PROBABILITY AND STATISTICS
GRADES 11-12

THE EWING PUBLIC SCHOOLS
2099 Pennington Road
Ewing, NJ 08618

Board Approval Date: December 16, 2019 Michael Nitti
Written by: EHS Mathematics Department Superintendent

In accordance with The Ewing Public Schools' Policy 2230, Course Guides, this curriculum has been reviewed and found to be in compliance with all policies and all affirmative action criteria.

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Course Description and Rationale

In this course students will explore Probability and Statistics. They will learn how to use this specialized area of mathematics to analyze and predict real world scenarios. They will be able to apply mathematics to analyze data and come to their own conclusions. There are those who go along with what they are told or who take blind chances. There are those who can figure out for themselves. This course gives students the tools to determine for themselves.

The Ewing Public Schools' Math Vision

The Ewing Public Schools will deliver an instructional program in mathematics where students are actively engaged in the discovery of math concepts and are applying these concepts in ways that they find meaningful and relevant.

Ewing students will be mathematical thinkers who can reason, communicate and solve problems.

Ultimately, Ewing students will master and will be able to utilize these math concepts and skills throughout their lives.

21st Century Skills - During this course, students will work on developing, to an age appropriate level, the following 21st century skills:

Career Readiness Pathways:

- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP12. Work productively in teams while using cultural global competence.

Learning and Innovation Skills

Creativity and Innovation

Think Creatively

- Elaborate, refine, analyze and evaluate their own ideas in order to improve and maximize creative efforts

Work Creatively with Others

- View failure as an opportunity to learn; understand that creativity and innovation is a long-term, cyclical process of small successes and frequent mistakes

CRITICAL THINKING AND PROBLEM SOLVING

Reason Effectively

- Use various types of reasoning (inductive, deductive, etc.) as appropriate to the situation

Use Systems Thinking

- Analyze how parts of a whole interact with each other to produce overall outcomes in complex systems

Make Judgments and Decisions

- Effectively analyze and evaluate evidence, arguments, claims and beliefs
- Synthesize and make connections between information and arguments
- Interpret information and draw conclusions based on the best analysis

Solve Problems

- Identify and ask significant questions that clarify various points of view and lead to better solutions

COMMUNICATION AND COLLABORATION

Communicate Clearly

- Articulate thoughts and ideas effectively using oral, written and nonverbal communication skills in a variety of forms and contexts
- Listen effectively to decipher meaning, including knowledge, values, attitudes and intentions
- Use communication for a range of purposes (e.g. to inform, instruct, motivate and persuade)
- Utilize multiple media and technologies, and know how to judge their effectiveness a priori as well as assess their impact
- Communicate effectively in diverse environments (including multi-lingual)

Collaborate with Others

- Assume shared responsibility for collaborative work, and value the individual contributions made by each team member

Information, Media, and Technology Skills

Informational Literacy

Access and Evaluate Information

- Evaluate information critically and competently

Use and Manage Information

- Use information accurately and creatively for the issue or problem at hand

Life and Career Skills

Social and Cross-Cultural Skills

Interact Effectively with Others

- Know when it is appropriate to listen and when to speak

Work Effectively in Diverse Teams

- Respond open-mindedly to different ideas and values

Be Responsible to Others

- Act responsibly with the interests of the larger community in mind

Technology Integration

8.1 Educational Technology
All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and create and communicate knowledge.

ELA Integration:

SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one-on- one, in groups, and teacher-led) with peers on *grades 11–12 topics, texts, and issues,* building on others' ideas and expressing their own clearly and persuasively.

A. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well reasoned exchange of ideas.
B. Collaborate with peers to promote civil, democratic discussions and decision-making, set clear goals and assessments (e.g. student developed rubrics), and establish individual roles as needed.
C. Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives.
D. Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task. SL.11-12.2. Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.11-12.3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used.

SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

Unit 1: Descriptive Statistics (10 Days)

Why Is This Unit Important?

In this unit, students will learn ways to organize and describe data sets. The goal will be to make data easier to understand by making it easier to see trends, averages and variations.

Enduring Understandings: Students will understand:

- How to analyze sets of data
- How to interpret and graphically represent frequency distributions
- How to represent data sets both qualitative and quantitative
- How to identify and interpret measures of central tendency
- How to analyze and interpret measures of variation
- How to utilize measures of position to represent and interpret data sets

Essential Questions:

- What are the various ways we can graphically represent frequency distributions? How do they differ? How are they similar?
- What determines whether data is qualitative or quantitative?
- What are the measures of central tendency? How are they determined? How are they similar? How do they differ?
- How can measures of variation be determined and interpreted?

Acquired Knowledge: After studying the material of this unit, students will be able to:

- Define midpoint, relative frequency, cumulative frequency, frequency histogram
- Identify distribution shapes as symmetric, uniform or skewed
- Differentiate between qualitative and quantitative data
- Define mean, median, mode, bimodal and weighted mean
- Identify and interpret symmetric, uniform and skewed distributions
- Define range, deviation, population variance, standard deviation
- Understand the Empirical Rule
- Understand Chebyshev's Theorem

Acquired Skills: After studying the material of this unit, students will be able to:

- Construct a frequency distribution including midpoints, relative frequencies and cumulative frequencies
- Construct a frequency histogram
- Construct a frequency polygon
- Construct a relative frequency histogram
- Graph a quantitative data set using a stem-and–leaf plot
- Graph a quantitative data set using a dot plot
- Graph a qualitative data set using a pie chart
- Graph a qualitative data set using a Pareto chart
- Graph paired data sets using scatter plots
- Graph paired data sets using time series plots
- Determine mean, median, and mode of a population and sample
- Find the weighted mean and the mean of a frequency distribution
- Find the range of a data set
- Find the variance and standard deviation of a population and a sample
- Interpret standard deviation using the Empirical Rule and Chebyshev's Theorem
- Find the first, second and third quartiles of a data set
- Find the interquartile range of a data set
- Represent a data set using a box and whisker plot

Differentiation:

Enrichment:

- Construct an ogive

Supplement:

- Data distribution software

Assessments:

Formative Assessments:

- Assessment Checklist for Frequency Distribution Representation
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks:

- Descriptive Statistics Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

List of Applicable CCSS and Standards/CPIs Covered in This Unit:

- NJSLS.N-Q.2-3
- NJSLS.S-IC.3
- NJSLS.S-ID.1-5
- NJSLS.MP.1-8

Suggested Learning Experiences and Instructional Activities:

- Frequency distribution constructions of collected data
- Interpreting frequency distributions
- Graphing data sets
- Interpreting graphically represented data sets
- Working with measures of central tendency
- Using measure of position to represent and interpret data sets

Unit 2: Probability (15 Days)

Why Is This Unit Important?

In this unit, students will learn how to use data to predict the probability that an event will occur.

Enduring Understandings: Students will understand:

- How to distinguish between types of probability
- How to find the probability of two events occurring in sequence
- How to find the probability that at least one of two events will occur
- How to count the number of ways an event can occur

Essential Questions:

- How can one differentiate between the three types of probability?
- What is conditional probability?
- How can one determine if two events will occur in sequence?
- How can one determine if two events are mutually exclusive?

Acquired Knowledge: After studying the material of this unit, students will be able to:

- Differentiate between classical, empirical and subjective probability
- Understand the Law of Large Numbers
- Distinguish between independent and dependent events
- Understand what a conditional probability is
- Understand the multiplication rule
- Differentiate between inclusive and exclusive events
- Understand the Fundamental Counting Principal
- Differentiate between permutation and combinations

Acquired Skills: After studying the material of this unit, students will be able to:

- Identify the sample space of a probability experiment
- Identify simple events
- Find a classical probability
- Find an empirical probability
- Use frequency distributions to find probabilities
- Find the probability of a complement of an event
- Find the probability of an event given that another event has occurred
- Find conditional probabilities using the multiplication rule
- Find the probability of two events occurring in sequence using the multiplication rule
- Find probabilities using the addition rule
- Find probabilities of mutually-exclusive events

- How to use the Fundamental Counting Principle to find the number of ways two or more events can occur
- Find the number of ways a group of objects can be arranged in order
- Find the number of ways to choose several objects from a group without regard to order
- Use counting principles to find probabilities

Differentiation:

Enrichment:

- The Locker Problem

Supplement:

- Utilize tree diagrams to list events

Assessments:

Formative Assessments:

- Assessment Checklist for Event Probability
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks

- Probability Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

List of Applicable CCSS and Standards/CPIs Covered in This Unit:

- NJSLS.MP.1-8
- NJSLS.N-Q.2-3
- NJSLS.S-CP.1-9

Suggested Learning Experiences and Instructional Activities:

- Probability and Parking Lot Strategies
- Composing Mozart Variations with Dice
- Event Sequences
- Applications of the Addition Rule

Unit 3: Discrete Probability Distributions (13 Days)

Why Is This Unit Important?

This unit will serve to develop the skills for creating and using probability distributions.

Enduring Understandings: Students will understand:

- How to represent and interpret discrete probability distributions
- How to represent and interpret binomial probability distributions

Essential Questions:

- How can we differentiate between discrete and continuous random variables? How are they similar? How do they differ?
- What does a discrete probability distribution list?
- What determines if an experiment is binomial?

Acquired Knowledge: After studying the material of this unit, students will be able to:

- Differentiate between discrete and continuous
- Understand and use the notation for binomial experiments
- Understand the binomial probability formula
- Understand the population parameters of a binomial distribution

Acquired Skills: After studying the material of this unit, students will be able to:

- Construct discrete probability distributions and their graphs
- Find the mean, variance and standard deviation of a discrete probability distribution
- Find the expected value of a probability distribution
- Find binomial probabilities using the binomial probability formula
- Find binomial probabilities using a binomial probability table
- Construct a binomial distribution and its graph
- Find the mean, variance and standard deviation of a binomial probability distribution
- Find probabilities using geometric distribution
- Find probabilities using the Poisson distribution

Differentiation:

Enrichment:

- Using Poisson Distributions as Queuing Models

Supplement:

- Graphing Normal Distributions

Assessments:

Formative Assessments:

- Assessment Checklist for Discrete Versus Continuous Events
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks

- Discrete Probability Distributions Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

List of Applicable CCSS and Standards/CPIs Covered in This Unit:

- NJSLS.MP.1-8
- NJSLS.N-Q.2-3
- NJSLS.S-ID.1-3
- NJSLS.S-MP.1-4

Suggested Learning Experiences and Instructional Activities:

- Case Study on the Distribution of Airplane Accidents
- Heads or Tails
- Discrete Probability Distributions
- Constructing Binomial Distributions
- Delving into Geometric Distributions

Unit 4: Normal Probability Distributions (13 Days)

Why Is This Unit Important?

This unit will serve to develop the skills of recognizing normal distributions and how to use their properties in real-life applications.

Enduring Understandings: Students will understand:

- The area under a curve of a normal distribution represents the probability of a random variable falling within a specified range of values
- The Central Limit Theorem describes the relationship between the sampling distribution of sample means and the population from which the samples are taken
- A normal distribution can be used to approximate a binomial distribution

Essential Questions:

- How can one identify if a distribution is normal?
- What does the area under a curve of a normal distribution describe?

Acquired Knowledge: After studying the material of this unit, students will be able to:

- Understand the properties of a normal distribution
- Application of the Empirical Rule for normal distributions
- Understand what a standard or z-score represents
- Compare data from two normal distributions
- Define what sampling distribution of sample means
- Decide when the normal distribution can approximate the binomial distribution

Acquired Skills: After studying the material of this unit, students will be able to:

- Interpret graphs of normal probability distributions
- Find the probability for random variables with normal distributions using the area under a curve
- Find and interpret z-scores
- Find the value of a variable when its z-score is given
- Find the area under a standard normal curve
- Find the probability of a sample mean by applying the Central Limit Theorem
- Find the correction for continuity

Differentiation:

Enrichment:

- Investigate and report on the reasons for creating calculus

Supplement:

- Case Study on Birth Rates in America

Assessments:

Formative Assessments:

- Assessment Checklist for Normal Distribution Representations
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks:

- Normal Probability Distributions Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

List of Applicable CCSS and Standards/CPIs Covered in This Unit:

- NJSLS.MP.1-8
- NJSLS.N-Q.2-3
- NJSLS.S-ID.1-4
- NJSLS.S-MD.1-5

Suggested Learning Experiences and Instructional Activities:

- Data Collection to Determine Approaching Normal Distributions
- Applications of Normal Distributions Presentation
- Age Distribution in the United States
- Global Age Distribution

Unit 5: Confidence Intervals (13 Days)

Why Is This Unit Important?

In this unit, students will begin their study of inferential statistics by making more meaningful estimates by specifying an interval of values together with a statement of confidence that a given interval contains the population parameter.

Enduring Understandings: Students will understand:

- Confidence intervals can be utilized to describe the level of likelihood for an interval to contain the population mean
- The confidence interval for a population mean can be constructed, even if the standard deviation is unknown and sampling is impractical, if a random variable is normally distributed
- Population proportions can be estimated using confidence intervals
- The ability to control the amount of variation is augmented by the ability to measure it

Essential Questions:

- How can we express the level of confidence we have for a value falling within a certain range of values?
- What is the chi-square distribution?

Acquired Knowledge: After studying the material of this unit, students will be able to:

- Define a point estimate, interval estimate and level of confidence
- Determine degrees of freedom
- Understand the c-confidence interval

Acquired Skills: After studying the material of this unit, students will be able to:

- Find a point estimate and a maximum error of estimate
- Construct and interpret confidence intervals for the population mean
- Determine the required minimum sample size when estimating the population mean
- Interpret a t-distribution
- Construct confidence intervals when $n < 30$ or the population mean is unknown
- Find a sample proportion
- Construct a confidence interval for a population proportion
- Determine a minimum sampling size when estimating a population proportion
- Interpret the chi-square distribution
- Use a chi-square distribution to construct a confidence interval

Differentiation:

Enrichment:

- Shell Lengths in Loggerhead Sea Turtles

Supplement:

- Determining Degrees of Freedom

Assessments:

Formative Assessments:

- Assessment Checklist for Confidence Intervals
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks

- Confidence Intervals Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

List of Applicable CCSS and Standards/CPIs Covered in This Unit:

- NJSLS.MP.1-8
- NJSLS.N-Q.2-3
- NJSLS.S-IC.1-4
- NJSLS.S-ID.1-5
- NJSLS.S-MD.1-4

Suggested Learning Experiences and Instructional Activities:

- Chi-Square Value Interpretation
- History of Statistics-Timeline
- Most Admired Polls
- Traffic Safety Facts
- Crash tests

Unit 6: Hypothesis Testing (12 Days)

Why Is This Unit Important?

In this unit, students will continue their study of inferential statistics. Instead of making an estimate about a population parameter, they will now learn how to test a claim about a parameter.

Enduring Understandings: Students will understand:

- Statistical testing can be used to test a claim about the value of a population parameter
- Statistical testing can be used to determine difference in means between populations

Essential Questions:

- What differentiates a hypothesis and a statement?
- What is a null hypothesis? An alternative hypothesis?
- How can we identify types of errors and their significance?
- What factors determine how many tailed tests are used?
- What do 'critical values' represent?

Acquired Knowledge: After studying the material of this unit, students will be able to:

- Differentiate between a null hypothesis and an alternative hypothesis
- Identify type I and type II errors, interpreting levels of significance
- Determine whether a one-tailed or two-tailed statistical test should be used
- Find critical values in a normal distribution
- Find critical values in a t-distribution

Acquired Skills: After studying the material of this unit, students will be able to:

- Write a claim for a hypothesis
- Use a z-test to test a mean
- Find and use P-values to test a mean
- Use a t-test to test a mean
- Use a t-test to test a population proportion
- Perform a two sample z-test for the difference between two means using large independent samples
- Perform a t-test for the difference between two population means using small independent samples

Differentiation:

Enrichment:

- The Case of the Vanishing Woman

Supplement:

- Tails Over Heads

Assessments:

Formative Assessments:

- Assessment Checklist for Statistical Testing
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks

- Hypothesis Testing Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

List of Applicable CCSS and Standards/CPIs Covered in This Unit:

- NJSLS.MP.1-8
- NJSLS.N-Q.2-3
- NJSLS.S-IC.1-6
- NJSLS.S-ID.1-5
- NJSLS.S-MD.1-7

Suggested Learning Experiences and Instructional Activities:

- Writing a Hypothesis
- Human Body Temperature: What's Normal?
- Ascertaining Appropriate Sample Size
- Oat Bran and Cholesterol Level

Unit 7: Correlation and Regression (8 Days)

Why Is This Unit Important?

In this unit, students will study how to describe and test the significance of relationships between two variables when data are presented as ordered pairs.

Enduring Understandings: Students will understand:

- The level of correlation between two pairs of ordered data can be evaluated
- Correlation does not signify causation
- High levels of linear correlation can be used to predict values for non-sampled situations using linear regression techniques

Essential Questions:

- When looking at data collected between two variables, how can we judge their correlations?
- How are correlation and causation related to each other?
- If two variables are correlated, how can we predict one variable's value given the value of the other variable?

Acquired Knowledge: After studying the material of this unit, students will be able to:

- Define correlation, explanatory variable and response variable
- Determine whether a t-test can be used to test whether the correlation between two variables is significant
- Distinguish between correlation and causation
- Understand what a line of best fit represents

Acquired Skills: After studying the material of this unit, students will be able to:

- Construct a scatter plot
- Find a correlation coefficient
- Perform a hypothesis test for a population correlation coefficient
- Find the equation of a regression line
- Predict y-values using a regression equation
- Use a t-test to test a population proportion
- Perform a two sample z-test for the difference between two means using large independent samples
- Perform a t-test for the difference between two population means using small independent samples

Differentiation:

Enrichment:

- Predicting and Presenting Correlated Data Sets

Supplement:

- Graphing software used for interpretation and evaluation of data sets

Assessments:

Formative Assessments:

- Assessment Checklist for Linear Regression
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks

- Correlation and Regression Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

List of Applicable CCSS and Standards/CPIs Covered in This Unit:

- NJSLS.MP.1-8
- NJSLS.N-Q.2-3
- NJSLS.S-ID.1-9

Suggested Learning Experiences and Instructional Activities:

- Correlation of Body Measurements
- Tar, Nicotine and Carbon Monoxide
- Predicting Future Occurrences

Sample Standards Integration

21st Century Skills & Career Readiness Practices

CRP4. Communicate clearly and effectively and with reason.

For example, in Unit 7 students will justify their reasoning in determining significance of relationship between variables.

CRP6. Demonstrate creativity and innovation.

For example, in Unit 2 students will use probability analysis to solve real world complex volume problems.

CRP7. Employ valid and reliable research strategies.

For example, in Unit 5 students will analyze and interpret data plots for real world scenarios for their confidence intervals.

CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.

For example, in Unit 4 students will work to solve and understand real world applications utilizing normalized data.

CRP12. Work productively in teams while using cultural global competence.

For example, in Unit 6 students will work in small teams to develop models for hypothesis testing.

8.1 Educational Technology

All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and create and communicate knowledge.

For example, in Unit 3 students will access, manage, evaluate, and synthesize information to develop models for probability distributions.

Interdisciplinary Connections

SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one-on- one, in groups, and teacher-led) with peers on *grades 11–12 topics, texts, and issues,* building on others' ideas and expressing their own clearly and persuasively.

A. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well reasoned exchange of ideas.
B. Collaborate with peers to promote civil, democratic discussions and decision-making, set clear goals and assessments (e.g. student developed rubrics), and establish individual roles as needed.
C. Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives.
D. Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task. SL.11-12.2. Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.11-12.3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used.

SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

These standards are met throughout the course. For example, in Unit 1 students will discuss their solutions to a variety of real world scenarios justifying their argument descriptive statistics.