Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

Course Name: AP Pre-Calculus
Course Number: 034250

Updated: June 2024

PART I: UNIT RATIONALE

<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 1A: Polynomial Functions

In Unit 1A, students develop an understanding of two key function concepts while exploring polynomial functions. The first concept is covariation, or how output values change in tandem with changing input values. The second concept is rates of change, including average rate of change, rate of change at a point, and changing rates of change. The central idea of a function as a rule for relating two simultaneously changing sets of values provides students with a vital tool that has many applications, in nature, human society, and business and industry. For example, the idea of crop yield increasing but at a decreasing rate or the efficacy of medicine decreasing but at an increasing rate are important understandings that inform critical decisions.

Essential Questions	Learning Targets/Objectives
1. How do we model the intensity of light from its source? 2. How can I use data and graphs to figure out the best time to purchase event tickets? 3. How can we adjust known projectile motion models to account for changes in conditions?	Students will be able to: ● Understand how output values change in tandem with changing input values in polynomial functions. ● Construct a graph representing two quantities that vary with respect to each other in a contextual scenario. ● Compare the rates of change at two points using average rates of change near the points. ● Describe how two quantities vary together at different points and over different intervals of a function. ● Determine the average rates of change for linear and quadratic

	sequences and functions. ● Determine the change of average rates of change for linear and quadratic functions. ● Identify key characteristics of polynomial functions related to rates of change. ● Identify key characteristics of a polynomial function related to its zeros when suitable factorizations are available or with technology. ● Determine if a polynomial is even or odd. ● Describe end behaviors of polynomial functions.
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
Function, input, output, domain, range, independent variable, dependent variable, change in tandem, zeros, x-intercepts, y-intercept, concavity, inflection point, intervals of increasing, intervals of decreasing, average rate of change, rate of change at a point, slope, secant line, linear function, quadratic function, polynomial, relative extrema, global (absolute) extrema, multiplicity, complex zeros, odd and even polynomials, end behavior, and degree of a polynomial	Average Rate of Change, Rate of Change at a point, polynomial functions

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. A.APR.B.3	1. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
2. F.IF.A.1	2. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then f(x) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation y = f(x).
3. F.IF.A.2	3. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
4. F.IF.B.5	4. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. **Climate Change Example:** Students will relate the domain of a function c(m) representing the amount of carbon dioxide produced by burning m molecules of ethane (gasoline), to its graph in order to determine the appropriate domain for c(m).
5. F.IF.B.6	5. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **Climate Change Example**: Students may calculate the average rate of change of a function c(m) presented symbolically or as a table, where c(m) represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).
6. F.IF.C.7.c	6. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
7. F.IF.B.4	7. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.

NJSLS	Interdisciplinary Connections
1. L.KL.9-10.2.A	1. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level.
2. SL.PE.9-10.1.D	2. Respond thoughtfully to various perspectives, summarize points of agreement and disagreement, and justify own views. Make new connections in light of the evidence and reasoning presented.
3. SL.PI.9-10.4	3. Present information, findings, and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
4. HS-PS1-5	4. Apply scientific principles and evidence to provide an explanation about the effects of changing the temperature or concentration of the reacting particles on the rate at which a reaction occurs.
2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills	
1. 9.4.12.CI.1	1. Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
2. 9.4.5.DC.4	2. Model safe, legal, and ethical behavior when using online or offline technology
3. 9.4.12.TL.3	3. Analyze the effectiveness of the process and quality of collaborative environments.
4. 9.4.12.CT.2	4. Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
2020 New Jersey Student Learning Standards for Computer Science and Design Thinking	
1. 8.1.2.CS.1	1. Select and operate computing devices that perform a variety of tasks accurately and quickly based on user needs and preferences.
2. 8.1.2.DA.1	2. Collect and present data, including climate change data, in various visual formats.
3. 8.1.2.DA.3	3. Identify and describe patterns in data visualizations.
4. 8.1.2.DA.4	4. Make predictions based on data using charts or graphs.
5. 8.1.2.AP.4	5. Break down a task into a sequence of steps.
6. 8.1.5.DA.5	6. Propose cause and effect relationships, predict outcomes, or communicate ideas using data.

The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Definitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures

- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately

- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook:

Pre-Calculus with Limits, A Graphing Approach 7th edition – Ron Larson, 2016

Online Resources

- AP Central
- Web Assign
- Desmos Activities
- Pear Assessment
- IXL
- Quizizz
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- Canva
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- Inside Mathematics
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- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub
- Math Space
- Math Medic
- Albert IO AP Pre-Calc

Videos

- 1.1 Change in Tandem
- 1.2 Rates of Change
- 1.3 Rates of Change in Linear and Quadratic Functions
- 1.4 Polynomial Functions and Rates of Change
- 1.5A Polynomial Functions and Complex Zeros
- 1.5B Even and Odd Polynomials
- 1.6 Polynomial Functions and End Behavior

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- *Web Assign online program*
- Devices:
    - Chromebooks
    - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 1.1: Change in Tandem		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
● Understand how output values change in tandem with changing input values in polynomial functions. ● Construct a graph representing two quantities that vary with respect to each other in a contextual scenario.	**Cumulative Practice**: determine the domain, range, interval of increasing, decreasing, concavity and constant, approximate zeros, inflection points and the y-intercept **Prerequisite Skills Practice:** A function is a mathematical relation that maps a set of input values to a set of output values such that each input value is mapped to exactly one output value. The set of input values is called the domain of the function, and the set of output values is called the range of the function. The variable representing input values is called the independent variable, and the variable representing output values is called the dependent variable.	1.1 Change in Tandem

Section 1.2: Rates of Change		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Compare the rates of change at two points using average rates of change near the points. • Describe how two quantities vary together at different points and over different intervals of a function.	**Cumulative Practice**: Given two points on a parabola find the average rate of change between the two points.. **Prerequisite Skills Practice:** The average rate of change of a function over an interval of the function's domain is the constant rate of change that yields the same change in the output values as the function yielded on that interval of the function's domain. It is the ratio of the change in the output values to the change in input values over that interval	1.2 Rates of Change

Section 1.3: Rates of Change in Linear and Quadratic Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Determine the average rates of change for linear and quadratic sequences and functions. • Determine the change of average rates of change for linear and quadratic functions.	**Cumulative Practice**: Find the average rate of change on a parabola on a closed interval using a secant line. **Prerequisite Skills Practice:** For a linear function, the average rate of change over any length input-value interval is constant. For a quadratic function, the average rates of change over consecutive equal-length input-value intervals can be given by a linear function.	1.3 Rates of Change in Linear and Quadratic Functions

Section 1.4: Polynomial Functions and Rates of Change		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Identify key characteristics of polynomial functions related to rates of change.	**Cumulative Practice**: Determine the leading term, degree, and leading coefficient of a polynomial. **Prerequisite Skills Practice:** A nonconstant polynomial function of x is any function representation that is equivalent to the analytical form $p(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} + \ldots + a_2 x^2 + a_1 x^1 + a_0$, where n is a positive integer, a_i is a real number for each i from 1 to n, and a_n is nonzero. The polynomial has degree n, the leading term is $a_n x^n$, and the leading coefficient is a_n. A constant is also a polynomial function of degree zero.	1.4 Polynomial Functions and Rates of Change

Section 1.5: Polynomial Functions and Complex Zeros		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Identify key characteristics of a polynomial function related to its zeros when suitable factorizations are available or with technology. • Determine if a polynomial is even or odd.	**Cumulative Practice**:Find all athe x-intercepts of the function. **Prerequisite Skills Practice:** If a is a complex number and p(a) = 0, then a is called a zero of the polynomial function p, or a root of p(x) = 0. If a is a real number, then (x a −) is a linear factor of p if and only if a is a zero of p.	1.5A Polynomial Functions and Complex Zeros 1.5B Even and Odd Polynomials

Section 1.6: Polynomial Functions and End Behavior		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Describe end behaviors of polynomial functions.	**Cumulative Practice**: Determine if a polynomial goes “up” or “down” as x approaches negative infinity, then determine if the same polynomial goes “up” or “down” as x approaches infinity. **Prerequisite Skills Practice:** As input values of a nonconstant polynomial function increase without bound, the output values will either increase or decrease without bound. The corresponding mathematical notation is $\lim_{x \to -\infty} p(x) = \infty$ or $\lim_{x \to -\infty} p(x) = -\infty$.	1.6 Polynomial Functions and End Behavior

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

<table>
<tr><th colspan="3">Assessments</th></tr>
<tr><th>Summative</th><th>Formative</th><th>Performance</th></tr>
<tr>
<td>The following assessments will be used to evaluate student learning, skill acquisition, and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.

● Diagnostic Pre-Test
● Chapter Tests
● Standardized Tests</td>
<td>The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to:

● Teacher observations
● Self-Assessments
● Student record-keeping
● Quizzes
● Warm-ups
● Exit Tickets
● Participation in class discussions
● Independent practice</td>
<td>The following assessments require students to utilize various strands of mathematics.

● Projects
● Performance Tasks
● Homework
● Classwork</td>
</tr>
<tr>
<td colspan="3">List of Accommodations and Modifications
● Special Education
● 504 Students
● At Risk Students
● MLL
● Gifted and Talented</td>
</tr>
</table>

<table>
<tr><th>State Mandates and Resources</th></tr>
<tr><td>● New Jersey Student Learning Standards
● Standards for Mathematical Practices</td></tr>
</table>

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Course Name: AP Pre-Calculus
Course Number: 034250

Updated: June 2024

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 1B: Rational Functions

In Unit 1B, students develop an understanding of two key function concepts while exploring rational functions. The first concept is covariation, or how output values change in tandem with changing input values. The second concept is rates of change, including average rate of change, rate of change at a point, and changing rates of change. The central idea of a function as a rule for relating two simultaneously changing sets of values provides students with a vital tool that has many applications, in nature, human society, and business and industry. For example, the idea of crop yield increasing but at a decreasing rate or the efficacy of medicine decreasing but at an increasing rate are important understandings that inform critical decisions

Essential Questions	Learning Targets/Objectives
1. What defines a rational function? 2. How do we identify the domain of a rational function? 3. What are the characteristics of vertical and horizontal asymptotes? 4. What role do zeros and poles play in rational functions? 5. How can we simplify and analyze rational functions?	Students will be able to: • Describe end behavior of rational functions. • Determine the zeros of rational functions. • Determine vertical asymptotes of graphs of rational functions. • Determine holes of graphs of rational functions. • Rewrite polynomial and rational expressions in equivalent forms

6. What are the implications of vertical and horizontal shifts, stretches, and compressions on rational functions? 7. How do we find intercepts of rational functions? 8. What is the significance of end behavior in rational functions? 9. What are the real-world applications of rational functions? 10. How do rational functions relate to polynomial and exponential functions? Investigate connections and differences between these types of functions.	• Rewrite the repeated product of binomials using the binomial theorem. • Determine the quotient of two polynomial functions using long division.
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
Rational function, end behavior, domain, range, horizontal asymptote, vertical asymptote, slant asymptote, holes, interval notation, limits, average rate of change, zeros, multiplicity, factored form, standard form, Binomial Theorem, long division	Rational functions, asymptotes, holes, limits, rate of change

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. A.REI.A.2	1. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
2. A.APR.C.5	2. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
3. A.APR.D.6	3. Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x)*, where *a(x), b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system.
4. F.BF.A.1	4. Write a function that describes a relationship between two quantities.
5. F.IF.B.4	5. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
6. F.IF.C.7.d	6. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
7. F.IF.C.8	7. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
NJSLS	**Interdisciplinary Connections**
1. L.KL.9-10.2.A	1. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level.
2. SL.PE.9-10.1.D	2. Respond thoughtfully to various perspectives, summarize points of agreement and disagreement, and justify own views. Make new connections in light of the evidence and reasoning presented.

3. SL.PI.9-10.4	3. Present information, findings, and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills	
1. 9.2.12.CAP.5	1. Assess and modify a personal plan to support current interests and postsecondary plans.
2. 9.4.12.CI.1	2. Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
3. 9.4.5.DC.4	3. Model safe, legal, and ethical behavior when using online or offline technology
4. 9.4.12.TL.3	4. Analyze the effectiveness of the process and quality of collaborative environments.
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2020 New Jersey Student Learning Standards for Computer Science and Design Thinking	
1. 8.1.2.CS.1	1. Select and operate computing devices that perform a variety of tasks accurately and quickly based on user needs and preferences.
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Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
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Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

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- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Definitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples

- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
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- Analyze Relationships
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- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
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Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

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- Math Medic

Videos

- 1.7A Rational Functions and End Behavior
- 1.7B Rational Functions and End Behavior
- 1.8 Rational Functions and Zeros
- 1.9 Rational Functions and Vertical Asymptotes
- 1.10 Rational Functions and Holes
- 1.11A Equivalent Expressions and Binomial Theorem
- 1.11B Polynomial Long Division and Slant Asymptotes

Integrated Technology

• Google Suite: Google Classroom, Docs, Drive, Mail, etc...
• *Web Assign online program*
• Devices:
  ○ Chromebooks
  ○ Texas Instrument TI-84 Plus Graphing Calculator

ML Resources
• Multi-Language Glossary
Gifted & Talented Resources
• Leveled Assessments • Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 1.7: Rational Functions and End Behavior		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Describe the end behavior of rational functions.	**Cumulative Practice**: Determine the domain and range of a rational function. **Prerequisite Skills Practice:** A rational function is analytically represented as a quotient of two polynomial functions and gives a measure of the relative size of the polynomial function in the	1.7A Rational Functions and End Behavior 1.7B Rational Functions and End Behavior

	numerator compared to the polynomial function in the denominator for each value in the rational function's domain.	

Section 1.8: Rational Functions and Zeros		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Determine the zeros of rational functions.	**Cumulative Practice**: Determine the zeros and domain from the graph of a rational function. **Prerequisite Skills Practice:** The real zeros of a rational function correspond to the real zeros of the numerator for such values in its domain	1.8 Rational Functions and Zeros

Section 1.9: Rational Functions and Vertical Asymptotes		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Determine vertical asymptotes of graphs of rational functions.	**Cumulative Practice**: Find the x-values where the rational function is undefined. Determine the left and right behavior of the graph at the vertical asymptotes. **Prerequisite Skills Practice:** Near a vertical asymptote, x = a, of a rational function, the values of the polynomial function in the denominator are	1.9 Rational Functions and Vertical Asymptotes

	arbitrarily close to zero, so the values of the rational function r increase or decrease without bound.	

Section 1.10: Rational Functions and Holes		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Determine holes of graphs of rational functions.	**Cumulative Practice**: Determine the values for x that are undefined. Identify if those values represent asymptotes or holes. **Prerequisite Skills Practice:** If the multiplicity of a real zero in the numerator is greater than or equal to its multiplicity in the denominator, then the graph of the rational function has a hole at the corresponding input value	1.10 Rational Functions and Holes

Section 1.11: Equivalent Representations of Polynomial and Rational Expressions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Rewrite polynomial and rational expressions in equivalent forms • Rewrite the repeated product of binomials using the binomial theorem. • Determine the quotient of two polynomial functions using long division.	**Cumulative Practice**: Write the polynomial in factored form. **Prerequisite Skills Practice:** Because the factored form of a polynomial or rational function readily provides information about real zeros, it can reveal information about x-intercepts, asymptotes, holes, domain, and range	1.11A Equivalent Expressions and Binomial Theorem 1.11B Polynomial Long Division and Slant Asymptotes

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition, and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. ● Diagnostic Pre-Test ● Chapter Tests ● Standardized Tests	The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to: ● Teacher observations ● Self-Assessments ● Student record-keeping ● Quizzes ● Warm-ups ● Exit Tickets ● Participation in class discussions ● Independent Practice	The following assessments require students to utilize various strands of mathematics. ● Projects ● Performance Tasks ● Homework ● Classwork
List of Accommodations and Modifications ● Special Education ● 504 Students ● At Risk Students ● MLL ● Gifted and Talented		

State Mandates and Resources
● New Jersey Student Learning Standards ● Standards for Mathematical Practices

Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

Course Name: AP Pre-Calculus
Course Number: 034250

Updated: June 2024

PART I: UNIT RATIONALE

<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 1C: Transformations and Modeling

In Unit 1C, students construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology. Students describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools. Students identify information from graphical, numerical, analytical, and verbal representations to answer a question or construct a model, with and without technology. They support conclusions or choices with a logical rationale or appropriate data and construct new functions, using transformations, compositions, inverses, or regressions, that may be useful in modeling contexts, criteria, or data, with and without technology.

Essential Questions	Learning Targets/Objectives
1. How do additive transformations impact the key features of a function? 2. How do multiplicative transformations influence the characteristics of a function? 3. How do dilations impact the domain and range of a function? Investigate how scaling affects the spread of the function's values. 4. How do you restrict the domain of a function for real-world	Students will be able to: • Construct a function that is an additive and/or multiplicative transformation of another function. • Construct a linear, quadratic, cubic, quartic, polynomial of degree n, or related piecewise-defined function model. • Construct a rational function model based on a context. • Apply a function model to answer questions about a data set or

applications?	contextual scenario. • Identify an appropriate function type to construct a function model for a given scenario. • Describe assumptions and restrictions related to building a function model.
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
Translations, additive translations, multiplicative transformations, dilations, domain, range, linear, quadratic, cubic, restricting the domain, restricting the range, piecewise functions, regression, quartic, exponential, logarithmic, logistic, sine, inversely proportional	Translations, dilations, linear, quadratic, cubic, quartic, exponential, logarithmic, logistic, sinusoidal

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PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. F.IF.B.4	1. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
2. F.BF.A.1	2. Write a function that describes a relationship between two quantities.
3. F.BF.B.3	3. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an

	explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
4. F.LE.A.2	4. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
5. F.LE.B.5	5. Interpret the parameters in a linear or exponential function in terms of a context.
NJSLS	**Interdisciplinary Connections**
1. L.KL.9-10.2.A	1. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level.
2. SL.PE.9-10.1.D	2. Respond thoughtfully to various perspectives, summarize points of agreement and disagreement, and justify own views. Make new connections in light of the evidence and reasoning presented.
3. SL.PI.9-10.4	3. Present information, findings, and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills	
1. 9.4.12.CI.1	1. Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
2. 9.4.5.DC.4	2. Model safe, legal, and ethical behavior when using online or offline technology
3. 9.4.12.TL.3	3. Analyze the effectiveness of the process and quality of collaborative environments.
4. 9.4.12.CT.2	4. Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
2020 New Jersey Student Learning Standards for Computer Science and Design Thinking	
1. 8.1.2.CS.1	1. Select and operate computing devices that perform a variety of tasks accurately and quickly based on user needs and preferences.
2. 8.1.2.DA.1	2. Collect and present data, including climate change data, in various visual formats.
3. 8.1.2.DA.3	3. Identify and describe patterns in data visualizations.
4. 8.1.2.DA.4	4. Make predictions based on data using charts or graphs.

5. 8.1.2.AP.4 6. 8.1.5.DA.5	5. Break down a task into a sequence of steps. 6. Propose cause and effect relationships, predict outcomes, or communicate ideas using data.

The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships

- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Definitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources

- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

Look for and make use of structure: Look closely to see the structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook:

Pre-Calculus with Limits, A Graphing Approach 7th edition – Ron Larson, 2016

Online Resources

- AP Central
- Web Assign
- Desmos Activities
- Pear Assessment
- IXL
- Quizizz
- EdPuzzle
- Canva
- Khan Academy
- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub
- Math Space
- Math Medic

Videos

- 1.12A Translations of Functions
- 1.12B Dilations of Functions
- 1.13 Function Model Selection and Articulation Assumption
- 1.14 Function Model Construction and Application

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- *Web Assign online program*

• Devices:
  ○ Chromebooks
  ○ Texas Instrument TI-84 Plus Graphing Calculator

ML Resources
• Multi-Language Glossary
Gifted & Talented Resources
• Leveled Assessments • Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 1.12: Transformations of Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct a function that is an additive and/or multiplicative transformation of another function.	**Cumulative Practice**: Given $f(x) = x^3$, $graph\ f(x) + 2$ and $f(x - 2)$	1.12A Translations of Functions

	Prerequisite Skills Practice: The function g (x) = f (x) + k is an additive transformation of the function f that results in a vertical translation of the graph of f by k units. The function g (x) = f (x + h) is an additive transformation of the function f that results in a horizontal translation of the graph of f by −h units. The function g (x) = a f (x), where a ≠ 0, is a multiplicative transformation of the function f that results in a vertical dilation of the graph of f by a factor of, the transformation involves a reflection over the x-axis. The function g (x) = f (bx), where b ≠ 0, is a multiplicative transformation of the function f that results in a horizontal dilation of the graph of f by a factor of, the transformation involves a reflection over the y-axis	1.12B Dilations of Functions

Section 1.13: Function Model Selection & Assumption Articulation		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Identify an appropriate function type to construct a function model for a given scenario. • Describe assumptions and restrictions related to building a function model.	**Cumulative Practice**: From a table determine the appropriate function to model the situation. **Prerequisite Skills Practice:** Linear functions model data sets or aspects of contextual scenarios that demonstrate	1.13 Function Model Selection and Articulation Assumption

	roughly constant rates of change. Quadratic functions model data sets or aspects of contextual scenarios that demonstrate roughly linear rates of change, or data sets that are roughly symmetric with a unique maximum or minimum value. Geometric contexts involving area or two dimensions can often be modeled by quadratic functions. Geometric contexts involving volume or three dimensions can often be modeled by cubic functions.	

Section 1.14: Function Model Construction and Application		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct a linear, quadratic, cubic, quartic, polynomial of degree n, or related piecewise-defined function model. • Construct a rational function model based on a context. • Apply a function model to answer questions about a data set or contextual scenario.	**Cumulative Practice**: Use regression to write an equation for the best-fit curve. **Prerequisite Skills Practice:** A model can be constructed based on restrictions identified in a mathematical or contextual scenario. A model of a data set or a contextual scenario can be constructed using transformations of the parent function. A model of a data set can be constructed using technology and regressions, including linear, quadratic, cubic, and quartic regressions. A piecewise-defined function model can be constructed through a combination of modeling techniques. Construct a rational function model based on a context. Data sets and aspects of contextual scenarios involving quantities that are inverses.	1.14 Function Model Construction and Application

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

<table>
<tr><th colspan="3">Assessments</th></tr>
<tr><th>Summative</th><th>Formative</th><th>Performance</th></tr>
<tr>
<td>The following assessments will be used to evaluate student learning, skill acquisition, and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period.

● Diagnostic Pre-Test
● Chapter Tests
● Standardized Tests</td>
<td>The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to:

● Teacher observations
● Self-Assessments
● Student record-keeping
● Quizzes
● Warm-ups
● Exit Tickets
● Participation in class discussions
● Independent Practice</td>
<td>The following assessments require students to utilize various strands of mathematics.

● Projects
● Performance Tasks
● Homework
● Classwork</td>
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● Special Education
● 504 Students
● At Risk Students
● MLL
● Gifted and Talented</td></tr>
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Course Name: AP Pre-Calculus
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PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 2A: Exponential Functions

In Unit 2A, students build an understanding of exponential functions. Exponential function models are widespread in the natural and social sciences. When an aspect of a phenomenon changes proportionally to the existing amount, exponential models are employed to harness the information. Exponential functions are key to modeling population growth, radioactive decay, interest rates, and the amount of medication in a patient. The study of this function type touches careers in business, medicine, chemistry, physics, education, and human geography, among others.

Essential Questions	Learning Targets/Objectives
1. How can I make a single model that merges the interest I earn from my bank with the taxes that are due so I can know how much I will have in the end? 2. How can we adjust the scale of distance for a model of planets in the solar system so the relationships among the planets are easier to see? 3. If different functions can be used to model data, how do we pick which one is best? 4. How can exponential functions be used to model real-world problems? 5. How can exponential functions be used to model growth and decay? 6. How do you write, graph, and interpret an exponential growth or decay function?	Students will be able to: • Express arithmetic sequences found in mathematical and contextual scenarios as functions of whole numbers. • Express geometric sequences found in mathematical and contextual scenarios as functions of whole numbers. • Construct functions of the real numbers that are comparable to arithmetic and geometric sequences. • Describe similarities and differences between linear and exponential functions. • Identify key characteristics of exponential functions. • Rewrite exponential expressions in equivalent forms. • Construct a model for situations involving proportional output values over equal-length input-value intervals. • Apply exponential models to answer questions about a data set or

7. How do you recognize, describe, and compare linear and exponential functions? 8. How does the graph of ( ) = change when a and b are changed?	contextual scenario.
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
base, variable, exponent, growth, decay, constant, change, domain, range, increasing, decreasing, data, sequences, notation, principal, interest, y-intercept, asymptote, shift, reflection, transformation, inverse	exponential growth, exponential decay, arithmetic sequences, geometric sequences, regression

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. A.SSE.B.3	1. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
2. A.SSE.B.3.c	2. Use the properties of exponents to transform expressions for exponential functions.
3. A.SSE.B.4	3. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.
4. F.BF.A.1	4. Write a function that describes a relationship between two quantities.
5. F.BF.A.2	5. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
6. F.BF.B.3	6. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
7. F.IF.A.3	7. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
8. F.BF.A	8. Build a function that models a relationship between two quantities. (I think standard needs to be removed)

9. F.IF.B.4	9. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
10. F.IF.C.8.b	10. Use the properties of exponents to interpret expressions for exponential functions.
11. F.IF.C.9	11. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
12. F.LE.A.1	12. Distinguish between situations that can be modeled with linear functions and with exponential functions.
13. F.LE.A.1.a	13. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by equal factors over equal intervals.
14. F.LE.A.1.b	14. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
15. F.LE.A.1.c	15. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
16. F.LE.A.2	16. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
17. F.LE.B.5	17. Interpret the parameters in a linear or exponential function in terms of a context.
NJSLS	**Interdisciplinary Connections**
1. L.KL.9-10.2.A	1. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level.
2. SL.PE.9-10.1.D	2. Respond thoughtfully to various perspectives, summarize points of agreement and disagreement, and justify own views. Make new connections in light of the evidence and reasoning presented.
3. SL.PI.9-10.4	3. Present information, findings, and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
4. HS-LS2-1	4. Use mathematical and/or computational representations to support explanations of factors that affect carrying capacity of ecosystems at different scales.
2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills	
1. 9.1.12.PB.6	1. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level.
2. 9.1.12.PB.1	2. Respond thoughtfully to various perspectives, summarize points of agreement and disagreement, and justify own

	views. Make new connections in light of the evidence and reasoning presented.
3. 9.4.12.CI.1	3. Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
4. 9.4.5.DC.4	4. Model safe, legal, and ethical behavior when using online or offline technology.
5. 9.4.12.TL.3	5. Analyze the effectiveness of the process and quality of collaborative environments.
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1. 8.1.2.CS.1	1. Select and operate computing devices that perform a variety of tasks accurately and quickly based on user needs and preferences.
2. 8.1.2.DA.1	2. Collect and present data, including climate change data, in various visual formats.
3. 8.1.2.DA.3	3. Identify and describe patterns in data visualizations.
4. 8.1.2.DA.4	4. Make predictions based on data using charts or graphs.
5. 8.1.2.AP.4	5. Break down a task into a sequence of steps.
6. 8.1.5.DA.5	6. Propose cause and effect relationships, predict outcomes, or communicate ideas using data.

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Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
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- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

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- Use Equations
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- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Defi nitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
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- Use a Graph
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- Analyze Relationships
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- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
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Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
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Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
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Resources

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Videos

- 2.1 Change in Arithmetic and Geometric Sequences
- 2.2 Change in Linear and Exponential Functions
- 2.3 Exponential Functions
- 2.4 Exponential Function Manipulation
- 2.5A Exponential Function Context and Data Modeling
- 2.5B Exponential Function Context and Data Modeling

Integrated Technology

• Google Suite: Google Classroom, Docs, Drive, Mail, etc...
• *Web Assign online program*
• Devices:
  ○ Chromebooks
  ○ Texas Instrument TI-84 Plus Graphing Calculator

ML Resources
• Multi-Language Glossary
Gifted & Talented Resources
• Leveled Assessments • Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 2.1: Change in Arithmetic and Geometric Sequences		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Express arithmetic sequences found in mathematical and contextual scenarios as functions of the whole numbers. • Express geometric sequences found in mathematical and contextual scenarios as functions of the whole numbers.	**Cumulative Practice**: From a table, determine the function to model the situation. **Prerequisite Skills Practice**: Simplifying and solving functions to find all the zeros.	2.1 Change in Arithmetic and Geometric Sequences

Section 2.2: Change in Linear and Exponential Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct functions of the real numbers that are comparable to arithmetic and geometric sequences. • Describe similarities and differences between linear and exponential functions.	**Cumulative Practice**: Find the domain of the function. **Prerequisite Skills Practice**: Sketching graphs of a variety of functions.	2.2 Change in Linear and Exponential Functions

Section 2.3: Exponential Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Identify key characteristics of exponential functions.	**Cumulative Practice**: Find characteristics of functions using intercepts, asymptotes, domain, range, etc. **Prerequisite Skills Practice**: Using characteristics to graph functions.	2.3 Exponential Functions

Section 2.4: Exponential Function Manipulation		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Rewrite exponential expressions in equivalent forms.	**Cumulative Practice**: Use vocabulary to understand and simplify expressions. **Prerequisite Skills Practice**: Simplifying and analyzing algebraic expressions.	2.4 Exponential Function Manipulation

Section 2.5: Exponential Function Context and Data Modeling		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct a model for situations involving proportional output values over equal-length input-value intervals. • Apply exponential models to answer questions about a data set or contextual scenario.	**Cumulative Practice**: Transform the function from its parent function. **Prerequisite Skills Practice**: Understanding and using shifts and reflections to transform functions.	2.5A Exponential Function Context and Data Modeling 2.5B Exponential Function Context and Data Modeling

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. • Diagnostic Pre-Test • Chapter Tests • Standardized Tests	The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to: • Teacher observations • Self-Assessments • Student record-keeping • Quizzes • Warm-ups • Exit Tickets • Participation in class discussions • Independent practice	The following assessments require students to utilize various strands of mathematics. • Projects • Performance Tasks • Homework • Classwork

List of Accommodations and Modifications

- Special Education
- 504 Students
- At Risk Students
- MLL
- Gifted and Talented

State Mandates and Resources

- New Jersey Student Learning Standards
- Standards for Mathematical Practices

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PART I: UNIT RATIONALE

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Unit 2B: Function Validation

In Unit 2B, students build an understanding of functions. Function models are widespread in the natural and social sciences. When an aspect of a phenomenon changes proportionally to the existing amount, exponential and logarithmic models are employed to harness the information. These functions are key to modeling population growth, radioactive decay, interest rates, and the amount of medication in a patient and are useful in modeling sound intensity and frequency, the magnitude of earthquakes, the pH scale in chemistry, and the working memory in humans. The study of function types touches careers in business, medicine, chemistry, physics, education, and human geography, among others.

Essential Questions	Learning Targets/Objectives
1. How can I make a single model that merges the interest I earn from my bank with the taxes that are due so I can know how much I will have in the end? 2. How can we adjust the scale of distance for a model of planets in the solar system so the relationships among the planets are easier to see? 3. If different functions can be used to model data, how do we pick which one is best? 4. What is function composition? 5. How do you compose functions? 6. How do you evaluate composite functions? 7. How do you find the inverse of a function?	Students will be able to: • Construct linear, quadratic, and exponential models based on a data set. • Validate a model constructed from a data set. • Evaluate the composition of two or more functions for given values. • Construct a representation of the composition of two or more functions. • Rewrite a given function as a composition of two or more functions. • Determine the input-output pairs of the inverse of a function. • Determine the inverse of a function on an invertible domain.

Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
function, linear, quadratic, exponential, composition, inverse, domain, range, input, output, relationship, model, data set, interval, rate of change, constant, quantities	invertible, parameters, competing, composition, exponential, logarithmic

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

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New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. A.SSE.B.3	1. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
2. F.BF.A	2. Build a function that models a relationship between two quantities.
3. F.BF.A.1	3. Write a function that describes a relationship between two quantities.
4. F.BF.A.1.c	4. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
5. F.BF.B.4	5. Find inverse functions.
6. F.BF.B.4.b	6. (+) Verify by composition that one function is the inverse of another.
7. F.BF.B.4.c	7. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
8. F.BF.B.4.d	8. (+) Produce an invertible function from a non-invertible function by restricting the domain.
9. F.IF.B.4	9. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
10. F.IF.C.9	10. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).
11. F.LE.A.1.a	11. Prove that linear functions grow by equal differences over equal intervals, and that exponential functions grow by

	equal factors over equal intervals.
12. F.LE.A.1.b	12. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
13. F.LE.A.1.c	13. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
14. F.LE.A.2	14. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
15. F.LE.B.5	15. Interpret the parameters in a linear or exponential function in terms of a context.
NJSLS	**Interdisciplinary Connections**
1. L.KL.9-10.2.A	1. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level.
2. SL.PE.9-10.1.D	2. Respond thoughtfully to various perspectives, summarize points of agreement and disagreement, and justify own views. Make new connections in light of the evidence and reasoning presented.
3. SL.PI.9-10.4	3. Present information, findings, and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
4. HS-LS2-1	4. Use mathematical and/or computational representations to support explanations of factors that affect carrying capacity of ecosystems at different scales.
2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills	
1. 9.4.12.CI.1	1. Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
2. 9.4.5.DC.4	2. Model safe, legal, and ethical behavior when using online or offline technology.
3. 9.4.12.TL.3	3. Analyze the effectiveness of the process and quality of collaborative environments.
4. 9.4.12.CT.2	4. Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
2020 New Jersey Student Learning Standards for Computer Science and Design Thinking	
1. 8.1.2.CS.1	1. Select and operate computing devices that perform a variety of tasks accurately and quickly based on user needs and preferences.
2. 8.1.2.DA.1	2. Collect and present data, including climate change data, in various visual formats.

3. 8.1.2.DA.3 4. 8.1.2.DA.4 5. 8.1.2.AP.4 6. 8.1.5.DA.5	3. Identify and describe patterns in data visualizations. 4. Make predictions based on data using charts or graphs. 5. Break down a task into a sequence of steps. 6. Propose cause and effect relationships, predict outcomes, or communicate ideas using data.

The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions

- Use Defi nitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely

- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook:

Pre-Calculus with Limits, A Graphing Approach 7th edition – Ron Larson, 2016

Online Resources

- AP Central
- Web Assign
- Desmos Activities
- Pear Assessment
- IXL
- Quizizz
- EdPuzzle
- Canva

- Khan Academy
- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub
- Math Space
- Math Medic

Videos

- 2.6 Competing Function Model Validation
- 2.7A Composition of Functions
- 2.7B Composition of Functions
- 2.8 Inverse Functions

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- *Web Assign online program*
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 2.6: Competing Function Model Validation		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct linear, quadratic, and exponential models based on a data set. • Validate a model constructed from a data set.	**Cumulative Practice**: From a table, determine the function to model the situation. **Prerequisite Skills Practice**: Graphing functions using data tables.	2.6 Competing Function Model Validation

Section 2.7: Composition of Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Evaluate the composition of two or more functions for given values. • Construct a representation of the composition of two or more functions. • Rewrite a given function as a composition of two or more functions.	**Cumulative Practice**: Use vocabulary to understand and simplify expressions. **Prerequisite Skills Practice**: Simplifying functions using a variety of algebraic skills.	2.7A Composition of Functions 2.7B Composition of Functions

Section 2.8: Inverse Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Determine the input-output pairs of the inverse of a function. • Determine the inverse of a function on an invertible domain.	**Cumulative Practice**: Find the domain and range of functions. **Prerequisite Skills Practice**: Simplifying and solving function expressions and equations.	2.8 Inverse Functions

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. ● Diagnostic Pre-Test ● Chapter Tests ● Standardized Tests	The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to: ● Teacher observations ● Self-Assessments ● Student record-keeping ● Quizzes ● Warm-ups ● Exit Tickets ● Participation in class discussions ● Independent practice	The following assessments require students to utilize various strands of mathematics. ● Projects ● Performance Tasks ● Homework ● Classwork
List of Accommodations and Modifications ● Special Education ● 504 Students ● At Risk Students ● MLL ● Gifted and Talented		

State Mandates and Resources
● New Jersey Student Learning Standards ● Standards for Mathematical Practices

Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

Course Name: AP Pre-Calculus
Course Number: 034250

Updated: June 2024

PART I: UNIT RATIONALE

<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 2C: Exponential and Logarithmic Functions

In Unit 2C, students build an understanding of exponential and logarithmic functions. Exponential and logarithmic function models are widespread in the natural and social sciences. When an aspect of a phenomenon changes proportionally to the existing amount, exponential and logarithmic models are employed to harness the information. Exponential functions are key to modeling population growth, radioactive decay, interest rates, and the amount of medication in a patient. Logarithmic functions are useful in modeling sound intensity and frequency, the magnitude of earthquakes, the pH scale in chemistry, and the working memory in humans. The study of these two function types touches careers in business, medicine, chemistry, physics, education, and human geography, among others.

Essential Questions	Learning Targets/Objectives
1. How can I make a single model that merges the interest I earn from my bank with the taxes that are due so I can know how much I will have in the end? 2. How can we adjust the scale of distance for a model of planets in the solar system so the relationships among the planets are easier to see? 3. If different functions can be used to model data, how do we pick which one is best? 4. How can you write an exponential expression as a logarithm? 5. How can you recognize, describe, and compare linear and exponential functions?	Students will be able to: • Evaluate logarithmic expressions. • Construct representations of the inverse of an exponential function with an initial value of 1. • Identify key characteristics of logarithmic functions. • Rewrite logarithmic expressions in equivalent forms. • Solve exponential and logarithmic equations and inequalities. • Construct the inverse function for exponential and logarithmic functions. • Construct a logarithmic function model. • Determine if an exponential model is appropriate by examining a

6. When can you use properties of exponents and logarithms to manipulate functions? 7. How do you evaluate exponential and logarithmic functions? ... 8. When and why are the properties of logarithms used in expanding and condensing logarithmic expressions?	semi-log plot of a data set. • Construct the linearization of exponential data.
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
base, exponent, logarithmic expressions, logarithmic functions, exponential expressions, exponential functions, inverses, inequalities, modeling, distance, growth, decay, transformations, compositions	linearization, precision, logical rationale, regression, numerical, analytical, verbal representations

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. A.REI.D.11	1. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.★
2. A.SSE.B.3	2. Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. ★
3. F.BF.A.1	3. Write a function that describes a relationship between two quantities.★
4. F.BF.B.5	4. (+) Use the inverse relationship between exponents and logarithms to solve problems involving logarithms and exponents.
5. F.IF.C.7	5. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.★
6. F.IF.C.7.e	6. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.

7. F.LE.A.2	7. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
8. F.LE.A.4	8. Understand the inverse relationship between exponents and logarithms. For exponential models, express as a logarithm the solution to abct=d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
9. F.LE.B.5	9. Interpret the parameters in a linear or exponential function in terms of a context.
NJSLS	**Interdisciplinary Connections**
1. L.KL.9-10.2.A	1. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level.
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4. HS-LS2-1	4. Use mathematical and/or computational representations to support explanations of factors that affect carrying capacity of ecosystems at different scales.
2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills	
1. 9.3.12.AG-PST.1	1. Apply physical science principles and engineering applications to solve problems and improve performance in AFNR power, structural and technical systems.
2. 9.4.12.CI.1	2. Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
3. 9.4.5.DC.4	3. Model safe, legal, and ethical behavior when using online or offline technology.
4. 9.4.12.TL.3	4. Analyze the effectiveness of the process and quality of collaborative environments.
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2. 8.1.2.DA.1 3. 8.1.2.DA.3 4. 8.1.2.DA.4 5. 8.1.2.AP.4 6. 8.1.5.DA.5	2. Collect and present data, including climate change data, in various visual formats. 3. Identify and describe patterns in data visualizations. 4. Make predictions based on data using charts or graphs. 5. Break down a task into a sequence of steps. 6. Propose cause and effect relationships, predict outcomes, or communicate ideas using data.

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- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

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- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

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- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
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- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook:

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Online Resources

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- Web Assign
- Desmos Activities
- Pear Assessment
- IXL
- Quizizz
- EdPuzzle
- Canva
- Khan Academy
- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub
- Math Space
- Math Medic

Videos

- 2.9 Logarithmic Expressions
- 2.10 Inverses of Exponential Functions
- 2.11 Logarithmic Functions
- 2.12 Logarithmic Function Manipulation
- 2.13A Exponential and Logarithmic Equations and Inequalities
- 2.13B Exponential and Logarithmic Equations and Inequalities
- 2.14 Logarithmic Function Context and Data Modeling
- 2.15 Semi-Log Plots

Integrated Technology

• Google Suite: Google Classroom, Docs, Drive, Mail, etc...
• *Web Assign online program*
• Devices:
  ○ Chromebooks
  ○ Texas Instrument TI-84 Plus Graphing Calculator

ML Resources
• Multi-Language Glossary
Gifted & Talented Resources
• Leveled Assessments • Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 2.9: Logarithmic Expressions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Evaluate logarithmic expressions.	**Cumulative Practice**: Evaluate each linear, quadratic, cubic, square root and rational function. **Prerequisite Skills Practice**: Evaluating linear, quadratic, cubic, square root and rational functions in a variety of ways.	2.9 Logarithmic Expressions

Section 2.10: Inverses of Exponential Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct representations of the inverse of an exponential function with an initial value of 1.	**Cumulative Practice**: Find the inverse of each linear, quadratic, cubic, square root and rational function. **Prerequisite Skills Practice**: Finding the inverse of linear, quadratic, cubic, square root and rational functions.	2.10 Inverses of Exponential Functions

Section 2.11: Logarithmic Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Identify key characteristics of logarithmic functions.	**Cumulative Practice**: Find the intercept and asymptote of each exponential function. **Prerequisite Skills Practice**: Finding characteristics of linear, quadratic, cubic, square root and rational functions.	2.11 Logarithmic Functions

Section 2.12: Logarithmic Function Manipulation		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Rewrite logarithmic expressions in equivalent forms.	**Cumulative Practice**: Rewrite the exponential function as a logarithmic function. **Prerequisite Skills Practice**: Rewriting functions in different forms. Finding inverses of different functions.	2.12 Logarithmic Function Manipulation

Section 2.13: Exponential and Logarithmic Equations and Inequalities		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Solve exponential and logarithmic equations and inequalities. • Construct the inverse function for exponential and logarithmic functions.	**Cumulative Practice**: Solve each linear, quadratic, cubic, square root and rational equation and inequality. Find the inverse of each function. **Prerequisite Skills Practice**: Solving linear quadratic, cubic, square root and rational, exponential and logarithmic equations and inequalities. FInding inverses of a variety of functions.	2.13A Exponential and Logarithmic Equations and Inequalities 2.13B Exponential and Logarithmic Equations and Inequalities

Section 2.14: Logarithmic Function Context and Data Modeling		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct a logarithmic function model.	**Cumulative Practice**: Graph each exponential growth and decay model **Prerequisite Skills Practice**: Graphing and evaluating exponential and logarithmic functions.	2.14 Logarithmic Function Context and Data Modeling

Section 2.15: Semi-Log Plots		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Determine if an exponential model is appropriate by examining a semi-log plot of a data set. • Construct the linearization of exponential data.	**Cumulative Practice**: Examine the graph of each linear, quadratic, cubic, square root and rational function. **Prerequisite Skills Practice**: Determining characteristics of different function models.	2.15 Semi-Log Plots

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. ● Diagnostic Pre-Test ● Chapter Tests ● Standardized Tests	The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to: ● Teacher observations ● Self-Assessments ● Student record-keeping ● Quizzes ● Warm-ups ● Exit Tickets ● Participation in class discussions ● Independent practice	The following assessments require students to utilize various strands of mathematics. ● Projects ● Performance Tasks ● Homework ● Classwork
List of Accommodations and Modifications ● Special Education ● 504 Students ● At Risk Students ● MLL ● Gifted and Talented		

State Mandates and Resources
● New Jersey Student Learning Standards ● Standards for Mathematical Practices

Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

Course Name: AP Pre-Calculus
Course Number: 034250

Updated: June 2024

PART I: UNIT RATIONALE

<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 3A: Periodic Trigonometric Functions

In Unit 3A, students explore trigonometric functions and their relation to the angles and arcs of a circle. Since their output values repeat with every full revolution around the circle, trigonometric functions are ideal for modeling periodic, or repeated pattern phenomena, such as: the highs and lows of a wave, the blood pressure produced by a heart, and the angle from the North Pole to the Sun year to year. Furthermore, periodicity is found in human inventions and social phenomena. For example, moving parts of an analog clock are modeled by a trigonometric function with respect to each other or with respect to time; traffic flow at an intersection over the course of a week demonstrates daily periodicity; and demand for a particular product over the course of a year falls into an annually repeating pattern. Trigonometry serves as the bridge between the two systems.

Essential Questions	Learning Targets/Objectives
1. Since energy usage goes up and down through the year, how can I use trends in data to predict my monthly electricity bills when I get my first apartment? 2. How does right triangle trigonometry from geometry relate to trigonometric functions? 3. How do you sketch the graphs of sine, cosine and tangent functions? 4. How do you evaluate trigonometric functions by using the unit circle? 5. How do you use trigonometry to find unknown side lengths and angle measures in right triangles? 6. How do you evaluate trigonometric functions of any angle?	Students will be able to: • Construct graphs of periodic relationships based on verbal representations. • Describe key characteristics of a periodic function based on a verbal representation. • Determine the sine, cosine, and tangent of an angle using the unit circle. • Determine coordinates of points on a circle centered at the origin. • Construct representations of the sine and cosine functions using the unit circle. • Identify key characteristics of the sine and cosine functions. • Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function.

	• Construct sinusoidal function models of periodic phenomena. • Construct representations of the tangent function using the unit circle. • Describe key characteristics of the tangent function. • Describe additive and multiplicative transformations involving the tangent function.
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
sine, cosine, tangent, unit circle, degrees, radians, trigonometric function, periodic function, concavity, increasing, decreasing, period, amplitude, arc, maximum, minimum, sinusoidal, asymptote, phase shift, vertical shift, frequency, midline	sinusoidal, periodic function, unit circle, midline

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. N.Q.A.1  2. F.BF.B.3	1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **Climate Change Example**: Students may use units to guide the solution of multi-step problems about how variations in the flow of energy into and out of the Earth's systems result in climate change. Note: Changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution. 2. Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of

	the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
3. F.TF.A.1	3. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
4. F.TF.A.2	4. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
5. F.TF.A.3(+)	5. Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for πx, π+x, and 2π–x in terms of their values for x, where x is any real number.
6. F.TF.B.5	6. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
NJSLS	**Interdisciplinary Connections**
1. L.KL.9-10.2.A	1. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level.
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4. HS-PS4-1	4. Use mathematical representations to support a claim regarding relationships among the frequency, wavelength, and speed of waves traveling in various media.
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1. 9.4.12.CI.1	1. Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
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	and preferences.
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The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships

- Reason Abstractly

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- Use Assumptions
- Use Defi nitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

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- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

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- Use Other Resources

- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

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Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
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Resources
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Videos

- 3.1 Periodic Phenomena
- 3.2A Radians
- 3.2B Sine, Cosine, and Tangent
- 3.3A Sine and Cosine Function Values
- 3.3B Sine and Cosine Function Values
- 3.4 Sine and Cosine Function Graphs
- 3.5 Sinusoidal Functions
- 3.6A Sinusoidal Function Transformations
- 3.6B Sinusoidal Function Transformations
- 3.7 Sinusoidal Function Context and Data Modeling
- 3.8 The Tangent Function

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- *Web Assign online program*
- Devices:
    - Chromebooks

○ Texas Instrument TI-84 Plus Graphing Calculator
ML Resources
● Multi-Language Glossary
Gifted & Talented Resources
● Leveled Assessments ● Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 3.1: Periodic Phenomena		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct graphs of periodic relationships based on verbal representations. • Describe key characteristics of a periodic function based on a verbal representation.	**Cumulative Practice**: analyze a polynomial function and determine on what intervals the function is increasing or decreasing, concavity, and any relative extrema. **Prerequisite Skills Practice**: determining on what intervals a polynomial function is increasing or decreasing, concavity, and any relative extrema.	3.1 Periodic Phenomena

Section 3.2: Sine, Cosine, and Tangent		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Determine the sine, cosine, and tangent of an angle using the unit circle.	**Cumulative Practice**: analyze a periodic function and determine on what intervals the function is increasing or decreasing, concavity, and any relative extrema. **Prerequisite Skills Practice**: determining the characteristics of a periodic function.	3.2A Radians 3.2B Sine, Cosine, and Tangent

Section 3.3: Sine and Cosine Function Values		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Determine coordinates of points on a circle centered at the origin.	**Cumulative Practice**: use the arc length formula to find the missing value of the angle, radius or arc length. **Prerequisite Skills Practice**: using the arc length formula to find the missing value.	3.3A Sine and Cosine Function Values 3.3B Sine and Cosine Function Values

Section 3.4: Sine and Cosine Function Graphs		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct representations of the sine and cosine functions using the unit circle.	**Cumulative Practice**: evaluate trigonometric functions of any angle. **Prerequisite Skills Practice**: understand trigonometric ratios of special right triangles and of quadrant angles.	3.4 Sine and Cosine Function Graphs

Section 3.5: Sinusoidal Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Identify key characteristics of the sine and cosine functions.	**Cumulative Practice**: describe the concavity of a cosine function and if it is increasing or decreasing. **Prerequisite Skills Practice**:analyzing a polynomial function and determining its concavity and whether it is increasing or decreasing.	3.5 Sinusoidal Functions

Section 3.6: Sinusoidal Function Transformations		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function.	**Cumulative Practice**: graph a sine curve. **Prerequisite Skills Practice**: plotting points on a coordinate plane and evaluating expressions.	3.6A Sinusoidal Function Transformations 3.6B Sinusoidal Function Transformations

Section 3.7: Sinusoidal Function Context and Data Modeling		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct sinusoidal function models of periodic phenomena.	**Cumulative Practice**: graph a sine curve with transformations. **Prerequisite Skills Practice**: plotting points on a coordinate plane, vertex form (h,k), and evaluating expressions.	3.7 Sinusoidal Function Context and Data Modeling

Section 3.8: The Tangent Function		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct representations of the tangent function using the unit circle. • Describe key characteristics of the tangent function. • Describe additive and multiplicative transformations involving the tangent function.	**Cumulative Practice**: estimate the period, frequency, vertical shift, and amplitude of a sinusoidal function given a table. **Prerequisite Skills Practice**: understand what a max or min point looks like and recall the definitions of amplitude and vertical shift.	3.8 The Tangent Function

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. ● Diagnostic Pre-Test ● Chapter Tests ● Standardized Tests	The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to: ● Teacher observations ● Self-Assessments ● Student record-keeping ● Quizzes ● Warm-ups ● Exit Tickets ● Participation in class discussions ● Independent practice	The following assessments require students to utilize various strands of mathematics. ● Projects ● Performance Tasks ● Homework ● Classwork
List of Accommodations and Modifications ● Special Education ● 504 Students ● At Risk Students ● MLL ● Gifted and Talented		

State Mandates and Resources
● New Jersey Student Learning Standards ● Standards for Mathematical Practices

Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

Course Name: AP Pre-Calculus
Course Number: 034250

Updated: June 2024

PART I: UNIT RATIONALE

<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 3B: Other Trigonometric Functions

In Unit 3B, Constructing new functions, using transformations, compositions, inverses, or regressions may be useful in modeling contexts, criteria or data, with and without technology. Solving equations and inequalities can be represented analytically, with and without technology. Identifying information from graphical, numerical, analytical, and verbal representations can be used to answer a question or construct a model, with and without technology. One can describe the characteristics of a function with varying levels of precision, depending on the function representation and available mathematical tools. Constructing equivalent graphical, numerical, analytical, and verbal representations of functions are useful in a given mathematical or applied context, with and without technology.

Essential Questions	Learning Targets/Objectives
1. How does right triangle trigonometry from geometry relate to trigonometric functions? 2. How do you evaluate and graph the inverses of trigonometric functions? 3. How do trigonometric equations differ from algebraic equations, and what strategies can we use to solve them? 4. What are the fundamental trigonometric identities, and how do we use them to simplify trigonometric expressions and equations? 5. How do we solve trigonometric equations involving multiple angles, such as double-angle or half-angle identities?	Students will be able to: • Construct analytical and graphical representations of the inverse of the sine, cosine, and tangent functions over a restricted domain. • Solve equations and inequalities involving trigonometric functions. • Identify key characteristics of functions that involve quotients of the sine and cosine functions. • Rewrite trigonometric expressions in equivalent forms with the Pythagorean identity.

6. What are the general solutions to trigonometric equations, and how do we find them using the unit circle or other methods? 7. What techniques can we use to solve trigonometric inequalities, and how do we graph their solutions on the unit circle or the coordinate plane? 8. How do we graph the secant, cosecant, and cotangent functions, and what are their key characteristics such as asymptotes and periodicity? 9. What are the transformations of secant, cosecant, and cotangent graphs, and how do they affect their equations and graphs? 10. How do we apply secant, cosecant, and cotangent functions to real-world problems, such as modeling periodic phenomena or analyzing oscillatory motion?	
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
Inverse, arc, domain, range, restricted domain, principal value, cosecant, secant, cotangent, asymptote, trigonometric identities, pythagorean identities, sum and difference identities, double angles	principal value, trigonometric identities, pythagorean identities, sum and difference identities, double angles

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. N.Q.A.1	1. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.  **Climate Change Example**: Students may use units to guide the solution of multi-step problems about how variations in the flow of energy into and out of the Earth's systems result in climate change. Note: Changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.
2. F-TF.B.5	2. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
3. F-TF.B.6	3. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
4. F-TF.B.7	4. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
5. F-TF.C.8	5. Prove the Pythagorean identity sin^2 (θ) + cos^2 (θ) = 1 and use it to find sin(θ), cos(θ),or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
6. F-TF.C.9	6. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
NJSLS	**Interdisciplinary Connections**
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Videos

- 3.9 Inverse Trigonometric Functions
- 3.10 Trigonometric Equations and Inequalities
- 3.11 The Secant, Cosecant, and Cotangent Functions
- 3.12A Equivalent Representations of Trigonometric Functions
- 3.12B Equivalent Representations of Trigonometric Functions

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- *Web Assign online program*
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 3.9: Inverse Trigonometric Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct analytical and graphical representations of the inverse of the sine, cosine, and tangent functions over a restricted domain	**Cumulative Practice**: evaluate the trigonometric functions of a 30-60-90 and a 45-45-90 triangle. **Prerequisite Skills Practice**: recall the sides of a special right triangle	3.9 Inverse Trigonometric Functions

Section 3.10: Trigonometric Equations and Inequalities		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Solve equations and inequalities involving trigonometric functions.	**Cumulative Practice**: find the inverse of a trigonometric expression. **Prerequisite Skills Practice**: labeling the sides of a special right triangle	3.10 Trigonometric Equations and Inequalities

Section 3.11: The Secant, Cosecant, and Cotangent Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Identify key characteristics of functions that involve quotients of the sine and cosine functions.	**Cumulative Practice**: solve a trigonometric equation **Prerequisite Skills Practice**: inverse operations, combining like terms, taking the square root	3.11 The Secant, Cosecant, and Cotangent Functions

Section 3.12: Equivalent Representations of Trigonometric Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Rewrite trigonometric expressions in equivalent forms with the Pythagorean identity.	**Cumulative Practice**: graph a sine curve. **Prerequisite Skills Practice**: finding the period, amplitude, phase shift and vertical shift of a periodic function.	3.12A Equivalent Representations of Trigonometric Functions 3.12B Equivalent Representations of Trigonometric Functions

PART IV: EVIDENCE OF LEARNING

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Assessments		
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Where inspiring excellence is our standard, and student achievement is the result.

Course Name: AP Pre-Calculus
Course Number: 034250

Updated: June 2024

PART I: UNIT RATIONALE

<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 3C: Polar Functions

In Unit 3C, students explore trigonometric functions and their relation to the angles and arcs of a circle. Since their output values repeat with every full revolution around the circle, trigonometric functions are ideal for modeling periodic, or repeated pattern phenomena, such as: the highs and lows of a wave, the blood pressure produced by a heart, and the angle from the North Pole to the Sun year to year. Furthermore, periodicity is found in human inventions and social phenomena. For example, moving parts of an analog clock are modeled by a trigonometric function with respect to each other or with respect to time; traffic flow at an intersection over the course of a week demonstrates daily periodicity; and demand for a particular product over the course of a year falls into an annually repeating pattern. Polar functions, which are also explored in this unit, have deep ties to trigonometric functions as they are both based on the circle. Polar functions are defined on the polar coordinate system that uses the circular concepts of radii and angles to describe location instead of rectangular concepts of left-right and up-down, which students have worked with previously. Trigonometry serves as the bridge between the two systems.

Essential Questions	Learning Targets/Objectives
1. How do we model aspects of circular and spinning objects without using complex equations from the x-y rectangular-based coordinate system? 2. What are polar coordinates, and how do they differ from Cartesian coordinates? 3. How do we convert between polar and Cartesian coordinates, and vice versa? 4. What are the polar coordinate representations of points on the Cartesian coordinate plane? 5. What are polar graphs, and how do they differ from Cartesian	Students will be able to: • Determine the location of a point in the plane using both rectangular and polar coordinates. • Construct graphs of polar functions. • Describe characteristics of the graph of a polar function.

graphs? 6. What are the basic polar graphs of functions? 7. What is the rate of change in a polar function, and how does it differ from Cartesian functions? 8. How do we calculate the rate of change of a polar function?	
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
polar coordinate, rectangular coordinate, pole, polar form, complex number, complex plane, rose curve, cycle, limacon, cardioid, archimedes' spiral, average rate of change	polar coordinate, pole, polar form, rose curve, limacon, cardioid, archimedes' spiral,

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. N-CN.B.4	1. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
2. F-TF.B.5	2. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
3. F-TF.C.8	3. Prove the Pythagorean identity sin^2 (θ) + cos^2 (θ) = 1 and use it to find sin(θ), cos(θ),or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
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Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Defi nitions
- Use Prior Results
- Make Conjectures
- Build Arguments

- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes

- Calculate Accurately
- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook:

Pre-Calculus with Limits, A Graphing Approach 7th edition – Ron Larson, 2016

Online Resources

- AP Central
- Web Assign
- Desmos Activities
- Pear Assessment
- IXL
- Quizizz
- EdPuzzle
- Canva
- Khan Academy
- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub
- Math Space
- Math Medic

Videos

- 3.13 Trigonometry and Polar Coordinates
- 3.14A Polar Function Graphs
- 3.14B Polar Function Graphs
- 3.15 Rates of Change in Polar Functions

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- *Web Assign online program*
- Devices:

○ Chromebooks ○ Texas Instrument TI-84 Plus Graphing Calculator
ML Resources
● Multi-Language Glossary
Gifted & Talented Resources
● Leveled Assessments ● Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 3.13: Trigonometry and Polar Coordinates		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Determine the location of a point in the plane using both rectangular and polar coordinates.	**Cumulative Practice**: simplify a trigonometric expression. **Prerequisite Skills Practice**: knowledge of trigonometric identities and algebra skills.	3.13 Trigonometry and Polar Coordinates

Section 3.14: Polar Function Graphs		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct graphs of polar functions.	**Cumulative Practice**: convert a polar coordinate to a rectangular coordinate. **Prerequisite Skills Practice**: evaluate a trigonometric function of an angle.	3.14A Polar Function Graphs 3.14B Polar Function Graphs

Section 3.15: Rates of Change in Polar Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Describe characteristics of the graph of a polar function.	**Cumulative Practice**: write the equation of a polar function. **Prerequisite Skills Practice**: knowledge of the characteristics of a rose curve, limacon, cardioid, and archimedes' spiral.	3.15 Rates of Change in Polar Functions

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. ● Diagnostic Pre-Test ● Chapter Tests ● Standardized Tests	The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to: ● Teacher observations ● Self-Assessments ● Student record-keeping ● Quizzes ● Warm-ups ● Exit Tickets ● Participation in class discussions ● Independent practice	The following assessments require students to utilize various strands of mathematics. ● Projects ● Performance Tasks ● Homework ● Classwork
List of Accommodations and Modifications ● Special Education ● 504 Students ● At Risk Students ● MLL ● Gifted and Talented		

State Mandates and Resources
● New Jersey Student Learning Standards ● Standards for Mathematical Practices

Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

Course Name: AP Pre-Calculus
Course Number: 034250

Updated: June 2024

PART I: UNIT RATIONALE

<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 4A: Cumulative Review of Units 1, 2 and 3

Unit 4A is a review of the first three units. Applications of topics learned in Units 1: Polynomial and Rational Functions, Unit 2: Exponential and Logarithmic Functions, and Unit 3: Trigonometric and Polar Functions, along with test prep for the AP exam will be the focus of Unit 4A. This two week review will conclude with a practice AP Test.

Essential Questions	Learning Targets/Objectives
1. How do we model the intensity of light from its source? 2. How can I use data and graphs to figure out the best time to purchase event tickets? 3. How can we adjust known projectile motion models to account for changes in conditions? 4. How do rational functions relate to polynomial and exponential functions? Investigate connections and differences between these types of functions? 5. How do additive transformations impact the key features of a function? 6. How do multiplicative transformations influence the characteristics of a function? 7. How do dilations impact the domain and range of a function? Investigate how scaling affects the spread of the function's	Students will be able to: • Understand how output values change in tandem with changing input values in polynomial functions. • Construct a graph representing two quantities that vary with respect to each other in a contextual scenario. • Compare the rates of change at two points using average rates of change near the points. • Describe how two quantities vary together at different points and over different intervals of a function. • Determine the average rates of change for linear and quadratic sequences and functions. • Determine the change of average rates of change for linear and quadratic functions. • Identify key characteristics of polynomial functions related to rates of

values.

8. How do you restrict the domain of a function for real-world applications?
9. How can I make a single model that merges the interest I earn from my bank with the taxes that are due so I can know how much I will have in the end?
10. How can we adjust the scale of distance for a model of planets in the solar system so the relationships among the planets are easier to see?
11. If different functions can be used to model data, how do we pick which one is best?
12. How can exponential functions be used to model real-world problems?
13. How can exponential functions be used to model growth and decay?
14. How do you write, graph, and interpret an exponential growth or decay function?
15. How can I make a single model that merges the interest I earn from my bank with the taxes that are due so I can know how much I will have in the end?
16. How can we adjust the scale of distance for a model of planets in the solar system so the relationships among the planets are easier to see?
17. If different functions can be used to model data, how do we pick which one is best?
18. What is function composition?
19. How do you compose functions?
20. How do you evaluate composite functions?
21. How do you find the inverse of a function?
22. Since energy usage goes up and down through the year, how can I use trends in data to predict my monthly electricity bills when I get my first apartment?
23. How does right triangle trigonometry from geometry relate to trigonometric functions?
24. How do you sketch the graphs of sine, cosine and tangent functions?
25. How do you evaluate trigonometric functions by using the unit circle?
26. How do you use trigonometry to find unknown side lengths and angle measures in right triangles?
27. How do you evaluate trigonometric functions of

change.

- Identify key characteristics of a polynomial function related to its zeros when suitable factorizations are available or with technology.
- Determine if a polynomial is even or odd.
- Describe end behaviors of polynomial functions.
- Describe the end behavior of rational functions.
- Determine the zeros of rational functions.
- Determine vertical asymptotes of graphs of rational functions.
- Determine holes of graphs of rational functions.
- Rewrite polynomial and rational expressions in equivalent forms
- Rewrite the repeated product of binomials using the binomial theorem.
- Determine the quotient of two polynomial functions using long division.
- Construct a function that is an additive and/or multiplicative transformation of another function.
- Construct a linear, quadratic, cubic, quartic, polynomial of degree n, or related piecewise-defined function model.
- Construct a rational function model based on a context.
- Apply a function model to answer questions about a data set or contextual scenario.
- Identify an appropriate function type to construct a function model for a given scenario.
- Describe assumptions and restrictions related to building a function model.
- Express arithmetic sequences found in mathematical and contextual scenarios as functions of whole numbers.
- Express geometric sequences found in mathematical and contextual scenarios as functions of whole numbers.
- Construct functions of the real numbers that are comparable to arithmetic and geometric sequences.
- Describe similarities and differences between linear and exponential functions.
- Identify key characteristics of exponential functions.
- Rewrite exponential expressions in equivalent forms.
- Construct a model for situations involving proportional output values over equal-length input-value intervals.
- Apply exponential models to answer questions about a data set or contextual scenario.
- Construct linear, quadratic, and exponential models based on a data set.

any angle? 28. How do we model aspects of circular and spinning objects without using complex equations from the x-y rectangular-based coordinate system? 29. What are polar coordinates, and how do they differ from Cartesian coordinates? 30. How do we convert between polar and Cartesian coordinates, and vice versa? 31. What are the polar coordinate representations of points on the Cartesian coordinate plane? 32. What are polar graphs, and how do they differ from Cartesian graphs? 33. What are the basic polar graphs of functions? 34. What is the rate of change in a polar function, and how does it differ from Cartesian functions? 35. How do we calculate the rate of change of a polar function?	• Validate a model constructed from a data set. • Evaluate the composition of two or more functions for given values. • Construct a representation of the composition of two or more functions. • Rewrite a given function as a composition of two or more functions. • Determine the input-output pairs of the inverse of a function. • Determine the inverse of a function on an invertible domain. • Construct graphs of periodic relationships based on verbal representations. • Describe key characteristics of a periodic function based on a verbal representation. • Determine the sine, cosine, and tangent of an angle using the unit circle. • Determine coordinates of points on a circle centered at the origin. • Construct representations of the sine and cosine functions using the unit circle. • Identify key characteristics of the sine and cosine functions. • Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function. • Construct analytical and graphical representations of the inverse of the sine, cosine, and tangent functions over a restricted domain. • Solve equations and inequalities involving trigonometric functions. • Identify key characteristics of functions that involve quotients of the sine and cosine functions. • Rewrite trigonometric expressions in equivalent forms with the Pythagorean identity. • Determine the location of a point in the plane using both rectangular and polar coordinates. • Construct graphs of polar functions. • Describe characteristics of the graph of a polar function.
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
Function, input, output, domain, range, independent variable, dependent variable, change in tandem, zeros, x-intercepts, y-intercept, concavity, inflection point, intervals of increasing, intervals of decreasing, average rate of change, rate of change at a point, slope, secant line, linear function, quadratic function, polynomial, relative extrema, global (absolute) extrema, multiplicity, complex zeros, odd and	Average Rate of Change, Rate of Change at a point, polynomial functions, Rational functions, asymptotes, holes, limits, Translations, dilations, linear, quadratic, cubic, quartic, exponential, logarithmic, logistic, sinusoidal, exponential growth, exponential decay, arithmetic sequences, geometric sequences, regression, invertible, parameters, competing, composition,

even polynomials, end behavior, and degree of a polynomial, Rational function, end behavior, domain, range, horizontal asymptote, vertical asymptote, slant asymptote, holes, interval notation, limits, multiplicity, factored form, standard form, Binomial Theorem, long division, Translations, additive translations, multiplicative transformations, dilations, domain, range, linear, quadratic, cubic, restricting the domain, restricting the range, piecewise functions, regression, quartic, exponential, logarithmic, logistic, sine, inversely proportional, base, variable, exponent, growth, decay, constant, change, domain, range, increasing, decreasing, data, sequences, notation, principal, interest, inverse,	

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. A.APR.B.3	1. Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
2. A.APR.C.5	2. Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.
3. A.APR.D.6	3. Rewrite simple rational expressions in different forms; write *a(x)/b(x)* in the form *q(x) + r(x)/b(x)*, where *a(x), b(x), q(x),* and *r(x)* are polynomials with the degree of *r(x)* less than the degree of *b(x)*, using inspection, long division, or, for the more complicated examples, a computer algebra system.
4. A.REI.A.2	4. Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.

5. A.REI.D.11	5. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
6. A.SSE.B.3	6. Choose and produce an equivalent form of an expression to reveal and explain the properties of the quantity represented by the expression.
7. A.SSE.B.3.c	7. Use the properties of exponents to transform expressions for exponential functions.
8. A.SSE.B.4	8. Derive and/or explain the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments.
9. F.BF.A.1	9. Write a function that describes a relationship between two quantities.
10. F.BF.A.1.c	10. (+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
11. F.BF.A.2	11. Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
12. F.BF.B.3	12. Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $kf(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
13. F.BF.B.4	13. Find inverse functions.
14. F.BF.B.4.b	14. (+) Verify by composition that one function is the inverse of another.
15. F.BF.B.4.c	15. (+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
16. F.BF.B.4.d	16. (+) Produce an invertible function from a non-invertible function by restricting the domain.
17. F.IF.A.1	17. Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If *f* is a function and *x* is an element of its domain, then f(x) denotes the output of *f* corresponding to the input *x*. The graph of *f* is the graph of the equation y = f(x).

18. F.IF.A.2	18. Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of context.
19. F.IF.A.3	19. Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers.
20. F.IF.B.4	20. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
21. F.IF.B.5 	21. Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. **Climate Change Example:** Students will relate the domain of a function c(m) representing the amount of carbon dioxide produced by burning m molecules of ethane (gasoline), to its graph in order to determine the appropriate domain for c(m).
22. F.IF.B.6 	22. Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. **Climate Change Example**: Students may calculate the average rate of change of a function c(m) presented symbolically or as a table, where c(m) represents the amount of carbon dioxide produced by burning a given number of molecules of ethane (gasoline).
23. F.IF.C.7	23. Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.
24. F.IF.C.7.e	24. Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
25. F.IF.C.7.c	25. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
26. F.IF.C.7.d	26. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.
27. F.IF.C.8	27. Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
28. F.IF.C.9	28. Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

29. F.LE.A.1	29. Distinguish between situations that can be modeled with linear functions and with exponential functions.
30. F.LE.A.1.a	30. Prove that linear functions grow by equal differences over equal intervals and that exponential functions grow by equal factors over equal intervals.
31. F.LE.A.1.b	31. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
32. F.LE.A.1.c	32. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
33. F.LE.A.2	33. Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (including reading these from a table).
34. F.LE.B.5	34. Interpret the parameters in a linear or exponential function in terms of a context.
35. F.TF.A.1	35. Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
36. F.TF.A.2	36. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
37. F.TF.A.3(+)	37. Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosines, and tangent for πx, π+x, and 2π–x in terms of their values for x, where x is any real number.
38. F.TF.B.5	38. Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.
39. F-TF.B.6	39. Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
40. F-TF.B.7	40. Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.
41. F-TF.C.8	41. Prove the Pythagorean identity sin^2 (θ) + cos^2 (θ) = 1 and use it to find sin(θ), cos(θ),or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
42. F-TF.C.9	42. Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
43. N-CN.B.4	43. Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.

44. N-Q.A.1	44. Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **Climate Change Example**: Students may use units to guide the solution of multi-step problems about how variations in the flow of energy into and out of the Earth's systems result in climate change. Note: Changes in climate are limited to changes in surface temperatures, precipitation patterns, glacial ice volumes, sea levels, and biosphere distribution.
NJSLS	**Interdisciplinary Connections**
1. L.KL.9-10.2.A	1. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level.
2. SL.PE.9-10.1.D	2. Respond thoughtfully to various perspectives, summarize points of agreement and disagreement, and justify own views. Make new connections in light of the evidence and reasoning presented.
3. SL.PI.9-10.4	3. Present information, findings, and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills	
1. 9.4.12.CI.1	1. Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
2. 9.4.5.DC.4	2. Model safe, legal, and ethical behavior when using online or offline technology
3. 9.4.12.TL.3	3. Analyze the effectiveness of the process and quality of collaborative environments.
4. 9.4.12.CT.2	4. Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
2020 New Jersey Student Learning Standards for Computer Science and Design Thinking	
1. 8.1.2.CS.1	1. Select and operate computing devices that perform a variety of tasks accurately and quickly based on user needs and preferences.
2. 8.1.2.DA.1	2. Collect and present data, including climate change data, in various visual formats.
3. 8.1.2.DA.3	3. Identify and describe patterns in data visualizations.
4. 8.1.2.DA.4	4. Make predictions based on data using charts or graphs.
5. 8.1.2.AP.4	5. Break down a task into a sequence of steps.

6. 8.1.5.DA.5	6. Propose cause and effect relationships, predict outcomes, or communicate ideas using data.

The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:
Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to: ● Explain the Meaning ● Find Entry Points ● Analyze Givens ● Interpret a Solution ● Make a Plan ● Consider Similar Problems ● Check Progress ● Consider Simpler Forms ● Problem Solve **Reason abstractly and quantitatively**: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to: ● Make Sense of Quantities ● Use Equations ● Use Expressions ● Understand Quantities ● Use Operations ● Contextualize ● Relationships ● Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Definitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

Look for and make use of structure: Look closely to see the structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook:

Pre-Calculus with Limits, A Graphing Approach 7th edition – Ron Larson, 2016

Online Resources

- AP Central
- Web Assign
- Desmos Activities
- Pear Assessment
- IXL
- Quizizz
- EdPuzzle
- Canva
- Khan Academy
- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub
- Math Space
- Math Medic
- Albert IO AP Pre-Calc

Videos

- What is on the AP Precalculus Exam
- Unit 1 Review with Practice Questions
- Unit 2 Review with Practice Questions
- Unit 3 Review with Practice Questions

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- *Web Assign online program*
- Devices:
  - Chromebooks

○ Texas Instrument TI-84 Plus Graphing Calculator
ML Resources
● Multi-Language Glossary
Gifted & Talented Resources
● Leveled Assessments ● Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 1: Unit 1 Review		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
● Understand how output values change in tandem with changing input values in polynomial functions. ● Construct a graph representing two quantities that vary with respect to each other in a contextual scenario.	**Cumulative Practice**: AP Test Prep review. **Prerequisite Skills Practice:** Students need to recall information	AP Pre-Calc Mutli Choice Unit 1 Test Prep Albert IO AP Pre-Calc Free Response Test Prep

• Compare the rates of change at two points using average rates of change near the points. • Describe how two quantities vary together at different points and over different intervals of a function. • Determine the average rates of change for linear and quadratic sequences and functions. • Determine the change of average rates of change for linear and quadratic functions. • Identify key characteristics of polynomial functions related to rates of change. • Identify key characteristics of a polynomial function related to its zeros when suitable factorizations are available or with technology. • Determine if a polynomial is even or odd. • Describe end behaviors of polynomial functions. • Describe end behavior of rational functions. • Determine the zeros of rational functions. • Determine vertical asymptotes of graphs of rational functions. • Determine holes of graphs of rational functions. • Rewrite polynomial and rational expressions in equivalent forms • Rewrite the repeated product of binomials using the binomial theorem. • Determine the quotient of two polynomial functions using long division. • Determine the location of a point in the plane using both rectangular and polar coordinates. • Construct graphs of polar functions. • Describe the characteristics of the graph of a polar function.	from Unit 1: Polynomials, Rationals, Transformations, and Modeling in order to complete practice AP Pre-Calculus exam questions.	

Section 2: Unit 2 Review		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Express arithmetic sequences found in mathematical and contextual scenarios as functions of whole numbers. • Express geometric sequences found in mathematical and contextual scenarios as functions of whole numbers. • Construct functions of the real numbers that are comparable to arithmetic and geometric sequences. • Describe similarities and differences between linear and exponential functions. • Identify key characteristics of exponential functions. • Rewrite exponential expressions in equivalent forms. • Construct a model for situations involving proportional output values over equal-length input-value intervals. • Apply exponential models to answer questions about a data set or contextual scenario. • Construct linear, quadratic, and exponential models based on a data set. • Validate a model constructed from a data set. • Evaluate the composition of two or more functions for given values. • Construct a representation of the composition of two or more functions. • Rewrite a given function as a composition of two or more functions. • Determine the input-output pairs of the inverse of a function. • Determine the inverse of a function on an invertible domain.	**Cumulative Practice**: AP Test Prep review. **Prerequisite Skills Practice:** Students need to recall information from Unit 2: Rational Functions to complete practice AP Pre-Calculus exam questions.	AP Pre-Calc Mult Choice Unit 2 Test Prep Albert IO AP Pre-Calc Free Response Test Prep

Section 3: Unit 3 Review		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct graphs of periodic relationships based on verbal representations. • Describe key characteristics of a periodic function based on a verbal representation. • Determine the sine, cosine, and tangent of an angle using the unit circle. • Determine the coordinates of points on a circle centered at the origin. • Construct representations of the sine and cosine functions using the unit circle. • Identify key characteristics of the sine and cosine functions. • Identify the amplitude, vertical shift, period, and phase shift of a sinusoidal function. • Construct analytical and graphical representations of the inverse of the sine, cosine, and tangent functions over a restricted domain. • Solve equations and inequalities involving trigonometric functions. • Identify key characteristics of functions that involve quotients of the sine and cosine functions. • Rewrite trigonometric expressions in equivalent forms with the Pythagorean identity. • How does right triangle trigonometry from geometry relate to trigonometric functions? • How do you evaluate and graph the inverses of trigonometric functions? • How do trigonometric equations differ from algebraic equations, and what strategies can we use to solve them? • What are the fundamental trigonometric identities, and how do we use them to simplify trigonometric expressions and equations?	**Cumulative Practice**: AP Test Prep review. **Prerequisite Skills Practice:** Students need to recall information from Unit 1: Polynomials to complete practice AP Pre-Calculus exam questions.	AP Pre-Calc Mult Choice Unit 3 Test Prep Albert IO AP Pre-Calc Free Response Test Prep

• How do we solve trigonometric equations involving multiple angles, such as double-angle or half-angle identities? • What are the general solutions to trigonometric equations, and how do we find them using the unit circle or other methods? • What techniques can we use to solve trigonometric inequalities, and how do we graph their solutions on the unit circle or the coordinate plane? • How do we graph the secant, cosecant, and cotangent functions, and what are their key characteristics such as asymptotes and periodicity? • What are the transformations of secant, cosecant, and cotangent graphs, and how do they affect their equations and graphs? • How do we apply secant, cosecant, and cotangent functions to real-world problems, such as modeling periodic phenomena or analyzing oscillatory motion?		

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey	The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to:	The following assessments require students to utilize various strands of mathematics. ● Projects

Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. ● Diagnostic Pre-Test ● Chapter Tests ● Standardized Tests	● Teacher observations ● Self-Assessments ● Student record-keeping ● Quizzes ● Warm-ups ● Exit Tickets ● Participation in class discussions ● Independent practice	● Performance Tasks ● Homework ● Classwork
List of Accommodations and Modifications ● Special Education ● 504 Students ● At Risk Students ● MLL ● Gifted and Talented		

State Mandates and Resources

- New Jersey Student Learning Standards
- Standards for Mathematical Practices

Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

Course Name: AP Pre-Calculus
Course Number: 034250

Updated: June 2024

PART I: UNIT RATIONALE

WHY ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 4B: Conic Sections

In Unit 4B, students explore function types that expand their understanding of the function concept. A conic section, also known as a conic or quadratic curve, is a curve that results from the intersection of a plane and a right circular cone. The type of conic section that forms depends on the angle of the plane relative to the cone. The four basic types of conic sections are: Circle: Formed when the plane is perpendicular to the axis of the cone, Ellipse: Formed when the plane intersects one of the cone's pieces and its axis, but is not perpendicular to the axis, Parabola: Formed when all points on the curve are the same distance from a given line, called the directrix, and a point not on the line, called the focus, Hyperbola: One of the three types of conic sections. This component-based understanding is important not only in calculus but in all fields of the natural and social sciences where we seek to understand one aspect of a phenomenon independent of other confounding aspects.

Essential Questions	Learning Targets/Objectives
1. How can we determine when the populations of species in an ecosystem will be relatively steady? 2. How can we analyze the vertical and horizontal aspects of motion independently? 3. How does high resolution computer generated imaging achieve smooth and realistic motion on screen with so many Pixels? 4. How do I identify the characteristics of parabolas graphically and algebraically centered on the origin? 5. How do I identify the characteristics of ellipses graphically and algebraically centered on the origin?	Students will be able to: • Represent conic sections with horizontal or vertical symmetry analytically. • Represent a curve in the plane parametrically. • Represent conic sections parametrically.

6. How can one part of a circle help determine the measure of another part? 7. How are area and circumference connected? How can we determine area, given circumference? 8. Can we determine diameter or radius, given area or circumference? 9. What role do foci play in the definition of conic quadratic relations? 10. How can ellipses be defined in relation to their foci?	
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
conic sections, center, diameter, circumference, radius, circle, parabola, standard form, ellipse, focus, foci, vertex, vertices, co-vertices, curve, plane, pythagorean theorem, distance, constant, directrix, sum, difference, plane, curve, horizontal, vertical, symmetric, parametric	symmetrical, parametric, analytically, conic section, foci, directrix, radius, center, vertices, co-vertices

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. F.IF.B.4 2. G.GPE.A.1 3. G.GPE.A.2 4. G.GPE.A.3	1. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. 2. (+) Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. 3. (+) Derive the equation of a parabola given a focus and directrix. 4. (+) Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant.
NJSLS	**Interdisciplinary Connections**

1. L.KL.9-10.2.A	1. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level.
2. SL.PE.9-10.1.D	2. Respond thoughtfully to various perspectives, summarize points of agreement and disagreement, and justify own views. Make new connections in light of the evidence and reasoning presented.
3. SL.PI.9-10.4	3. Present information, findings, and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.
4. HS-ESS1-4	4. Use mathematical or computational representations to predict the motion of orbiting objects in the solar system.
2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills	
1. 9.4.12.CI.1	1. Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
2. 9.4.5.DC.4	2. Model safe, legal, and ethical behavior when using online or offline technology.
3. 9.4.12.TL.3	3. Analyze the effectiveness of the process and quality of collaborative environments.
4. 9.4.12.CT.2	4. Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
2020 New Jersey Student Learning Standards for Computer Science and Design Thinking	
1. 8.1.2.CS.1	1. Select and operate computing devices that perform a variety of tasks accurately and quickly based on user needs and preferences.
2. 8.1.2.DA.1	2. Collect and present data, including climate change data, in various visual formats.
3. 8.1.2.DA.3	3. Identify and describe patterns in data visualizations.
4. 8.1.2.DA.4	4. Make predictions based on data using charts or graphs.
5. 8.1.2.AP.4	5. Break down a task into a sequence of steps.
6. 8.1.5.DA.5	6. Propose cause and effect relationships, predict outcomes, or communicate ideas using data.

The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

Make sense of problems and persevere in solving them: Take time to analyze the given information and what the problem is asking to help you to plan a solution pathway. Throughout the unit students are given problems that require them to:

- Explain the Meaning
- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Defi nitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples
- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions

- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components

- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
- Evaluate Results

Resources

Textbook:

Pre-Calculus with Limits, A Graphing Approach 7th edition – Ron Larson, 2016

Online Resources

- AP Central
- Web Assign
- Desmos Activities
- Pear Assessment
- IXL
- Quizizz
- EdPuzzle
- Canva
- Khan Academy
- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub
- Math Space
- Math Medic

Videos

- 4.6 Conic Sections
- 4.7 Parametrization of Implicitly Defined Functions

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- *Web Assign online program*
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

<u>How</u> will students uncover content and build skills?

Section 4.6: Conic Sections		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Represent conic sections with horizontal or vertical symmetry analytically.	**Cumulative Practice**: Find the center and radius of the circle. **Prerequisite Skills Practice**: Finding key characteristics of the graphs of a variety of functions.	4.6 Conic Sections

Section 4.7: Parametrization of Implicitly Defined Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Represent a curve in the plane parametrically. • Represent conic sections parametrically.	**Cumulative Practice**: sketch and analyze the characteristics of a circle, parabola, ellipse and hyperbola. **Prerequisite Skills Practice**: Knowledge of center, radius, vertices, co-vertices, foci, directrix an asymptotes of conic sections.	4.7 Parametrization of Implicitly Defined Functions

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. • Diagnostic Pre-Test • Chapter Tests • Standardized Tests	The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to: • Teacher observations • Self-Assessments • Student record-keeping • Quizzes • Warm-ups • Exit Tickets • Participation in class discussions • Independent practice	The following assessments require students to utilize various strands of mathematics. • Projects • Performance Tasks • Homework • Classwork
List of Accommodations and Modifications • Special Education • 504 Students		

- At Risk Students
- MLL
- Gifted and Talented

State Mandates and Resources

- New Jersey Student Learning Standards
- Standards for Mathematical Practices

Black Horse Pike Regional School District

Where inspiring excellence is our standard, and student achievement is the result.

Course Name: AP Pre-Calculus
Course Number: 034250

Updated: June 2024

PART I: UNIT RATIONALE

<u>WHY</u> ARE STUDENTS LEARNING THIS CONTENT AND THESE SKILLS?

Unit 4C: Parametric Functions

In Unit 4C, students explore function types that expand their understanding of the function concept. Parametric functions have multiple dependent variables' values paired with a single input variable or parameter. Modeling scenarios with parametric functions allows students to explore change in terms of components. This component-based understanding is important not only in calculus but in all fields of the natural and social sciences where we seek to understand one aspect of a phenomenon independent of other confounding aspects. Another major function type in this unit involves matrices mapping a set of input vectors to output vectors. The capacity to map large quantities of vectors instantaneously is the basis for vector-based computer graphics. While students may see their favorite video game character trip and fall or seemingly move closer or farther, matrices implement a rotation on a set of vectors or a dilation on a set of vectors. The power of matrices to map vectors is not limited to graphics but to any system that can be expressed in terms of components of vectors such as electrical systems, network connections, and regional population distribution changes over time. Vectors and matrices are also powerful tools of data science as they can be used to model aspects of complex scientific and social science phenomena.

Essential Questions	Learning Targets/Objectives
1. How can we determine when the populations of species in an ecosystem will be relatively steady? 2. How can we analyze the vertical and horizontal aspects of motion independently? 3. How does high resolution computer generated imaging achieve smooth and realistic motion on screen with so many pixels? 4. What are parametric functions?	Students will be able to: • Construct a graph or table of values for a parametric function represented analytically. • Identify key characteristics of a parametric planar motion function that are related to position. • Identify key characteristics of a parametric planar motion function that are related to direction and rate of change.

5. How do parametric functions differ from regular functions? 6. How do you convert parametric equations into Cartesian equations, and vice versa? 7. What are some common parametric curves and their properties (e.g., circles, ellipses, parabolas)? 8. How do implicit functions relate to parametric functions and Cartesian functions?	• Express motion around a circle or along a line segment parametrically. • Construct a graph of an equation involving two variables. • Determine how the two quantities related in an implicitly defined function vary together.
Tier 2 Vocabulary *High-frequency words used throughout the unit*	**Tier 3 Vocabulary** *Discipline-specific words used throughout the unit*
parametric function, horizontal and vertical extrema, x-intercept, y-intercept, planar motion, average rate of change, implicit function	parametric function, planar motion, implicit function

PART II: INSTRUCTIONAL STRATEGIES AND RESOURCES

DESCRIBE THE LEARNING TARGETS.

New Jersey Student Learning Standards That Support Learning Targets	
2023 New Jersey Student Learning Standards for Mathematics	
1. F.IF.B.4	1. For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
2. F.TF.A.2	2. Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
3. G.CO.B.6	3. Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

NJSLS	Interdisciplinary Connections
1. L.KL.9-10.2.A	1. Acquire and use accurately general academic and domain-specific words and phrases, sufficient for reading, writing, speaking, and listening at the college and career readiness level.
2. SL.PE.9-10.1.D	2. Respond thoughtfully to various perspectives, summarize points of agreement and disagreement, and justify own views. Make new connections in light of the evidence and reasoning presented.
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4. HS-ESS1-4	4. Use mathematical or computational representations to predict the motion of orbiting objects in the solar system.
2020 New Jersey Student Learning Standards for Career Readiness, Life Literacies, and Key Skills	
1. 9.4.12.CI.1	1. Demonstrate the ability to reflect, analyze, and use creative skills and ideas.
2. 9.4.5.DC.4	2. Model safe, legal, and ethical behavior when using online or offline technology.
3. 9.4.12.TL.3	3. Analyze the effectiveness of the process and quality of collaborative environments.
4. 9.4.12.CT.2	4. Explain the potential benefits of collaborating to enhance critical thinking and problem solving.
2020 New Jersey Student Learning Standards for Computer Science and Design Thinking	
1. 8.1.2.CS.1	1. Select and operate computing devices that perform a variety of tasks accurately and quickly based on user needs and preferences.
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3. 8.1.2.DA.3	3. Identify and describe patterns in data visualizations.
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5. 8.1.2.AP.4	5. Break down a task into a sequence of steps.
6. 8.1.5.DA.5	6. Propose cause and effect relationships, predict outcomes, or communicate ideas using data.

The 8 Mathematical Practices are embedded throughout the course and are evident in daily lessons, assignments, activities, assessments, and projects:

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- Find Entry Points
- Analyze Givens
- Interpret a Solution
- Make a Plan
- Consider Similar Problems
- Check Progress
- Consider Simpler Forms
- Problem Solve

Reason abstractly and quantitatively: Investigate specific examples and represent them symbolically, and observe the relationships in numbers or symbols to derive conclusions about a concrete instance. Throughout the unit students are given problems that require them to:

- Make Sense of Quantities
- Use Equations
- Use Expressions
- Understand Quantities
- Use Operations
- Contextualize
- Relationships
- Reason Abstractly

Construct viable arguments and critique the reasoning of others: Make and justify conclusions and decide whether others' arguments are correct or flawed. Throughout the unit students are given problems that require them to:

- Use Assumptions
- Use Defi nitions
- Use Prior Results
- Make Conjectures
- Build Arguments
- Analyze Conjectures
- Use Counterexamples

- Justify Conclusions
- Compare Arguments
- Construct Arguments
- Listen and Ask Questions
- Critique Reasoning
- Use Logic
- Error Analysis

Model with mathematics: Apply the mathematics to a real-life problem, and you interpret mathematical results in the context of the situation. Throughout the unit students are given problems that require them to:

- Apply Mathematics
- Simplify a Solution
- Use a Diagram
- Use a Table
- Use a Graph
- Use a Formula
- Analyze Relationships
- Interpret Results
- Model Real Life

Use appropriate tools strategically: Know what tools are available and think about how each tool might help solve a mathematical problem. Use a tool for its advantages, while being aware of its limitations. Throughout the unit students are given problems that require them to:

- Choose Tools
- Recognize Usefulness of Tools
- Use Other Resources
- Use Technology to Explore

Attend to precision: Develop a habit of being careful how you talk about concepts, label your work, and write your answers. Throughout the unit students are given problems that require them to:

- Communicate Precisely
- Use Clear Definitions
- State the Meaning of Symbols
- Specify Units
- Label Axes
- Calculate Accurately
- Understand Mathematical Terms

Look for and make use of structure: Look closely to see structure within a mathematical statement, or step back for an overview to see how individual parts make one single object. Throughout the unit students are given problems that require them to:

- View as Components
- Look for Patterns
- Look for Structure

Look for and express regularity in repeated reasoning: Notice patterns and make generalizations. Keeping in mind the goal of a problem helps you evaluate reasonableness of answers along the way. Throughout the unit students are given problems that require them to:

- Repeat Calculations
- Find General Methods
- Maintain Oversight
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- Quizizz
- EdPuzzle
- Canva
- Khan Academy
- Inside Mathematics
- NJDOE Digital Item Library
- New Jersey Center for Teaching and Learning
- New Jersey Climate Education Hub
- Math Space
- Math Medic

Videos

- 4.1 Parametric Functions
- 4.2 Parametric Function Modeling Planar Motion
- 4.3 Parametric Functions and Rates of Change
- 4.4 Parametrically Defined Circles and Lines
- 4.5 Implicitly Defined Functions

Integrated Technology

- Google Suite: Google Classroom, Docs, Drive, Mail, etc...
- *Web Assign online program*
- Devices:
  - Chromebooks
  - Texas Instrument TI-84 Plus Graphing Calculator

ML Resources

- Multi-Language Glossary

Gifted & Talented Resources

- Leveled Assessments
- Enrichment worksheets

PART III: TRANSFER OF KNOWLEDGE AND SKILLS

DESCRIBE THE LEARNING EXPERIENCE.

How will students uncover content and build skills?

Section 4.1: Parametric Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct a graph or table of values for a parametric function represented analytically.	**Cumulative Practice**: sketch a quadratic function. **Prerequisite Skills Practice**: making a table of values, plotting points.	4.1 Parametric Functions

Section 4.2: Parametric Function Modeling Planar Motion		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Identify key characteristics of a parametric	**Cumulative Practice**: sketch a	4.2 Parametric Function Modeling Planar

planar motion function that are related to position.	parametric function. **Prerequisite Skills Practice**: making a table of values, evaluating expressions, plotting points.	Motion

Section 4.3: Parametric Functions and Rates of Change		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Identify key characteristics of a parametric planar motion function that are related to direction and rate of change.	**Cumulative Practice**: sketch and analyze the characteristics of a parametric function. **Prerequisite Skills Practice**: making a table of values, evaluating expressions, plotting points, finding the x and y intercept	4.3 Parametric Functions and Rates of Change

Section 4.4: Parametrically Defined Circles and Lines		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Express motion around a circle or along a line segment parametrically.	**Cumulative Practice**: find the average rate of change of a parametric function. **Prerequisite Skills Practice**: knowledge of finding the average rate of change.	4.4 Parametrically Defined Circles and Lines

Section 4.5: Implicitly Defined Functions		
Specific Learning Objective	**Warm-Up/Starting Options**	**Practice & Apply Exercises**
• Construct a graph of an equation involving two variables. • Determine how the two quantities related in an implicitly defined function vary together.	**Cumulative Practice**: find the parametric equation for a circle given the center and radius. **Prerequisite Skills Practice**: knowledge of center and radius of a circle.	4.5 Implicitly Defined Functions

PART IV: EVIDENCE OF LEARNING

IDENTIFY THE METHODS BY WHICH STUDENTS WILL DEMONSTRATE THEIR UNDERSTANDING OF CONTENT AND THEIR ABILITY TO APPLY SKILLS.

Assessments		
Summative	**Formative**	**Performance**
The following assessments will be used to evaluate student learning, skill acquisition and academic achievement of the Standards of Mathematical Practice and the New Jersey Learning Standards for Mathematics listed under each chapter in the Algebra 1 curriculum/syllabus at the conclusion of an instructional time period. ● Diagnostic Pre-Test ● Chapter Tests	The effectiveness of the instructional program will be based on numerous activities and strategies including the following and are not limited to: ● Teacher observations ● Self-Assessments ● Student record-keeping ● Quizzes ● Warm-ups ● Exit Tickets	The following assessments require students to utilize various strands of mathematics. ● Projects ● Performance Tasks ● Homework ● Classwork

• Standardized Tests	• Participation in class discussions • Independent practice	

List of Accommodations and Modifications

- Special Education
- 504 Students
- At Risk Students
- MLL
- Gifted and Talented

State Mandates and Resources

- New Jersey Student Learning Standards
- Standards for Mathematical Practices