing technique, the Squeeze Theorem, continuity, discontinuity, removable discontinuity, nonremovable discontinuity, Intermediate Value Theorem, infinite limit, limits at infinity, logistic function,

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES
DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. A-SSE.A.2	1. Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.
2. F-IF.A.2	2. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. Climate Change Example: Students may use function notation to determine the amount of carbon dioxide

<p>3. F-IF.B.4</p> <p>4. F-IF.B.5</p> <p>5. F-IF.B.6</p> <p>6. F-IF.C.7b</p> <p>7. F-IF.C.7d</p> <p>8. F-IF.C.7f</p> <p>9. F-IF.C.8a</p> <p>10. F-TF.A.3</p>	<p>produced by burning a given number of molecules of ethane (gasoline), m, where $c(m)$ is the number of molecules of carbon dioxide.</p> <p>3. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</p> <p>4. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Climate Change Example: Students may relate the domain of a function $c(m)$ representing the amount of carbon dioxide produced by burning m molecules of ethane (gasoline), to its graph in order to determine the appropriate domain for $c(m)$.</p> <p>5. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Climate Change Example: Students may calculate the average rate of change of a function $c(m)$ presented symbolically or as a table, where $c(m)$ represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).</p> <p>6. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p> <p>7. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</p> <p>8. Graph trigonometric functions, showing period, midline, and amplitude.</p> <p>9. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p> <p>10. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\frac{\pi}{3}$, $\frac{\pi}{4}$ and $\frac{\pi}{6}$ and use the unit circle to express the values of sine, cosines, and tangent for $\pi - x$, $\pi + x$ and $2\pi - x$ in terms of their values for x, where x is any real number.</p>
<p>NJSLS</p>	<p>Interdisciplinary Connections</p>
<p>1. RI.CR.11–12.1</p>	<p>1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well</p>

<p>2. W.IW.11–12.2</p> <p>3. SL.PE.11–12.1</p> <p>4. HS-PS2-1</p> <p>5. HS-PS2-2</p> <p>6. HS-PS3-5</p>	<p>as interpretations of the text.</p> <p>2. Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.</p> <p>3. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others’ ideas and expressing their own clearly and persuasively.</p> <p>4. Analyze data to support the claim that Newton’s second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.</p> <p>5. Use mathematical representations to support the claim that the total momentum of a system of objects is conserved when there is no net force on the system.</p> <p>6. Develop and use a model of two objects interacting through electric or magnetic fields to illustrate the forces between objects and the changes in energy of the objects due to the interaction.</p>
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2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills

<p>1. 9.1.12.FI.3</p> <p>2. 9.1.12.CFR.6</p> <p>3. 9.3.12.AG-PST.1</p> <p>4. 9.4.12.CT.2</p> <p>5. 9.4.12.CI.1</p>	<p>1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g.,college, retirement).</p> <p>2. Identify and explain the consequences of breaking federal and/or state employment or financial laws.</p> <p>3. Apply physical science principles and engineering applications to solve problems and improve performance in AFNR power, structural and technical systems.</p> <p>4. Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).</p> <p>5. Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a).</p>
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2020 New Jersey Student Learning Standards for Computer Science and Design Thinking

<p>1. 8.1.12.AP.2</p> <p>2. 8.1.12.DA.1</p>	<p>1. Create generalized computational solutions using collections instead of repeatedly using simple variables.</p> <p>2. Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.</p>
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3. 8.2.12.ETW.3	3. Identify a complex, global environmental or climate change issue, develop a systematic plan of investigation, and propose an innovative sustainable solution.
4. 8.2.12.ED.6	4. Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations

- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Definitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook

Online Resources

- [CalcChat](#)
- [Desmos Activities](#)
- [Pear Assessment](#)
- [IXL](#)
- [Quizizz](#)
- [EdPuzzle](#)
- [Canva](#)
- [Khan Academy](#)
- [Inside Mathematics](#)
- [NJDOE Digital Item Library](#)
- [New Jersey Center for Teaching and Learning](#)
- [New Jersey Climate Education Hub](#)

Videos

- [CalcView](#) - Video Solutions of selected problems in the textbook
- [Khan Academy](#)

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
 - Chromebooks
 - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 1.1: A Preview of Calculus		
Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> • Understand what Calculus is and how it compares to precalculus. • Understand that the tangent line problem is basic to Calculus • Understand that area problem is also basic to Calculus 	Suggestions include but not limited to: Find slope between two points and the equation of a line connecting them Refer to Lesson Motivator in textbook	Section 1.1 p.63-64 # 5-14, 17, 18

Section 1.2: Finding Limits Graphically and Numerically

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Estimate a limit using a numerical or graphical approach. Learn different ways that a limit can fail to exist. Study and use a formal definition of limits. 	<p>Suggestions include but not limited to: Create a list of numbers that get closer and closer to a number without reaching it. Also refer to Lesson Motivator in textbook</p>	<p>Section 1.2 p. 72-75 #1-6 all, 8-16 even, 17-25 odd, 26, 28, 31, 39, 42, 55, 57, 61, 62, 69, 70</p>

Section 1.3: Evaluating Limits Analytically

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Evaluate a limit using properties of limits. Develop and use a strategy for finding limits. Evaluate a limit using the dividing out technique. Evaluate a limit using the rationalization technique. Evaluate a limit using the Squeeze (Sandwich) Theorem. 	<p>Suggestions include but not limited to: Have students calculate the limit of a polynomial $f(x)$ numerically around $x=0$ and compare to $f(0)$. Also refer to Lesson Motivator in textbook</p>	<p>Section 1.3 P. 84-86 # 1-46 (any), 53-68 (any), 73, 76, 77, 79, 82, 97-100 (any), 101, 102,</p>

Section 1.4: Continuity and One-Sided Limits

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Determine continuity at a point and continuity on an open interval. Determine one-sided limits and continuity on 	<p>Suggestions include but not limited to: Exploration on pg. 87. Also refer to Lesson Motivator in textbook</p>	<p>Section 1.4 p. 96-99 # 1-15 all, 17, 18, 19, 21, 23, 25, 30, 32, 34, 35, 39-53 odd, 63- 66 all, 69, 71, 89,</p>

<ul style="list-style-type: none"> a closed interval. Use properties of continuity. Understand and use the Intermediate Value Theorem. 		91, 93, 97, 117
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Section 1.5: Infinite Limits		
Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Determine infinite limits from the left and right. Find and sketch the vertical asymptotes of the graph of a function. 	Suggestions include but not limited to: Small Socratic Seminar about the nature of infinity. Also refer to Lesson Motivator in textbook	Section 1.5 p. 105-107 # 1,3, 5-8 all, 10, 11, 13, 15, 16, 18, 21, 22, 24, 25, 28, 31, 40-48 even, 49-61 odd, 67-68, 70

Section 1.6: Limits at Infinity		
Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Determine (finite) limits at infinity. Determine the horizontal asymptotes, if any, of the graph of a function. Determine infinite limits at infinity. 	Suggestions include but not limited to: Review methods of discovering end behavior for polynomials and rational function from precalculus. Also refer to Lesson Motivator in textbook	Section 1.6 p. 115-117, # 8-16 even, 19, 22, 24, 26, 27, 28, 31, 37, 39, 40, 41, 47, 56

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	Formative	Performance
<p>The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.</p> <ul style="list-style-type: none">• Diagnostic Pre- Test• Chapter Tests• Cumulative Semester Assessments	<p>The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to:</p> <ul style="list-style-type: none">• Teacher observations• Self-Assessments• Student record-keeping• Quizzes• Warm-ups• Exit Tickets• Participation in class discussions• Independent practice	<p>The following assessments require students to utilize various strands of mathematics.</p> <ul style="list-style-type: none">• Projects• Performance Tasks• Homework• Classwork

List of Accommodations and Modifications <ul style="list-style-type: none">• Special Education• 504 Students• At Risk Students• MLL• Gifted and Talented		

State Mandates and Resources		
<ul style="list-style-type: none">• New Jersey Student Learning Standards• Standards for Mathematical Practices		

Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

Course Name: Calculus

Course Number: 034100

Updated: June 2024

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit Title: Differentiation

*In Chapter 2, students will really get into the heart of calculus. Derivatives are a major concern of the course, so you will spend a significant amount of time on this unit. As you progress through the chapter, you will want to spend some time on the relationship between position, velocity, and acceleration, as well as related rates. This will most likely be the first time students will be asked to think deeply on a conceptual level, so they may struggle at first. To implement many of the methods introduced in the chapter, students will be required to use some calculus at first, then they will complete the methods using mostly algebra.
(Ch. 2.1, 2.2, 2.3, 2.4, 2.5, 2.6, 2.7, 2.8 (optional))*

Essential Questions

Learning Targets/Objectives

1. What is a derivative and what is its relationship to continuity?
2. How do you find the derivatives of basic algebraic functions, trigonometric functions, and exponential functions?
3. How do you find the derivatives of functions involving products and quotients?
4. How do you find the derivatives of composite functions, natural logarithmic functions, and exponential functions with bases other than e ?
5. How do you find the derivative of implicitly defined functions?
6. How do you find the derivatives of inverse functions, including inverse trigonometric functions?

Students will be able to:

- Find the slope of the tangent line to a curve at a point.
- Use the limit definition to find the derivative of a function.
- Understand the relationship between differentiability and continuity.
- Find the derivative of a function using a table or a graph.
- Find the derivative of a function using the Constant Rule.
- Find the derivative of a function using the Power Rule.
- Find the derivative of a function using the Constant Multiple Rule.
- Find the derivative of a function using the Sum and Difference Rules.
- Find the derivative of the sine and cosine functions.

<p>7. What is a related rate and how do you find it?</p> <p>8. (optional) How can you use derivatives to approximate the zero of a function?</p>	<ul style="list-style-type: none"> • Find the derivative of exponential functions. • Use derivatives to find rates of change. • Find the derivative of a function using the Product Rule. • Find the derivative of a function using the Quotient Rule. • Find the derivative of any trigonometric functions. • Find a higher-order derivative of a function. • Find the derivative of a composite function using the Chain Rule. • Find the derivative of a function using the General Power Rule. • Simplify the derivative of a function using algebra. • Find the derivative of a transcendental function using the Chain Rule. • Find the derivative of a function involving the natural logarithm function. • Define and differentiate exponential functions that have bases other than e. • Distinguish between functions written in implicit form and explicit form. • Use implicit differentiation to find the derivative of a function. • Find derivatives of functions using logarithmic differentiation. • Find the derivative of an inverse function. • Differentiate an inverse trigonometric function. • Find a related rate. • Use related rates to solve real-life applications. • (optional) Approximate a zero of a function using Newton's Method.
<p style="text-align: center;">Tier 2 Vocabulary <i>High-frequency words used throughout the unit</i></p>	<p style="text-align: center;">Tier 3 Vocabulary <i>Discipline-specific words used throughout the unit</i></p>
<p>Secant line, slope, change in x, change in y, tangent line, point-slope formula, position function, velocity, speed, acceleration, vertical asymptote, trigonometric function, logarithmic function, exponential function, natural base, inverse function, area, volume, angle of elevation, angle of depression, surface area</p>	<p>Difference quotient, differentiation, derivative, vertical tangent line, horizontal tangent line, instantaneous rate of change, average rate of change, open interval, closed interval, differentiability, continuity, cusp, power rule, constant multiple rule, sum and difference rule, product rule, quotient rule, second derivative, third derivative, higher-order derivative, chain rule, transcendental function, implicit differentiation, explicit form, related rate, Newton's Method</p>

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES
DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. N-RN.A.2	1. Rewrite expressions involving radicals and rational exponents using the properties of exponents.
2. N-RN.A.3	2. Simplify radicals, including algebraic radicals
3. N-Q.A.1	3. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. <i>Climate Change Example: Students may use units to guide the solution of multi-step problems about how variations in the flow of energy into and out of the Earth's systems result in climate change. Note: Changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.</i>
4. N-Q.A.2	4. Define appropriate quantities for the purpose of descriptive modeling. <i>Climate Change Example: Students may define appropriate quantities for a descriptive model of how variations in the flow of energy into and out of Earth's systems result in climate change. Note: changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.</i>
5. A-APR.C.5	5. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n , where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
6. A-CED.A.2	6. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
7. A-CED.A.4	7. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations <i>Climate Change Example: Students may rearrange formulas related to the economic impact of climate change to highlight a quantity of interest, using the same reasoning as in solving equations.</i>
8. A-SSE.B.3a	8. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.

<p>9. F-IF.A.2</p> <p>10. F-IF.B.5</p> <p>11. F-IF.B.6</p> <p>12. F-TF.A.3</p> <p>13. F-TF.B.7</p> <p>14. G-GMD.A.3</p> <p>15. G-MG.A.3</p> <p>16. G-SRT.C.8</p>	<p>9. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. Climate Change Example: Students may use function notation to determine the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), m, where $c(m)$ is the number of molecules of carbon dioxide.</p> <p>10. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Climate Change Example: Students may relate the domain of a function $c(m)$ representing the amount of carbon dioxide produced by burning m molecules of ethane (gasoline), to its graph in order to determine the appropriate domain for $c(m)$.</p> <p>11. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Climate Change Example: Students may calculate the average rate of change of a function $c(m)$ presented symbolically or as a table, where $c(m)$ represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).</p> <p>12. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number</p> <p>13. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.</p> <p>14. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems</p> <p>15. Apply geometric methods to solve design problems Climate Change Example: Students may apply geometric methods to solve design problems such as increasing access to green spaces in cities given physical and cost constraints.</p> <p>16. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.</p>
<p>NJSLS</p>	<p>Interdisciplinary Connections</p>
<p>1. RI.CR.11–12.1</p>	<p>1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well</p>

<p>2. W.IW.11–12.2</p> <p>3. SL.PE.11–12.1</p> <p>4. HS-PS2-1</p>	<p>as interpretations of the text.</p> <p>2. Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.</p> <p>3. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others’ ideas and expressing their own clearly and persuasively.</p> <p>4. Analyze data to support the claim that Newton’s second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.</p>
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2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills

<p>1. 9.1.12.FI.3</p> <p>2. 9.3.12.AG-PST.1</p> <p>3. 9.4.12.CT.2</p> <p>4. 9.4.12.CI.1</p>	<p>1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g.,college, retirement).</p> <p>2. Apply physical science principles and engineering applications to solve problems and improve performance in AFNR power, structural and technical systems.</p> <p>3. Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).</p> <p>4. Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a).</p>
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2020 New Jersey Student Learning Standards for Computer Science and Design Thinking

<p>1. 8.1.12.AP.2</p> <p>2. 8.1.12.DA.1</p> <p>3. 8.2.12.ETW.3</p> <p>4. 8.2.12.ED.6</p>	<p>1. Create generalized computational solutions using collections instead of repeatedly using simple variables.</p> <p>2. Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.</p> <p>3. Identify a complex, global environmental or climate change issue, develop a systematic plan of investigation, and propose an innovative sustainable solution.</p> <p>4. Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).</p>
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The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions

- Use Definitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation.

Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem.

Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely

- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook

Calculus for AP 2nd Edition: Larson and Battaglia

Online Resources

- [CalcChat](#)
- [Desmos Activities](#)
- [Pear Assessment](#)
- [IXL](#)
- [Quizizz](#)
- [EdPuzzle](#)

- [Canva](#)
- [Khan Academy](#)
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Videos

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Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
 - Chromebooks
 - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 2.1: The Derivative and the Tangent Line Problem

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none">• Find the slope of the tangent line to a curve at a point.• Use the limit definition to find the derivative of a function.• Understand the relationship between differentiability and continuity.• Find the derivative of a function given by a table or a graph.	<p>Suggestions include but not limited to: Have students work with the difference quotient on opening problems to refresh.</p> <p>Refer to Lesson Motivator in textbook</p>	<p>Section 2.1 p. 132-134 # 1,2,6-28 even, 29, 31, 37, 39, 43, 53, 61-65 odd, 69</p>

Section 2.2: Basic Differentiation Rules and Rates of Change

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none">• Find the derivative of a function using the following rules<ul style="list-style-type: none">○ Constant, Power, Constant Multiple, Sum, and Difference	<p>Suggestions include but not limited to: Complete Exploration-“Writing a Conjecture”, pg. 135 Also refer to Lesson Motivator in textbook</p>	<p>Section 2.2 p. 144-147 #1, 2, 4-26 even, 27, 29, 30, 32, 34, 37, 40, 41-53 odd, 56, 57-63 odd, 67, 71, 73, 79, 80, 99, 101, 110</p>

<ul style="list-style-type: none"> Find the derivatives of the sine and cosine functions and exponential functions Use derivatives to find rates of change.. 		
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Section 2.3: Product and Quotient Rules and Higher-Order Derivatives

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Find the derivative of a function using the Product and Quotient Rules. Find the derivative of a trigonometric function. Find a higher-order derivative of a function. 	Suggestions include but not limited to: Algebra review, pages A38 and A39 Also refer to Lesson Motivator in textbook	Section 2.3 p. 155-158, #2-20 even, 21, 23, 26, 28, 32, 35, 38, 39, 41, 51, 53-65 odd, 77-83 odd, 89, 95, 97, 100-108 even, 109, 111, 125(b),

Section 2.4: The Chain Rule

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Find the derivatives of composite functions and transcendental functions using the General Power Rule and Chain Rule. Simplify the derivative of a function using algebra. Find the derivative of a function involving the natural logarithmic function. Define and differentiate exponential functions that have bases other than e. 	Suggestions include but not limited to: Review composite function algebra Also refer to Lesson Motivator in textbook	Section 2.4 p. 169-173, #1-5 odd, 8-30 even, 41, 44, 45, 48, 49, 50, 53, 57, 61, 65, 68, 69, 71, 75, 82, 83, 88, 91, 99, 109, 111, 117, 119, 127, 129, 141, 142, 143, 149, 165, 167, 171, 172

Section 2.5: Implicit Differentiation

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises

<ul style="list-style-type: none"> • Distinguish between functions written in implicit form and explicit form. • Use implicit differentiation to find the derivative of a function. • Find derivatives of functions using logarithmic differentiation. 	<p>Suggestions include but not limited to: Discussion on how to write the equation of a conic section as a combination of functions. Also refer to Lesson Motivator in textbook</p>	<p>Section 2.5 p. 180-182 # 2-20 even, 21, 27-39 odd, 49, 51, 55, 56, 61, 63, 78</p>
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Section 2.7: Related Rates		
Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> • Find a related rate. • Use related rates to solve real life problems. 	<p>Suggestions include but not limited to: Have students complete the exploration on p. 190. Also refer to Lesson Motivator in textbook</p>	<p>Section 2.7 p. 195-198 #2-8 even, 9, 10, 13, 14, 16, 20, 21, 23, 24, 27, 28, 31, 35, 38, 43</p>

Section 2.8: Newton's Method (Optional; if time permits)		
Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> • Approximate a zero of a function using Newton's Method 	<p>Suggestions include but not limited to: Find x-intercept of a tangent line to a function. Also refer to Lesson Motivator in textbook</p>	<p>Section 2.8 p. 202-203 # 2-10 even, 17-23 odd, 35, 36, 40</p>

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	Formative	Performance
<p>The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.</p> <ul style="list-style-type: none"> ● Diagnostic Pre- Test ● Chapter Tests ● Cumulative Semester Assessments 	<p>The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to:</p> <ul style="list-style-type: none"> ● Teacher observations ● Self-Assessments ● Student record-keeping ● Quizzes ● Warm-ups ● Exit Tickets ● Participation in class discussions ● Independent practice 	<p>The following assessments require students to utilize various strands of mathematics.</p> <ul style="list-style-type: none"> ● Projects ● Performance Tasks ● Homework ● Classwork
<p>List of Accommodations and Modifications</p> <ul style="list-style-type: none"> ● Special Education ● 504 Students ● At Risk Students ● MLL ● Gifted and Talented 		

State Mandates and Resources

- [New Jersey Student Learning Standards](#)
- [Standards for Mathematical Practices](#)

Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

Course Name: Calculus

Course Number: 034100

Updated: June 2024

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit Title: Applications of Differentiation

Students will locate extrema on an interval, use Rolle's Theorem and the Mean Value Theorem. They will use the First and Second Derivative Test to find intervals of increase, decrease and concavity, inflection points, critical points and extreme values. Students will explore limits at infinity and analyze various types of functions. They will solve optimization problems and find differentials. Students will use L'Hôpital's Rule to solve problems. (Ch. 3.1, 3.2, 3.3, 3.4, 3.5, 3.6, 3.7, 7.7)

Essential Questions

1. What are extrema and how can you find them on open and closed intervals?
2. What is the Mean Value Theorem and how is it used?
3. How can you determine the intervals on which a function is increasing or decreasing and the location of the function's relative extrema?
4. How do you determine the concavity of a function and find its inflection points?
5. How do you analyze a function and sketch its graph?
6. How do you maximize or minimize quantities?
7. How are differentials used to explain the tangent line approximation?
8. How do you evaluate a limit when direct substitution produces

Learning Targets/Objectives

- Students will be able to:
- Understand the definition of extrema of a function on an interval.
 - Understand the definition of relative extrema of a function on an open interval.
 - Find extrema on a closed interval.
 - Understand and use Rolle's Theorem.
 - Understand and use the Mean Value Theorem.
 - Determine intervals on which a function is increasing and decreasing.
 - Apply the First Derivative Test to find relative extrema of a function.
 - Determine intervals on which a function is concave upward or concave downward.
 - Find any points of inflection of the graph of a function.

<p>an indeterminate form?</p>	<ul style="list-style-type: none"> • Apply the Second Derivative Test to find relative extrema of a function. • Analyze and sketch the graph of a function. • Solve applied maximum and minimum problems. • Understand the concept of a tangent line approximation. • Compare the value of the differential, dy, with the actual change in y, Δy. • Find the differential of a function using differentiation formula • Recognize limits that produce indeterminate forms. • Apply L'Hopital's Rule to evaluate a limit.
<p>Tier 2 Vocabulary <i>High-frequency words used throughout the unit</i></p>	<p>Tier 3 Vocabulary <i>Discipline-specific words used throughout the unit</i></p>
<p>Minimum, maximum, increasing, decreasing, constant, open interval, closed interval, derivative, average rate of change, instantaneous rate of change, horizontal tangent, secant line, x-intercept, y-intercept, symmetry, domain, range, continuity, vertical asymptote, horizontal asymptote, differentiability, cusp, first derivative, second derivative</p>	<p>Absolute minimum, absolute maximum, global minimum, global maximum, relative minimum, relative maximum, extreme value, critical number, Rolle's Theorem, Mean Value Theorem, First Derivative Test, concavity, concave up, concave down, Second Derivative Test, inflection point, optimization problems, primary equation, secondary equation, tangent line approximation (linear approximation), differential form, L'Hopital's Rule, indeterminate form</p>

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. N-Q.A.1	<p>1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. <i>Climate Change Example: Students may use units to guide the solution of multi-step problems about how variations in the flow of energy into and out of the Earth's systems result in climate change. Note: Changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.</i></p>
2. N-Q.A.2	<p>2. Define appropriate quantities for the purpose of descriptive modeling. <i>Climate Change Example: Students may define appropriate quantities for a descriptive model of how variations in the flow of energy into and out of Earth's systems result in climate change. Note: changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.</i></p>
3. A-APR.B.3	<p>3. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.</p>
4. A-CED.A.2	<p>4. Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.</p>
5. A-CED.A.4	<p>5. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations <i>Climate Change Example: Students may rearrange formulas related to the economic impact of climate change to highlight a quantity of interest, using the same reasoning as in solving equations.</i></p>
6. A-SSE.B.3a	<p>6. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.</p>
7. F-IF.A.2	<p>7. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. <i>Climate Change Example: Students may use function notation to determine the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), m, where $c(m)$ is the number of molecules of carbon dioxide.</i></p>

<p>8. F-IF.B.4</p> <p>9. F-IF.B.5</p> <p>10. F-IF.B.6</p> <p>11. F-IF.C.7a</p> <p>12. F-IF.C.7b</p> <p>13. F-IF.C.7c</p> <p>14. F-IF.C.7d</p> <p>15. F-IF.C.7e</p> <p>16. F-IF.C.8a</p> <p>17. G-GMD.A.3</p>	<p>8. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.</p> <p>9. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Climate Change Example: Students may relate the domain of a function $c(m)$ representing the amount of carbon dioxide produced by burning m molecules of ethane (gasoline), to its graph in order to determine the appropriate domain for $c(m)$.</p> <p>10. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Climate Change Example: Students may calculate the average rate of change of a function $c(m)$ presented symbolically or as a table, where $c(m)$ represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).</p> <p>11. Graph linear and quadratic functions and show intercepts, maxima, and minima.</p> <p>12. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions.</p> <p>13. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.</p> <p>14. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.</p> <p>15. Graph exponential and logarithmic functions, showing intercepts and end behavior.</p> <p>16. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.</p> <p>17. Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems</p>
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18. G-MG.A.3	18. Apply geometric methods to solve design problems Climate Change Example: Students may apply geometric methods to solve design problems such as increasing access to green spaces in cities given physical and cost constraints.
19. G-SRT.C.8	19. Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.
NJSLS	Interdisciplinary Connections
1. RI.CR.11–12.1	1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text.
2. W.IW.11–12.2	2. Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content.
3. SL.PE.11–12.1	3. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others’ ideas and expressing their own clearly and persuasively.
4. HS-PS2-1	4. Analyze data to support the claim that Newton’s second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.
2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills	
1. 9.1.12.FI.3	1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g., college, retirement).
2. 9.3.12.AG-PST.1	2. Apply physical science principles and engineering applications to solve problems and improve performance in AFNR power, structural and technical systems.
3. 9.4.12.CT.2	3. Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a).
4. 9.4.12.CI.1	4. Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a).
2020 New Jersey Student Learning Standards for Computer Science and Design Thinking	
1. 8.1.12.AP.2	1. Create generalized computational solutions using collections instead of repeatedly using simple variables.

2. 8.1.12.DA.1	2. Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change.
3. 8.2.12.ETW.3	3. Identify a complex, global environmental or climate change issue, develop a systematic plan of investigation, and propose an innovative sustainable solution.
4. 8.2.12.ED.6	4. Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Definitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph

- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook

Calculus for AP 2nd Edition: Larson and Battaglia

Online Resources

- [CalcChat](#)
- [Desmos Activities](#)
- [Pear Assessment](#)
- [IXL](#)
- [Quizizz](#)
- [EdPuzzle](#)
- [Canva](#)
- [Khan Academy](#)
- [Inside Mathematics](#)
- [NJDOE Digital Item Library](#)
- [New Jersey Center for Teaching and Learning](#)
- [New Jersey Climate Education Hub](#)

Videos

- [CalcView](#) - Video Solutions of selected problems in the textbook
- [Khan Academy](#)

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
 - Chromebooks
 - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 3.1: Extrema on an Interval		
Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> • Understand the definition of extrema of a function on an interval. • Understand the definition of relative extrema of a function on an open interval. • Find extrema on a closed interval 	Suggestions include but not limited to: Have students identify locations of a maximum using the feature on the calculator. Refer to Lesson Motivator in textbook	Section 3.1 p.217-219 # 2, 5, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 45, 48, 49, 62, 68, 75-78

Section 3.2: Rolle's Theorem and the Mean Value Theorem

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Understand and use Rolle's Theorem. Understand and use the Mean Value Theorem. 	<p>Suggestions include but not limited to: Complete Exploration-Extreme Values in a Closed Interval on p.220. Also refer to Lesson Motivator in textbook</p>	<p>Section 3.2 p. 224-226 #1, 2, 9-21 odd, 24-26, 29, 31, 35, 40-48 even, 52, 53, 64-66, 81-83</p>

Section 3.3: Product and Quotient Rules and Higher-Order Derivatives

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Determine intervals on which a function is increasing and decreasing. Apply the First Derivative Test to find relative extrema of a function 	<p>Suggestions include but not limited to: Have students identify intervals of increase and decrease by viewing a graph. Also refer to Lesson Motivator in textbook</p>	<p>Section 3.3 P 233-236 #10, 13, 16, 19, 22, 25, 28, 31, 34, 37, 40, 43, 46, 49, 52, 55, 57, 58, 59-65 odd, 75, 76, 80, 81-86, 90, 97, 99, 101, 115-117</p>

3.4: Concavity and Second Derivative Test

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Determine intervals on which a function is concave upward or concave downward. Find any points of inflection of the graph of a 	<p>Suggestions include but not limited to: Have students find the first two derivatives of a rational function.</p>	<p>Section 3.4 p. 242-244 # 3, 7, 12, 13, 18, 23, 26, 27, 31, 37, 41-51 odd, 59, 67, 69-71, 74, 80, 92-94</p>

function. <ul style="list-style-type: none"> Apply the Second Derivative Test to find relative extrema of a function. 	Also refer to Lesson Motivator in textbook	
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Section 3.5: A Summary of Curve Sketching

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Analyze and sketch the graph of a function 	Suggestions include but not limited to: Have students list all information needed to sketch a polynomial function precisely without a calculator. Also refer to Lesson Motivator in textbook	Section 3.5 P253-256 # 1, 12, 15, 22, 27, 31, 35, 36, 42, 43, 49, 53, 57, 64, 90, 99-102

Section 3.6: Optimization Problems

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Solve applied maximum and minimum problems 	Suggestions include but not limited to: Have students list possible dimensions for a rectangular yard given the area Also refer to Lesson Motivator in textbook	Section 3.6 P 262-266 # 2, 5, 9-23 odd, 29, 31, 37, 41, 48, 54-56

Section 3.7 Linear Approximation and Differentials

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises

<ul style="list-style-type: none"> Understand the concept of a tangent line approximation. Compare the value of the differential, dy, with the actual change in y, Δy. Find the differential of a function using differentiation formula 	<p>Suggestions include but not limited to: Have students complete the exploration on p267. Also refer to Lesson Motivator in textbook</p>	<p>Section 3.7 P 272-275 #3-5 9, 11, 14, 16-19, 25-28, 35, 40, 52-55</p>
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Section 7.7 Indeterminate Forms and L'Hopital's Rule

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Recognize limits that produce indeterminate forms. Apply L'Hopital's Rule to evaluate a limit 	<p>Suggestions include but not limited to: Find the limit of a rational function with an asymptote. Also refer to Lesson Motivator in textbook</p>	<p>Section 7.7 P513-516 # 1, 4, 5, 8-11, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 51, 54, 57, 60, 63, 65, 69-72, 80-82, 101, 102, 113-115</p>

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	Formative	Performance
<p>The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.</p> <ul style="list-style-type: none"> Diagnostic Pre-Test 	<p>The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to:</p> <ul style="list-style-type: none"> Teacher observations Self-Assessments Student record-keeping Quizzes Warm-ups 	<p>The following assessments require students to utilize various strands of mathematics.</p> <ul style="list-style-type: none"> Projects Performance Tasks Homework Classwork

<ul style="list-style-type: none">• Chapter Tests• Cumulative Semester Assessments	<ul style="list-style-type: none">• Exit Tickets• Participation in class discussions• Independent practice	
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<p>List of Accommodations and Modifications</p> <ul style="list-style-type: none">• Special Education• 504 Students• At Risk Students• MLL• Gifted and Talented
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State Mandates and Resources

<ul style="list-style-type: none">• New Jersey Student Learning Standards• Standards for Mathematical Practices
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Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

Course Name: Calculus
Course Number: 034100

Updated: June 2024

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit Title: Integration

*Students will find antiderivatives and indefinite integrals. They will estimate the area under the curve using Riemann sums to find the definite integral. Students will learn and apply the Fundamental Theorem of Calculus Parts 1 and 2. They will perform techniques of integration using u -substitution and various other rules.
(Ch. 4.1, 4.2, 4.3, 4.4, 4.6, 4.7, 6.1)*

Essential Questions

1. What are antiderivatives and how they are used?
2. How can you approximate the area of a plane figure?
3. What is the process of using Riemann sums?
4. What is the Fundamental Theorem of Calculus?
5. How do you integrate composite functions?
6. How do you integrate rational functions and trigonometric functions other than sine or cosine?
7. How do you find the area of a region between two curves?

Learning Targets/Objectives

- Students will be able to:
- Write the general solution of a differential equation and use indefinite integral notation for antiderivatives.
 - Use basic integration rules to find antiderivatives.
 - Find a particular solution of a differential equations.
 - Use sigma notation to write and evaluate a sum.
 - Understand the concept of area.
 - Approximate the area of a plan region.
 - Find the area of a plan region using limits.
 - Understand the definition of a Riemann sum.
 - Evaluate a definite integral using limits and geometric formulas and

	<p>using properties of definite integrals.</p> <ul style="list-style-type: none"> • Evaluate a definite integral using the Fundamental Theorem of Calculus. • Understand and use the Mean Value Theorem for Integrals. • Find the average value of a function over a closed interval. • Understand the use the Second Fundamental Theorem of Calculus. • Use pattern recognition to find an indefinite integral. • Use a change of variables to find an indefinite integral. • Use the General Power Rule for Integration to find an indefinite integral. • Use a change of variables to evaluate a definite integral. • Evaluate a definite integral involving an even or odd function. • Use the Log Rule for Integration to integrate a rational function. • Integrate trigonometric functions. • Find the area of a region between two curves using integration. • Find the area of a region between intersecting curves using integration. • Describe Integration as an accumulation process.
<p style="text-align: center;">Tier 2 Vocabulary <i>High-frequency words used throughout the unit</i></p>	<p style="text-align: center;">Tier 3 Vocabulary <i>Discipline-specific words used throughout the unit</i></p>
<p>Differential equation, initial condition, position, velocity, acceleration, area of a rectangle, area of a trapezoid, continuity, u-substitution, even function, odd function</p>	<p>Antiderivative, integration, indefinite integral, definite integral, particular solution, sigma notation, upper bound, lower bound, area under the curve, inscribed rectangle, circumscribe rectangle, lower sum, upper sum, Riemann sum, general partition, Trapezoidal Rule, Fundamental Theorem of Calculus, Mean Value Theorem for Integrals, average value, Second Fundamental Theorem of Calculus, change of variables, Power Rule for Integration, Log Rule for Integration, area between two curves</p>

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. N-RN.A.2	1. Rewrite expressions involving radicals and rational exponents using the properties of exponents.
2. N-RN.A.3	2. Simplify radicals, including algebraic radicals
3. N-Q.A.2	3. Define appropriate quantities for the purpose of descriptive modeling. Climate Change Example: Students may define appropriate quantities for a descriptive model of how variations in the flow of energy into and out of Earth's systems result in climate change. Note: changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.
4. A-CED.A.1	4. Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Climate Change Example: Students may create equations and/or inequalities to represent the economic impact of climate change
5. A-CED.A.4	5. Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations Climate Change Example: Students may rearrange formulas related to the economic impact of climate change to highlight a quantity of interest, using the same reasoning as in solving equations.
6. A-SSE.B.3a	6. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. Factor a quadratic expression to reveal the zeros of the function it defines.
7. A-SSE.B.4	7. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments
8. F-IF.A.2	8. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context. Climate Change Example: Students may use function notation to determine the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline), m , where $c(m)$ is the number of molecules of carbon dioxide.

9. F-IF.A.3	9. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
10. F-IF.B.4	10. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
11. F-IF.B.5	11. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. Climate Change Example: Students may relate the domain of a function $c(m)$ representing the amount of carbon dioxide produced by burning m molecules of ethane (gasoline), to its graph in order to determine the appropriate domain for $c(m)$.
12. F-IF.B.6	12. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Climate Change Example: Students may calculate the average rate of change of a function $c(m)$ presented symbolically or as a table, where $c(m)$ represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).
13. F-IF.C.7c	13. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
14. F-IF.C.8a	14. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
15. F-TF.A.3	15. Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x , where x is any real number
16. G-MG.A.2	16. Apply concepts of density based on area and volume in modeling situations Climate Change Example: Students may apply the concept of population density of different urban areas, including calculations of population density, and discuss different environmental factors (e.g., air and water quality, waste disposal, energy consumption) that might be exacerbated by increased population density.
17. G-MG.A.3	17. Apply geometric methods to solve design problems Climate Change Example: Students may apply geometric methods to solve design problems such as increasing access to green spaces in cities given physical and cost constraints.

NJSLS	Interdisciplinary Connections
<ol style="list-style-type: none"> 1. RI.CR.11–12.1 2. W.IW.11–12.2 3. SL.PE.11–12.1 4. HS-PS2-1 	<ol style="list-style-type: none"> 1. Accurately cite a range of thorough textual evidence and make relevant connections to strongly support a comprehensive analysis of multiple aspects of what an informational text says explicitly and inferentially, as well as interpretations of the text. 2. Write informative/explanatory texts (including the narration of historical events, scientific procedures/ experiments, or technical processes) to examine and convey complex ideas, concepts, and information clearly and accurately through the effective selection, organization, and analysis of content. 3. Initiate and participate effectively in a range of collaborative discussions (one-on-one, in groups, and teacher-led) with peers on grades 11–12 topics, texts, and issues, building on others’ ideas and expressing their own clearly and persuasively. 4. Analyze data to support the claim that Newton’s second law of motion describes the mathematical relationship among the net force on a macroscopic object, its mass, and its acceleration.
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<ol style="list-style-type: none"> 1. 9.1.12.FI.3 2. 9.3.12.AG-PST.1 3. 9.4.12.CT.2 4. 9.4.12.CI.1 	<ol style="list-style-type: none"> 1. Develop a plan that uses the services of various financial institutions to prepare for long term personal and family goals (e.g., college, retirement). 2. Apply physical science principles and engineering applications to solve problems and improve performance in AFNR power, structural and technical systems. 3. Explain the potential benefits of collaborating to enhance critical thinking and problem solving (e.g., 1.3E.12profCR3.a). 4. Demonstrate the ability to reflect, analyze, and use creative skills and ideas (e.g.1.1.12prof.CR3a).
2020 New Jersey Student Learning Standards for Computer Science and Design Thinking	
<ol style="list-style-type: none"> 1. 8.1.12.AP.2 2. 8.1.12.DA.1 3. 8.2.12.ETW.3 	<ol style="list-style-type: none"> 1. Create generalized computational solutions using collections instead of repeatedly using simple variables. 2. Create interactive data visualizations using software tools to help others better understand real world phenomena, including climate change. 3. Identify a complex, global environmental or climate change issue, develop a systematic plan of investigation, and propose an innovative sustainable solution.

4. 8.2.12.ED.6

4. Analyze the effects of changing resources when designing a specific product or system (e.g., materials, energy, tools, capital, labor).

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- Consider Simpler Forms
- Problem Solve

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- Understand Quantities
- Use Operations
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- Relationships
- Reason Abstractly

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- Use Definitions
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- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook

Calculus for AP 2nd Edition: Larson and Battaglia

Online Resources

- [CalcChat](#)

- [Desmos Activities](#)
- [Pear Assessment](#)
- [IXL](#)
- [Quizizz](#)
- [EdPuzzle](#)
- [Canva](#)
- [Khan Academy](#)
- [Inside Mathematics](#)
- [NJDOE Digital Item Library](#)
- [New Jersey Center for Teaching and Learning](#)
- [New Jersey Climate Education Hub](#)

Videos

- [CalcView](#) - Video Solutions of selected problems in the textbook
- [Khan Academy](#)

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- WebAssign
- Devices:
 - Chromebooks
 - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 4.1: Antiderivatives		
Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none">• Write the general solution of a differential equation and use indefinite integral notation for antiderivatives.• Use basic integration rules to find antiderivatives.• Find a particular solution of a differential equation.	Suggestions include but not limited to: Approximate Antiderivatives from Derivatives. Refer to Lesson Motivator in textbook	Section 4.1 p. 287-289 #2, 3, 9-11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 39, 42, 44, 48, 52, 53, 57, 65, 67, 74-77

Section 4.2: Area		
Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none">• Use sigma notation to write and evaluate a sum.	Suggestions include but not limited to: Discuss Archimedes method of	Section 4.2 p.299-301 # 1, 4, 7-35 odd, 41, 45, 48, 53, 58,

<ul style="list-style-type: none"> • Understand the concept of area. • Approximate the area of a plan region. • Find the area of a plan region using limits. 	<p>exhaustion for finding area. Also refer to Lesson Motivator in textbook</p>	<p>61, 68, 69, 75, 76, 78, 81-83</p>
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Section 4.3 Riemann Sums and Definite Integrals		
Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> • Understand the definition of a Riemann sum. • Evaluate a definite integral using limits and geometric formulas and using properties of definite integrals. 	<p>Suggestions include but not limited to: Model Riemann sum by drawing smaller rectangles and discuss trapezoid rule to approximate area under curve. Also refer to Lesson Motivator in textbook</p>	<p>Section 4.3 p. 312-316 #1-5 odd, 9-14, 18, 20, 25, 30, 31-35 odd, 40-50 even, 51, 53, 54, 57, 89, 92, 94, 114-117</p>

Section 4.4 The Fundamental Theorem of Calculus		
Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> • Evaluate a definite integral using the Fundamental Theorem of Calculus. • Understand and use the Mean Value Theorem for Integrals. • Find the average value of a function over a closed interval. • Understand and use the Second Fundamental Theorem of Calculus. 	<p>Suggestions include but not limited to: Discuss notation difference between the definite and Indefinite Integral. Also refer to Lesson Motivator in textbook</p>	<p>Section 4.4 p.326-328 #5-25 odd, 29, 32, 35-39 odd, 44, 45, 48, 50, 59, 65, 67, 70-72, 75, 76, 79, 80, 83, 84, 89-91</p>

Section 4.6 Integration

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> • Use pattern recognition to find an indefinite integral. • Use a change of variables to find an indefinite integral. • Use the General Power Rule for Integration to find an indefinite integral. • Use a change of variables to evaluate a definite integral. • Evaluate a definite integral involving an even or odd function. 	<p>Suggestions include but not limited to: Show how the need for a new technique arises with the introduction of composite functions. Also refer to Lesson Motivator in textbook</p>	<p>Section 4.6 p.343-346 # 1-4, 5, 8, 11, 14, 17, 20, 23, 26, 29, 32, 35, 38, 41, 44, 47, 50, 53, 59, 63, 67-79 odd, 82, 86, 91-95 odd, 117-120</p>

Section 4.7 The Natural Logarithmic Functions: Integration

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> • Use the Log Rule for Integration to integrate a rational function. • Integrate trigonometric functions 	<p>Suggestions include but not limited to: Give examples of Integrating Rational Functions. Also refer to Lesson Motivator in textbook</p>	<p>Section 4.7 p.353-355 #1, 5, 9, 13, 17, 21, 25, 29, 33, 37, 41, 43, 44, 47-49, 53, 55, 58, 68, 70, 71-77 odd, 81, 92, 94, 104-107.</p>

Section 6.1 Area of a Region Between Two Curves

Specific Learning Objective	Warm-Up/Starting Options	Practice & Apply Exercises
<ul style="list-style-type: none"> Find the area of a region between two curves using integration. Find the area of a region between intersecting curves using integration. Describe Integration as an accumulation process. 	<p>Suggestions include but not limited to: Explain subtraction of the area of regions and how it will pertain to $f(x)-g(x)$ where $f(x)>g(x)$ Also refer to Lesson Motivator in textbook</p>	<p>Section 6.1 P416-419 #3, 6, 7, 10, 11, 13, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48, 53-57 odd, 66, 78, 83-85.</p>

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments

Summative	Formative	Performance
<p>The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.</p> <ul style="list-style-type: none"> Diagnostic Pre- Test Chapter Tests Cumulative Semester Assessments 	<p>The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to:</p> <ul style="list-style-type: none"> Teacher observations Self-Assessments Student record-keeping Quizzes Warm-ups Exit Tickets Participation in class discussions Independent practice 	<p>The following assessments require students to utilize various strands of mathematics.</p> <ul style="list-style-type: none"> Projects Performance Tasks Homework Classwork

List of Accommodations and Modifications

- [Special Education](#)
- [504 Students](#)
- [At Risk Students](#)

- [MLL](#)
- [Gifted and Talented](#)

State Mandates and Resources

- [New Jersey Student Learning Standards](#)
- [Standards for Mathematical Practices](#)