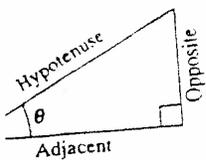


IMM Summer Assignment Reference Information

TRIGONOMETRY

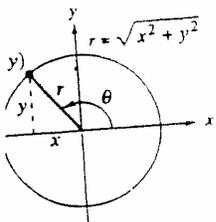
Definition of the Six Trigonometric Functions:

triangle definitions, where $0 < \theta < \pi/2$.

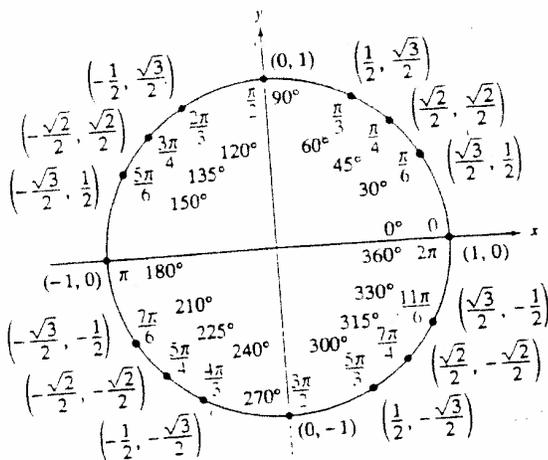


$$\begin{aligned} \sin \theta &= \frac{\text{opp}}{\text{hyp}} & \csc \theta &= \frac{\text{hyp}}{\text{opp}} \\ \cos \theta &= \frac{\text{adj}}{\text{hyp}} & \sec \theta &= \frac{\text{hyp}}{\text{adj}} \\ \tan \theta &= \frac{\text{opp}}{\text{adj}} & \cot \theta &= \frac{\text{adj}}{\text{opp}} \end{aligned}$$

circular function definitions, where θ is any angle.



$$\begin{aligned} \sin \theta &= \frac{y}{r} & \csc \theta &= \frac{r}{y} \\ \cos \theta &= \frac{x}{r} & \sec \theta &= \frac{r}{x} \\ \tan \theta &= \frac{y}{x} & \cot \theta &= \frac{x}{y} \end{aligned}$$



Reciprocal Identities:

$$\begin{aligned} \sin x &= \frac{1}{\csc x} & \sec x &= \frac{1}{\cos x} & \tan x &= \frac{1}{\cot x} \\ \csc x &= \frac{1}{\sin x} & \cos x &= \frac{1}{\sec x} & \cot x &= \frac{1}{\tan x} \end{aligned}$$

Tangent and Cotangent Identities:

$$\tan x = \frac{\sin x}{\cos x} \quad \cot x = \frac{\cos x}{\sin x}$$

Pythagorean Identities:

$$\begin{aligned} \sin^2 x + \cos^2 x &= 1 \\ 1 + \tan^2 x &= \sec^2 x & 1 + \cot^2 x &= \csc^2 x \end{aligned}$$

Cofunction Identities:

$$\begin{aligned} \sin\left(\frac{\pi}{2} - x\right) &= \cos x & \cos\left(\frac{\pi}{2} - x\right) &= \sin x \\ \csc\left(\frac{\pi}{2} - x\right) &= \sec x & \tan\left(\frac{\pi}{2} - x\right) &= \cot x \\ \sec\left(\frac{\pi}{2} - x\right) &= \csc x & \cot\left(\frac{\pi}{2} - x\right) &= \tan x \end{aligned}$$

Reduction Formulas:

$$\begin{aligned} \sin(-x) &= -\sin x & \cos(-x) &= \cos x \\ \csc(-x) &= -\csc x & \tan(-x) &= -\tan x \\ \sec(-x) &= \sec x & \cot(-x) &= -\cot x \end{aligned}$$

Sum and Difference Formulas:

$$\begin{aligned} \sin(u \pm v) &= \sin u \cos v \pm \cos u \sin v \\ \cos(u \pm v) &= \cos u \cos v \mp \sin u \sin v \\ \tan(u \pm v) &= \frac{\tan u \pm \tan v}{1 \mp \tan u \tan v} \end{aligned}$$

Double-Angle Formulas:

$$\begin{aligned} \sin 2u &= 2 \sin u \cos u \\ \cos 2u &= \cos^2 u - \sin^2 u = 2 \cos^2 u - 1 = 1 - 2 \sin^2 u \\ \tan 2u &= \frac{2 \tan u}{1 - \tan^2 u} \end{aligned}$$

Power-Reducing Formulas:

$$\begin{aligned} \sin^2 u &= \frac{1 - \cos 2u}{2} \\ \cos^2 u &= \frac{1 + \cos 2u}{2} \\ \tan^2 u &= \frac{1 - \cos 2u}{1 + \cos 2u} \end{aligned}$$

Law of Sines

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Law of Cosines

$$a^2 = b^2 + c^2 - 2bc \cos A$$

Sum-to-Product Formulas:

$$\begin{aligned} \sin u + \sin v &= 2 \sin\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \sin u - \sin v &= 2 \cos\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \\ \cos u + \cos v &= 2 \cos\left(\frac{u+v}{2}\right) \cos\left(\frac{u-v}{2}\right) \\ \cos u - \cos v &= -2 \sin\left(\frac{u+v}{2}\right) \sin\left(\frac{u-v}{2}\right) \end{aligned}$$

Product-to-Sum Formulas:

$$\begin{aligned} \sin u \sin v &= \frac{1}{2} [\cos(u-v) - \cos(u+v)] \\ \cos u \cos v &= \frac{1}{2} [\cos(u-v) + \cos(u+v)] \\ \sin u \cos v &= \frac{1}{2} [\sin(u+v) + \sin(u-v)] \\ \cos u \sin v &= \frac{1}{2} [\sin(u+v) - \sin(u-v)] \end{aligned}$$

Sequences:

$$\begin{aligned} a_n &= n\text{th term} \\ a_1 &= \text{first term} \end{aligned}$$

n = term #

d = common difference

r = common ratio

Arithmetic: $a_n = a_1 + (n-1)d$

Geometric: $a_n = a_1 r^{n-1}$

FUNCTIONS

For # 1-16, let:

$$f(x) = 3 - \sqrt{2x+5}$$

$$g(x) = \frac{4-x}{2x^2 - 5x - 42}$$

$$k(x) = 7x - 4$$

$$j(x) = \frac{3x-2}{2x+3}$$

1. Find the domains of $f(x)$, $g(x)$, $k(x)$, and $j(x)$.
2. Find the ranges of $f(x)$ and $k(x)$.
3. Find $(k+j)(x)$ and simplify.
4. Find $(\frac{j}{g})(x)$.
5. Find $(fg)(x)$.
6. Find $j(k(x))$ and simplify.
7. Find $k(j(x))$ and simplify.
8. Find $j(7x) - 4$ and simplify.
9. Find $7j(x) - 4$ and simplify.
10. Find $7j(x-4)$ and simplify.
11. Find $f(x+h)$.
12. Find $j^{-1}(x)$.
13. Find the zeros of $f(x)$, $g(x)$, $k(x)$, and $j(x)$.
14. Find the y-intercepts of $f(x)$, $g(x)$, $k(x)$, and $j(x)$.
15. Find the x-intercepts of $f(x)$, $g(x)$, $k(x)$, and $j(x)$.
16. Find $\frac{k(x+h) - k(x)}{h}$ and simplify.

COMPLEX NUMBERS

$i = i$
$i^2 = -1$
$i^3 = -i$
$i^4 = 1$
$i = \sqrt{-1}$

or #'s 17-20, write the complex number in standard form $(a+bi)$

7. $3 - \sqrt{-25}$

18. $6 + \sqrt{-4}$

19. $-5i + i^2$

20. $i^2 + 3i$

For #'s 21-25, perform the operation and write the result in standard form.

21. $(7 + 5i) + (-4 + 2i)$

24. $\frac{3 + 2i}{5 + i}$

22. $i(6 + i)(3 - 2i)$

25. $\frac{1}{2 + i} - \frac{5}{1 + 4i}$

23. $(10 - 8i)(2 - 3i)$

TRIGONOMETRY

For #'s 26 – 30, find the exact value. Write the answer in radical form. No decimals!

26. $\sin \frac{5\pi}{6}$

27. $\cot \frac{2\pi}{3}$

28. $\sec \frac{3\pi}{4}$

29. $\cos\left(-\frac{\pi}{6}\right)$

30. $\tan \frac{5\pi}{4}$

For #'s 31-33, graph each function.

31. $y = \sin x$

32. $y = \cos x$

33. $y = \tan x$

34. If $\cos \theta = -\frac{7}{25}$, and θ is in quadrant II, determine the exact value of $\sin \theta$.

35. Determine the exact values of $\sin \theta$, $\tan \theta$, $\csc \theta$, $\sec \theta$, and $\cot \theta$ if θ is an angle in quadrant III and $\cos \theta = -\frac{8}{17}$.

36. If the terminal side of an angle θ passes through the point $(-7, 24)$, determine the exact values of the six trigonometric functions of θ .

For #'s 37 and 38, find the exact values for all six trigonometric functions for each angle measurement.

37. -90°

38. $\frac{5\pi}{3}$

39. MULTIPLE CHOICE: The reciprocal function of $\csc \theta$ is

- A. $\cos \theta$ B. $\sin \theta$ C. $\frac{1}{\sin \theta}$ D. $\frac{1}{\cos \theta}$ E. none of these

40. MULTIPLE CHOICE: The ratio identity for $\tan \theta$ is

- A. $\cot \theta$ B. $\frac{\sin \theta}{\cos \theta}$ C. $\csc \theta$ D. $\frac{\cos \theta}{\sin \theta}$ E. none of these

For #'s 41-43, solve each triangle ABC described below. Angle C is the right angle.

41. $A = 63^\circ$, $a = 9.7$

42. $a = 2$, $b = 7$

43. $B = 83^\circ$, $b = \sqrt{31}$

For # 44 solve each triangle using the law of sines.

44. $A = 29^\circ$, $a = 12$, $b = 15$

For # 45 solve each triangle using the law of cosines.

45. $A = 51^\circ$, $b = 40$, $c = 45$

For #'s 46-48, verify that each of the following is an identity. Use a proof setup.

46. $\cos^2 x + \tan^2 x \cos^2 x = 1$

47. $\frac{(\sin^4 x - \cos^4 x)}{\sin^2 x} = 1 - \cot^2 x$

48. $\frac{\sin x}{\csc x} + \frac{\cos x}{\sec x} = 1$

For #'s 49-54, solve each equation, where $0 \leq x \leq 2\pi$. Answers should be exact and in radian form. No decimals!

49. $2\cos x - 1 = 0$

50. $2\cos x + \sqrt{3} = 0$

51. $\sin(3x) = 1$

52. $\sin(2x) = -1$

53. $\tan(2x) = 1$

54. $2\cos^2 x + 3\cos x - 2 = 0$ (hint: factor first)

ALGEBRA REVIEW

For #'s 55-58, solve each equation.

55. $6x^2 + 3x + 27 = 0$

56. $x^3 - 11x^2 + 18x = 0$

57. $\sqrt{x+4} = 6$

58. $2x^2 - 7x = 4$

59. Use a graphing calculator to estimate the real zeros of the function $f(x) = x^4 - 13x^2 + 40$

60. Rationalize the denominator: $\frac{5}{\sqrt{2}-1}$

61. Add: $\frac{3}{x-2} + \frac{x}{x^2+x}$

62. Simplify: $\frac{\frac{x}{x-1} - \frac{x}{x+1}}{\frac{x^2-1}{4x}}$

GRAPHING

For #'s 63 and 64, graph each system of inequalities.

63. $y > -2x$
 $y - 2x < 8$
 $x \leq 1$

$2x + y \geq 4$
64. $3x + 3y \geq 9$
 $x \geq 0, y \geq 0$

LOGARITHMS and EXPONENTS

For #'s 65 and 66, simplify.

65. $3x^2(3x)^{-2}$

66. $(2a)^{\frac{1}{3}}(a^2b)^{\frac{1}{3}}$

67. Write $\log_5 \frac{1}{125} = -3$ in exponential form.

For #'s 68 - 70, solve each equation.

68. $\log_3 81 = 4$

69. $\log_{\frac{1}{2}} x = -4$

70. $2.5^x = 65.7$

SEQUENCES AND SERIES

71. Find the next five terms of the sequence 3, 4.3, 5.6,
72. Find the 20th term of the arithmetic sequence for which $a_1 = 7$ and $d = -4$.
73. Form a sequence that has three arithmetic means between 6 and -4.
74. Find the next four terms of the sequence 343, 49, 7,
75. Find the 7th term of the geometric sequence for which $a_1 = 2.2$ and $r = 2$.