# PRE-CALCULUS
# GRADES 11-12

THE EWING PUBLIC SCHOOLS
2099 Pennington Road
Ewing, NJ 08618

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Written by: Crystal Tuccillo and Thomas Weber

Michael Nitti
Superintendent

In accordance with The Ewing Public Schools' Policy 2230, Course Guides, this curriculum has been reviewed and found to be in compliance with all policies and all affirmative action criteria.

## Table of Contents

## Course Description and Rationale

Pre-Calculus is a 'gateway' course to Calculus. This course strengthens the Algebra II foundation and serves as a bridge from Algebra II to Calculus.

In Pre-Calculus, students will lay down the foundation upon which they will build in Calculus.

## The Ewing Public Schools' Math Vision

*The Ewing Public Schools will deliver an instructional program in mathematics where students are actively engaged in the discovery of math concepts and are applying these concepts in ways that they find meaningful and relevant.*

*Ewing students will be mathematical thinkers who can reason, communicate and solve problems.*

*Ultimately, Ewing students will master and will be able to utilize these math concepts and skills throughout their lives.*

21st Century Skills - During this course, students will work on developing, to an age appropriate level, the following 21st century skills:

**Career Readiness Pathways**:

- CRP2. Apply appropriate academic and technical skills.
- CRP4. Communicate clearly and effectively and with reason.
- CRP6. Demonstrate creativity and innovation.
- CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.
- CRP12. Work productively in teams while using cultural global competence.

**Learning and Innovation Skills**

**Creativity and Innovation**

Think Creatively

- Elaborate, refine, analyze and evaluate their own ideas in order to improve and maximize creative efforts

Work Creatively with Others

- View failure as an opportunity to learn; understand that creativity and innovation is a long-term, cyclical process of small successes and frequent mistakes

**CRITICAL THINKING AND PROBLEM SOLVING**

Reason Effectively

- Use various types of reasoning (inductive, deductive, etc.) as appropriate to the situation

Use Systems Thinking

- Analyze how parts of a whole interact with each other to produce overall outcomes in complex systems

Make Judgments and Decisions

- Effectively analyze and evaluate evidence, arguments, claims and beliefs
- Synthesize and make connections between information and arguments
- Interpret information and draw conclusions based on the best analysis

Solve Problems

- Identify and ask significant questions that clarify various points of view and lead to better solutions

## COMMUNICATION AND COLLABORATION

Communicate Clearly

- Articulate thoughts and ideas effectively using oral, written and nonverbal communication skills in a variety of forms and contexts
- Listen effectively to decipher meaning, including knowledge, values, attitudes and intentions
- Use communication for a range of purposes (e.g. to inform, instruct, motivate and persuade)
- Utilize multiple media and technologies, and know how to judge their effectiveness a priori as well as assess their impact
- Communicate effectively in diverse environments (including multi-lingual)

Collaborate with Others

- Assume shared responsibility for collaborative work, and value the individual contributions made by each team member

## Information, Media, and Technology Skills

### Informational Literacy

Access and Evaluate Information

- Evaluate information critically and competently

Use and Manage Information

- Use information accurately and creatively for the issue or problem at hand

## Life and Career Skills

### Social and Cross-Cultural Skills

Interact Effectively with Others

- Know when it is appropriate to listen and when to speak

Work Effectively in Diverse Teams

- Respond open-mindedly to different ideas and values

Be Responsible to Others

- Act responsibly with the interests of the larger community in mind

**Technology Integration**

8.1 Educational Technology
All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and create and communicate knowledge.

**ELA Integration:**

SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one-on- one, in groups, and teacher-led) with peers on *grades 11–12 topics, texts, and issues,* building on others' ideas and expressing their own clearly and persuasively.

A. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well reasoned exchange of ideas.
B. Collaborate with peers to promote civil, democratic discussions and decision-making, set clear goals and assessments (e.g. student developed rubrics), and establish individual roles as needed.
C. Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives.
D. Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task. SL.11-12.2. Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

SL.11-12.3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used.

SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.

## Unit 1: Functions, Graphs and Applications (8 Days)

### Why Is This Unit Important?

In this unit, algebraic properties and strategies will be integrated with coordinate geometry to applications calling for the combined use in mathematical applications.

### Enduring Understandings:

- Use the idea of slope, relate to determining whether geometrical shapes are parallelograms, rhombuses, etc.
- Recognize when to use a specific linear function formula/equation when solving problems; apply the correct formula and determine why
- Find the line of best fit – explain and analyze it

### Skills:

- Determine if lines are parallel, perpendicular or neither
- Solving perpendicular bisectors

### Knowledge:

- Distance Formula
- Midpoint Formula
- Slope Formula

### Differentiation:

Enrichment:

- Temperatures

Supplement:

- Stacks of cups

### Assessments:

Formative Assessments:

- Assessment Checklist for Geometric Shape Identification
- Assessment Checklist for Best Line Fit
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks:

- Functions, Graphs, and Applications Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

**List of Applicable New Jersey Student Learning Standards (NJSLS) Covered in This Unit:**

- NJSLS.G-GPE.5-6
- NJSLS.MP.1-8

**Suggested Learning Experiences and Instructional Activities:**

- Plotting points in a plane
- Finding averages of 2 points
- Pythagorean Theorem

# Unit 2: Polynomial Functions (9 Days)

## Why Is This Unit Important?

In this unit, polynomial functions will be identified, evaluated and utilized in mathematical applications.

## Enduring Understandings:

- Evaluate polynomial functions using synthetic substitution and to determine its zeros
- Use synthetic division and apply the remainder and factor theorems
- Analyze a polynomial function for a given situation and to find the maximum or minimum value of the function
- Apply general theorems about polynomial equations

## Skills:

- Identify polynomial functions
- Graph a polynomial function and to determine an equation for a polynomial graph
- Use technology to approximate the real roots of a polynomial equation
- Solve polynomial equations by various methods

## Knowledge:

- Polynomial terms

## Differentiation:

Enrichment:

- Take rectangular paper and create box with max volume

Supplement:

- Graphical analysis

## Assessments:

Formative Assessments:

- Assessment Checklist for Maximum and Minimum Identification
- Assessment Checklist for Synthetic Substitution
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks:

- Polynomial Functions Assessment:

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

**List of Applicable New Jersey Student Learning Standards (NJSLS) Covered in This Unit:**

- NJSLS.A-APR.1-5
- NJSLS.F-IF.1-2
- NJSLS.MP.1-8

**Suggested Learning Experiences and Instructional Activities:**

- Sketch graphs
- Substitution

# Unit 3: Inequalities (6 Days)

## Why Is This Unit Important?

In real-life, solutions are sets of values encompassing a range of numbers rather than a set discrete value. In this unit, linear and polynomial inequalities will be identified, evaluated and utilized for mathematical applications where a range of values form the solution set.

## Enduring Understandings:

- Use linear programming to analyze real-world situations

## Skills:

- Solve and graph linear inequalities in one variable
- Solve and graph polynomial inequalities in one variable
- Graph polynomial inequalities in two variable and graph the solution set of a system of inequalities

## Knowledge:

- Corner-point principle

## Differentiation:

Enrichment:

- Geometric method of solving absolute-value equations
- Connecting linear programming to real-world problems

Supplement:

- Go over number line and terminology

## Assessments:

Formative Assessments:

- Assessment Checklist for Linear Inequalities
- Assessment Checklist for Polynomial Inequalities
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks:

- Inequalities Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

**List of Applicable CCSS and Standards/CPIs Covered in This Unit:**

- NJSLS.A-REI.1-5
- NJSLS.MP.1-8

**Suggested Learning Experiences and Instructional Activities:**

- Solving linear equation
- Sign change activity
- Linear programming

# Unit 4: Functions (9 Days)

## Why Is This Unit Important?

Linear, quadratic and polynomial functions have been previously studied and utilized. In this unit, functions in general will be considered. Functions can be combined to produce new functions. Simple changes in a function's rule will change its graph. The ideas developed in this unit will be utilized later in the course with exponential, logarithmic and trigonometric functions.

**Enduring Understandings**:

- Explore the relationships between the graph of a function and an algebraic rule for the function
- Form a function of one variable from a verbal description and, when appropriate, determine the minimum or maximum value of the function

**Skills**:

- Identify a function to determine the domain, range and zeros of a function, and graph a function
- Perform operations on functions and determine the domains of the resulting functions
- Reflect graphs and use symmetry to sketch graphs
- Determine periodicity and amplitude from graph, and perform transformations
- Find the inverse of a function, if the inverse exists
- Graph functions of two variable in a two-dimensional coordinate system and read such graphs

**Knowledge**:

- Operations on function formulas
- Periodic formula
- Amplitude formula
- Effects on graphs using various transformations.

**Differentiation:**

Enrichment:

- Proofs of functions
- Several composites
- Analyzing graphs of functions

Supplement:

- Graphing functions

**Assessments:**

Formative Assessments:

- Assessment Checklist for Function Identification
- Assessment Checklist for Function Formulation
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks

- Functions Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

**List of Applicable New Jersey Student Learning Standards (NJSLS) Covered in This Unit:**

- NJSLS.F-BF.1-5
- NJSLS.F-IF.1-9
- NJSLS.F-LE.1-3
- NJSLS.MP.1-8

**Suggested Learning Experiences and Instructional Activities:**

- Review of arithmetic rules
- Transforming graphs

# Unit 5: Exponents and Logarithms (9 Days)

## Why Is This Unit Important?

Mathematics is a powerful tool for representing, describing, interpreting and evaluating real world phenomena. Many such phenomena are not typically static or follow a constant linear growth or decline. Many of these situations found in the real world follow an exponential or logarithmic growth, decay or a cycle involving both. In this unit, students will learn strategies and techniques that will permit them to represent, describe, interpret and evaluate such situations.

**Enduring Understandings**:

- Investigate exponential growth and decay models
- Analyze the relationship between logarithmic form and exponential form
- Analyze real-world situations utilizing logarithms

**Skills**:

- Apply laws of exponents
- Apply laws of logarithms
- Solve exponential equations and change logarithms from one base to another

**Knowledge**:

- Laws of exponents
- Laws of logarithms
- Rule of 72

**Differentiation:**

Enrichment:

- Prove Rule of 72
- Save to buy a car assessment

Supplement:

- Save to buy a car activity

**Assessments:**

Formative Assessments:

- Assessment Checklist for Law of Exponents
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks:

- Exponents and Logarithms Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

**List of Applicable New Jersey Student Learning Standards (NJSLS) Covered in This Unit:**

- NJSLS.F-LE.4-5
- NJSLS.MP.1-8

**Suggested Learning Experiences and Instructional Activities:**

- Basic exponent problems
- Population growth

# Unit 6: Trigonometric Functions (9 Days)

## Why Is This Unit Important?

'Trigonometry' comes from two Greek words, *trigonon* and *metron* meaning 'triangle measurement'. The earliest use of trigonometry may have been for surveying land in ancient Egypt after the Nile River's annual flooding washed away property boundaries but, in this unit, more modern applications, such as the analysis of radio waves, will be investigated.

## Enduring Understandings:

- Evaluate graphs and equations of trigonometric functions
- Analyze real-world situations utilizing trigonometry while incorporating other areas of mathematics
- Obtain graphs of other trigonometric functions
- Determine domain, range, zeros and restrictions of trigonometric functions by analyzing graphs and equations

## Skills:

- Convert between degrees and radians
- Find arc length and areas of sectors of circles
- Apply trigonometric ratios
- Find values of trigonometric functions
- Solve simple trigonometric equations
- Find values of inverse trigonometric functions

## Knowledge:

- Trigonometric terms
- Arc length formulas
- Area of a sector of a circle formulas
- Trigonometric ratios

## Differentiation:

Enrichment:

- Graphing calculators
- Obtain 'other' trig functions from sine and cosine

Supplement:

- Bicycle Wheels

**Assessments:**

Formative Assessments:

- Assessment Checklist for Trigonometric Function Evaluation
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks:

- Trigonometric Functions Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

**List of Applicable New Jersey Student Learning Standards (NJSLS) Covered in This Unit:**

- NJSLS.F-TF.1-9
- NJSLS.MP.1-8

**Suggested Learning Experiences and Instructional Activities:**

- SOH CAH TOA
- Building special triangles from regular triangle/quadrilateral
- Construct helpful graphs (CAST)

## Unit 7: Trigonometric Equations and Applications (8 Days)

### Why Is This Unit Important?

In this unit, students will learn how to use fundamental trigonometric identities to evaluate functions, simplify expressions, develop additional identities and solve equations.

### Enduring Understandings:

- Apply periodic functions to a wide variety of fields
- Incorporate solutions to problems from analytic geometry
- Prove trigonometric identities
- Explore changes in the sine and cosine curves

### Skills:

- Solve simple trigonometric equations
- Find equations of different sine and cosine curves and apply these equations
- Use trigonometric functions to model periodic behavior
- Use technology to solve trigonometric equations

### Knowledge:

- Trigonometric Identities

### Differentiation:

Enrichment:

- Proving trig identities
- Daylight hours
- Relate Pythagorean Theorem with identities

Supplement:

- Drawing graphs
- Solving algebraic equations

### Assessments:

Formative Assessments:

- Assessment Checklist for Trigonometric Identities
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks:

- Trigonometric Equations and Applications Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

**List of Applicable New Jersey Student Learning Standards (NJSLS) Covered in This Unit:**

- NJSLS.F-TF.1-9
- NJSLS.MP.1-8

**Suggested Learning Experiences and Instructional Activities:**

- Solve algebraic equation
- Formulating trig graphs

# Unit 8: Triangle Trigonometry (9 Days)

## Why Is This Unit Important?

In this unit, the application of trigonometric properties and functions are focused on and applied to triangles.

**Enduring Understandings**:

- Utilize the laws of trigonometry to solve problems from navigation and surveying
- Explore new methods of solving polygons with trigonometric laws
- Investigate ambiguous cases of solving for triangle measurements

**Skills**:

- Use trigonometry to find unknown sides or angles of a right triangle
- Find the area of a triangle given the lengths of two sides and the measure of the included angle
- Use the law of sines and cosines to find unknown parts of a triangle

**Knowledge**:

- Law of Sines
- Law of Cosines
- Area Formula

**Differentiation:**

Enrichment:

- Solve navigation problems
- Discus Throw
- Proofs

Supplement:

- Draw triangles
- Formulate area formula
- Connecting to geometry

**Assessments:**

Formative Assessments:

- Assessment Checklist for Triangle Analysis
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks:

- Triangle Trigonometry Assessment

Alternative Assessments:

- Modified tasks and assessment rubrics
- Performance-based assessment tasks

**List of Applicable New Jersey Student Learning Standards (NJSLS) Covered in This Unit:**

- NJSLS.G-SRT.6-11
- NJSLS.MP.1-8

**Suggested Learning Experiences and Instructional Activities:**

- Find area without height of triangle
- Relate right triangles to real world problems (angle of descent for planes, angle of elevation and height, etc.)

## Unit 9: Trigonometric Addition Formulas (8 Days)

### Why Is This Unit Important?

In this unit, students will learn how to use trigonometric addition formulas to evaluate functions, simplify expressions, develop additional identities and solve equations.

### Enduring Understandings:

- Simplify expressions based on skills with identities
- Prove identities
- Visualize effects of having various transformations performed on trigonometric functions

### Skills:

- Apply the trigonometric formulas
- Use identities to solve trigonometric equations
- Find exact values for the sine, cosine and tangent of certain angles
- Find the angle measurement between two lines

### Knowledge:

- Trigonometric addition, subtraction, double-angle and half-angle formulas

### Differentiation:

Enrichment:

- Prove identities
- Simplify trig expressions

Supplement:

- Draw graphs

### Assessments:

Formative Assessments:

- Assessment Checklist for Trigonometric Addition
- Teacher's observation of students at work; anecdotal records
- Individual conferences and group discussions
- Students' recording sheets

Summative Assessments:

- Teacher's observation of students at work
- Individual conferences
- Chapter Assessments

Benchmarks:

- Trigonometric Addition Formulas Assessment

Alternative Assessments:

- **Modified tasks and assessment rubrics**
- **Performance-based assessment tasks**

**List of Applicable New Jersey Student Learning Standards (NJSLS) Covered in This Unit:**

- NJSLS.F-TF.1-9
- NJSLS.MP.1-8

**Suggested Learning Experiences and Instructional Activities:**

- Slope discussion relating to trigonometry
- Compare equations/formulas with graphing calculator

## Sample Standards Integration

**21st Century Skills & Career Readiness Practices**

**CRP4. Communicate clearly and effectively and with reason.**

For example, in Unit 8 students will justify their reasoning in their choice of solution pathways involving triangles scenarios

**CRP6. Demonstrate creativity and innovation.**

For example, in Unit 6 students will apply trigonometry to solve real world modeling problems.

**CRP7. Employ valid and reliable research strategies.**

For example, in Unit 1 students learn how to use best line fit to analyze and interpret data plots for real world scenarios.

**CRP8. Utilize critical thinking to make sense of problems and persevere in solving them.**

For example, in Unit 4 students will work to solve and understand function boundaries.

**CRP12. Work productively in teams while using cultural global competence.**

For example, in Unit 3 students will work in small teams to develop solution models for scenarios involving inequalities.

**8.1 Educational Technology**

**All students will use digital tools to access, manage, evaluate, and synthesize information in order to solve problems individually and collaborate and create and communicate knowledge.**

For example, in Unit 2 students will access, manage, evaluate, and synthesize information to develop models for approximating real roots of a polynomial equation.

**Interdisciplinary Connections**

**SL.11-12.1. Initiate and participate effectively in a range of collaborative discussions (one-on- one, in groups, and teacher-led) with peers on *grades 11–12 topics, texts, and issues,* building on others' ideas and expressing their own clearly and persuasively.**

A. Come to discussions prepared, having read and researched material under study; explicitly draw on that preparation by referring to evidence from texts and other research on the topic or issue to stimulate a thoughtful, well reasoned exchange of ideas.
B. Collaborate with peers to promote civil, democratic discussions and decision-making, set clear goals and assessments (e.g. student developed rubrics), and establish individual roles as needed.
C. Propel conversations by posing and responding to questions that probe reasoning and evidence; ensure a hearing for a full range of positions on a topic or issue; clarify, verify, or challenge ideas and conclusions; and promote divergent and creative perspectives.
D. Respond thoughtfully to diverse perspectives; synthesize comments, claims, and evidence made on all sides of an issue; resolve contradictions when possible; and determine what additional information or research is required to deepen the investigation or complete the task. SL.11-12.2. Integrate multiple sources of information presented in diverse formats and media (e.g., visually, quantitatively, orally) in order to make informed decisions and solve problems, evaluating the credibility and accuracy of each source and noting any discrepancies among the data.

**SL.11-12.3. Evaluate a speaker's point of view, reasoning, and use of evidence and rhetoric, assessing the stance, premises, links among ideas, word choice, points of emphasis, and tone used.**

**SL.11-12.4 Present information, findings and supporting evidence clearly, concisely, and logically. The content, organization, development, and style are appropriate to task, purpose, and audience.**

These standards are met throughout the course. For example, in Unit 5 students will discuss their solutions to a variety of real world scenarios justifying their argument involving growth and/or decay.